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Robust H_∞ Filtering for Neutral Systems with Multi-Delay and Application in the Flexible System

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***Abstract**: The residual vibrations in flexible structure system model can cause errors. In addition, the parameters in the system are also changed. For the problem of residual vibration, robust H_∞ filter is designed for neutral systems with multi-delay. Based on Lyapunov stability theory, the sufficient condition for the existence of filter is given. For the permitted uncertainty and multi-delay, the designed filter can guarantee the robust asymptotically stability and satisfy H_∞ performance index for the filtering error dynamic system. Finally, the designed filter is applied to the flexible system, and the result shows that the filter is effective.*

***Keywords**: Neutral systems, robust H_∞ filtering, linear matrix inequality, flexible system.*

1. Introduction

Flexible systems are widely used in various production activities, such as flexible joint robots and flexible spacecraft. In the process of using flexible structure, the cost and weight of the equipment can be reduced. But due to the inherently low damping characteristics of the flexible structure, coupled with the relatively difficult control of the flexible system and the existence of various uncertain factors, the system will result in the vibrations provoked by various reasons, so flexible systems are difficult to attenuate rapidly in a short period of time. Therefore, the research on the vibration control of flexible structures is significant, and more and more scholars have begun to work on this aspect [1-5]. In [2], for the problem of medium-low frequency mechanical resonance caused by mismatched inertia in flexible systems, the inertia matching range of different port acceleration feedbacks is discussed by means of acceleration compensation. The purpose of suppressing resonance is achieved by reasonably matching the motor and the load inertia. Reference [3] is based on a flexible robot designed multi-modal input shaper to reduce residual vibration while reducing the time-delay of system response.

Various uncertainties and time-delays are common in the system and become the main factors causing system instability. In the actual control system, many control system models can be transformed into neutral systems. At present, the research on this type of system has achieved rich results. The stability of neutral systems is discussed in reference [6], and the design method of robust H_∞ filters is given in [7]. While the case of adding nonlinear perturbations are discussed in the reference [8, 9]. However, in many practical control systems, not only a single time-delay factor, but also multi-delay factors are involved, and it is of great significance to fully consider the influence of time-delay factors on system control effects. In this paper, for the multi-delay neutral system, combined with the uncertainty of the state and the designed filter parameters, the nonlinear part is assumed to satisfy the Lipschitz condition, and the filter design algorithm is given by the linear matrix inequality. A flexible system is used as the research model and a robust H_∞ filter is applied to the residual vibration of the system. The application analysis shows the effectiveness of the filter.

2. Robust H_∞ filter design

Consider the following uncertain multi-delay neutral system:

$$(1)\qquad \begin{cases} \dot{x}(t) - C\dot{x}(t-d) = (A_0 + \Delta A_0)x(t) + \\ +\sum_{i=1}^{m} A_i x(t-h_i) + Dw(t) + Gf(x(t)), \\ y(t) = B_0 x(t) + D_1 w(t), \end{cases}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is the measurement output, $w(t) \in R^p$ is the external disturbances input which belongs to $L_2[0,\infty)$, $L_2[0,\infty)$ represents the square integrable vector function space, d and h_i are the constant time-delays, $i = 1, 2, \cdots, m.$ $G, D, D_1, A_0, B_0, A_i,$ $i = 1, 2, \cdots, m.$ are constant matrices with appropriate dimensions. The following filter is designed:

$$(2)\qquad \begin{cases} \dot{\hat{x}}(t) - C\dot{\hat{x}}(t-d) = A_0\hat{x}(t) + \sum_{i=1}^{m} A_i\hat{x}(t-h_i) + \\ +Gf(\hat{x}(t)) + (K + \Delta K)[y(t) - B_0\hat{x}(t)], \\ \hat{z}(t) = L_0\hat{x}(t), \end{cases}$$

where $\hat{x}(t) \in R^n$, $\hat{z}(t) \in R^q$, K is a matrix of coefficients to be determined, L_0 is a constant coefficient matrix with appropriate dimensions,

$$(3)\qquad \Delta A_0 = M_1 FN, \Delta K = M_2 FN,$$

where ΔA_0 and ΔK are uncertainties, M_1, M_2 and N are constant matrices with appropriate dimensions, F is an uncertain matrix and satisfies $F^{\mathrm{T}}F \le I.$

$f(x(t))$ is a nonlinear function and for any x_1, x_2 :

$$(4)\qquad \|f(x_1) - f(x_2)\| \le \|H(x_1 - x_2)\|,$$

where $f(0) = 0,$ H is a known weight matrix, $\|\cdot\|$ denotes the Euclidean norm.

Definition $x_e(t) = x(t) - \hat{x}(t),$ then we can get the corresponding filter error dynamic augmentation system:

$$\text{(5)}\quad \begin{cases} \dot{\eta}(t) - \tilde{C}\dot{\eta}(t-d) = (\tilde{A}_0 + \Delta\tilde{A}_0)\eta(t) + \\ +\sum_{i=1}^{m} \tilde{A}_i \eta(t-h_i) + (\tilde{D} + \Delta\tilde{D})w(t) + \tilde{G}\tilde{f}(\tilde{x}(t)), \\ \tilde{z}(t) = \tilde{L}\eta(t), \end{cases}$$

where

$$\text{(6)}\quad \eta(t) = \begin{bmatrix} x(t) \\ x_e(t) \end{bmatrix}, \ \tilde{z} = z(t) - \hat{z}(t),$$

$$\text{(7)}\quad \tilde{f}(\tilde{x}(t)) = \begin{bmatrix} f(x) \\ f(x) - f(\hat{x}) \end{bmatrix}, \ \left\| \tilde{f}(\tilde{x}(t)) \right\| \le \left\| H\eta(t) \right\|,$$

$$\text{(8)}\quad \Delta\tilde{A}_0 = \begin{bmatrix} \Delta A_0 & 0 \\ \Delta A_0 & -\Delta K B_0 \end{bmatrix}, \ \Delta\tilde{D} = \begin{bmatrix} 0 & 0 \\ -\Delta K D_1 & 0 \end{bmatrix},$$

$$\text{(9)}\quad \tilde{A}_0 = \begin{bmatrix} A_0 & 0 \\ 0 & A_0 - KB_0 \end{bmatrix}, \ \tilde{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & A_i \end{bmatrix}, \ \tilde{C} = \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix},$$

$$\text{(10)}\quad \tilde{D} = \begin{bmatrix} D & 0 \\ D - KD_1 & 0 \end{bmatrix}, \ \tilde{G} = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}, \ \tilde{L} = [0 \quad L_0],$$

where $z(t) = L_0 x(t)$.

The following lemma is cited from the reference [10].

Lemma 1. Given the matrix *Y*, *D* and *E* with appropriate dimension, where *Y* is symmetric, then

$$\text{(11)}\quad Y + DF(t)E + E^{\mathrm{T}}F^{\mathrm{T}}(t)D^{\mathrm{T}} < 0.$$

for all $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\text{(12)}\quad Y + \varepsilon DD^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E < 0.$$

The main conclusions are given below.

Theorem 1. Given the scalar $\gamma > 0$, if there exists positive definite symmetry matrices $P > 0$, $W > 0$, $Z_i > 0$, $i = 1, 2, \cdots, m$, for the filtering error system (5), the following inequality holds

$$\text{(13)}\quad \Sigma_1 = \begin{bmatrix} \breve{\Sigma}_{111} & \breve{\Sigma}_{112} & P\tilde{A}_1 & \cdots & P\tilde{A}_m & P(\tilde{D} + \Delta\tilde{D}) \\ * & \breve{\Sigma}_{122} & 0 & \cdots & 0 & 0 \\ * & * & -Z_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & -Z_m & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0,$$

where

$$\breve{\Sigma}_{111} = P(\tilde{A}_0 + \Delta\tilde{A}_0) + (\tilde{A}_0 + \Delta\tilde{A}_0)^{\mathrm{T}} P + \tilde{L}^{\mathrm{T}}\tilde{L} + P\tilde{G}H + H^{\mathrm{T}}\tilde{G}^{\mathrm{T}}P + W + \sum_{i=1}^{m} Z_i,$$

$$\breve{\Sigma}_{112} = \tilde{L}^{\mathrm{T}}\tilde{L}\tilde{C} + P(\tilde{A}_0 + \Delta\tilde{A}_0)\tilde{C} + P\tilde{G}H\tilde{C} + (W + \sum_{i=1}^{m} Z_i)\tilde{C},$$

$$\breve{\Sigma}_{122} = \tilde{L}^{\mathrm{T}}\tilde{L}\tilde{C} + P(\tilde{A}_0 + \Delta\tilde{A}_0)\tilde{C} + P\tilde{G}H\tilde{C} + (W + \sum_{i=1}^{m} Z_i)\tilde{C}\tilde{C}^{\mathrm{T}}(W + \sum_{i=1}^{m} Z_i)\tilde{C} - \\ - W + \tilde{C}^{\mathrm{T}}\tilde{L}^{\mathrm{T}}\tilde{L}\tilde{C}.$$

Then the filtering error system is asymptotically stable and can meet H_∞ performance.

Proof: Let $D(\eta_t) = \eta_t - \tilde{C}\eta(t-d)$, the following Lyapunov function is defined:

(14) $$V(t) = V_1(t) + V_2(t) + V_3(t),$$

where

$$V_1(t) = D^{\mathrm{T}}(\eta_t)PD(\eta_t),\ V_2(t) = \int_{t-d}^{t} \eta^{\mathrm{T}}(s)W\eta(s)\mathrm{d}s,\ V_3(t) = \sum_{i=1}^{m}\int_{t-h_i}^{t} \eta^{\mathrm{T}}(s)Z_i\eta(s)\mathrm{d}s,$$

then

(15) $$\begin{cases} \dot{V}_1(t) = 2D^{\mathrm{T}}(\eta_t)P[(\tilde{A}_0 + \Delta\tilde{A}_0)\eta(t) + \\ +\sum_{i=1}^{m}\tilde{A}_i\eta(t-h_i) + (\tilde{D} + \Delta\tilde{D})w(t) + \tilde{G}\tilde{f}(\tilde{x}(t))], \\ \dot{V}_2(t) = \eta^{\mathrm{T}}(t)W\eta(t) - \eta^{\mathrm{T}}(t-d)W\eta(t-d), \\ \dot{V}_3(t) = \sum_{i=1}^{m}\eta^{\mathrm{T}}(t)Z_i\eta(t) - \sum_{i=1}^{m}\eta^{\mathrm{T}}(t-h_i)Z_i\eta(t-h_i). \end{cases}$$

H_∞ performance:

(16) $$J(w) = \int_0^\infty \left[\tilde{z}^{\mathrm{T}}(t)\tilde{z}(t) - \gamma^2 w^{\mathrm{T}}(t)w(t)\right]\mathrm{d}t = \\ = \int_0^\infty \left[\tilde{z}^{\mathrm{T}}(t)\tilde{z}(t) - \gamma^2 w^{\mathrm{T}}(t)w(t) + \dot{V}(t)\right]\mathrm{d}t - \int_0^\infty \dot{V}(t)\mathrm{d}t.$$

Under the zero initial conditions, the following matrix inequality holds

(17) $$J(w) \le \int_0^\infty \left[\Omega^{\mathrm{T}}(t)\Sigma_1\Omega(t)\right]\mathrm{d}t,$$

where

$$\Omega(t) = \left[D(\eta_t) \quad \eta(t-d) \quad \eta(t-h_1) \quad \cdots \quad \eta(t-h_m) \quad w(t)\right]^{\mathrm{T}},$$

when $\Sigma_1 < 0,$ then $J(w) < 0.$ This completes the proof.

Theorem 2. Given the scalar $\gamma > 0,$ if there exists positive definite symmetry matrices $P_1 > 0,$ $P_2 > 0,$ $W_1 > 0,$ $W_2 > 0,$ $Z_{i1} > 0,$ $Z_{i2} > 0,$ $i = 1, 2, \cdots, m,$ and the positive scalars ε_1, ε_2, such that the following matrix inequality holds

$$
\begin{bmatrix}
X_1 & \cdots & X_2 & X_3 & X_4 \\
* & \cdots & \vdots & \vdots & \vdots \\
* & \cdots & X_5 & 0 & X_6 \\
* & \cdots & * & X_7 & 0 \\
* & \cdots & * & * & X_8
\end{bmatrix} < 0, \tag{18}
$$

where

$$
X_1 = \begin{bmatrix}
\Phi_{11} & 0 & \Phi_{13} & 0 & P_1 A_1 & 0 \\
0 & \Phi_{22} & 0 & \Phi_{24} & 0 & P_2 A_1 \\
\Phi_{13}^{\mathrm{T}} & 0 & \Phi_{33} & 0 & 0 & 0 \\
0 & \Phi_{24}^{\mathrm{T}} & 0 & \Phi_{44} & 0 & 0 \\
A_1^{\mathrm{T}} P_1 & 0 & 0 & 0 & -Z_{11} & 0 \\
0 & A_1^{\mathrm{T}} P_2 & 0 & 0 & 0 & -Z_{12}
\end{bmatrix},
$$

$$
X_5 = \mathrm{diag}\{-Z_{m1}, -Z_{m2}, -\gamma^2 I, -\gamma^2 I, -I, -I\},
$$

$$
X_7 = \mathrm{diag}\{-\varepsilon_1^{-1} I, -\varepsilon_1^{-1} I, -\varepsilon_2^{-1} I, -\varepsilon_2^{-1} I, -\varepsilon_1 I, -\varepsilon_1 I\},
$$

$$
X_8 = \mathrm{diag}\{-I, -I, -\varepsilon_1 I, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_2 I\},
$$

$$
\Phi_{11} = P_1 A_0 + A_0^{\mathrm{T}} P_1 + P_1 G H_1 + H_1^{\mathrm{T}} G^{\mathrm{T}} P_1 + W_1 + \sum_{i=1}^{m} Z_{i1},
$$

$$
\Phi_{13} = (P_1 A_0 + P_1 G H_1 + W_1 + \sum_{i=1}^{m} Z_{i1}) C + \varepsilon_1^{-1} N^{\mathrm{T}} N C,
$$

$$
\Phi_{22} = P_2 A_0 - Y B_0 + A_0^{\mathrm{T}} P_2 - B_0^{\mathrm{T}} Y^{\mathrm{T}} + P_2 G H_2 + W_2 +
$$

$$
+ H_2^{\mathrm{T}} G^{\mathrm{T}} P_2 + \sum_{i=1}^{m} Z_{i2}\,, \quad \Phi_{33} = C^{\mathrm{T}} (W_1 + \sum_{i=1}^{m} Z_i) C - W_1,
$$

$$
\Phi_{24} = (L_0^{\mathrm{T}} L_0 + P_2 A_0 - Y B_0 + P_2 G H_2 + W_2 + \sum_{i=1}^{m} Z_{i2}) C +
$$

$$
+ \varepsilon_1^{-1} B_0^{\mathrm{T}} N^{\mathrm{T}} N B_0 C\,, \quad \Phi_{44} = C^{\mathrm{T}} (W_2 + \sum_{i=1}^{m} Z_i) C - W_2,
$$

$$
X_2 = \begin{bmatrix}
P_1 A_m & 0 \\
0 & P_2 A_m \\
P_1 D & P_2 D_1 - Y D_1 \\
0 & 0 \\
0 & L_0^{\mathrm{T}} \\
0 & 0
\end{bmatrix}^{\mathrm{T}}, \quad
X_3 = \begin{bmatrix}
P_1 M_1 & P_2 M_1 \\
0 & -P_2 M_2 \\
0 & -P_2 M_2 \\
0 & 0 \\
N^{\mathrm{T}} & 0 \\
0 & B_0^{\mathrm{T}} N^{\mathrm{T}}
\end{bmatrix}^{\mathrm{T}},
$$

$$X_4=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C^{\mathrm{T}}N^{\mathrm{T}} & 0 & 0 & 0 \\ C^{\mathrm{T}}L_0^{\mathrm{T}} & 0 & 0 & C^{\mathrm{T}}B_0^{\mathrm{T}}N^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$X_6=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & D_1^{\mathrm{T}}N^{\mathrm{T}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then under the zero initial conditions, for any $w(t)\in L_2\left[0\quad\infty\right)$ and all uncertainties, the filtering error system (5) is asymptotically stable and satisfies $J(w)<0$ and the filter parameter is $K=P_2^{-1}Y$.

P r o o f: According to Formula (13), we can get

$$\text{(19)}\qquad \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & P\tilde{A}_1 & \cdots & P\tilde{A}_m & P\tilde{D} \\ * & \Gamma_{22} & 0 & \cdots & 0 & 0 \\ * & * & -Z_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & -Z_m & 0 \\ * & * & * & \cdots & * & -\gamma^2 I \end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & P\Delta\tilde{D} \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \Delta\tilde{D}^{\mathrm{T}}P & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}+$$

$$+\begin{bmatrix} P\Delta\tilde{A}_0+\Delta\tilde{A}_0^{\mathrm{T}}P & P\Delta\tilde{A}_0\tilde{C} & 0 & \cdots & 0 & 0 \\ \tilde{C}^{\mathrm{T}}\Delta\tilde{A}_0^{\mathrm{T}}P & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}<0,$$

where

$$\text{(20)}\qquad \Delta\tilde{A}_0=\begin{bmatrix} \Delta A_0 & 0 \\ \Delta A_0 & -\Delta KB_0 \end{bmatrix}=\begin{bmatrix} M_1 & 0 \\ M_1 & -M_2 \end{bmatrix}F(t)\begin{bmatrix} N & 0 \\ 0 & NB_0 \end{bmatrix}=\tilde{M}_1F(t)\tilde{N}_1,$$

$$\text{(21)}\qquad \Delta\tilde{D}=\begin{bmatrix} 0 & 0 \\ -\Delta KD_1 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ -M_2 & 0 \end{bmatrix}F(t)\begin{bmatrix} ND_1 & 0 \\ 0 & 0 \end{bmatrix}=\tilde{M}_2F(t)\tilde{N}_2.$$

Substituting Formula (20), Formula (21) into Formula (19), and applying Lemma1, then we can get

(22)
$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & P\tilde{A}_1 & \cdots & P\tilde{A}_m & P\tilde{D} \\ * & \Xi_{22} & 0 & \cdots & 0 & 0 \\ * & * & -Z_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & -Z_m & 0 \\ * & * & * & \cdots & * & \Xi \end{bmatrix} < 0,$$

where

$$\Xi_{11} = P\tilde{A}_0 + \tilde{A}_0^{\mathrm{T}}P + \tilde{L}^{\mathrm{T}}\tilde{L} + P\tilde{G}H + H^{\mathrm{T}}\tilde{G}^{\mathrm{T}}P + W +$$
$$+\sum_{i=1}^{m} Z_i + \varepsilon_1 P\tilde{M}_1\tilde{M}_1^{\mathrm{T}}P + \varepsilon_1^{-1}\tilde{N}_1^{\mathrm{T}}\tilde{N}_1 + \varepsilon_2 P\tilde{M}_2\tilde{M}_2^{\mathrm{T}}P,$$
$$\Xi_{12} = \tilde{L}^{\mathrm{T}}\tilde{L}\tilde{C} + P\tilde{A}_0\tilde{C} + P\tilde{G}H\tilde{C} + (W + \sum_{i=1}^{m} Z_i)\tilde{C} + \varepsilon_1^{-1}\tilde{N}_1^{\mathrm{T}}\tilde{N}_1\tilde{C},$$
$$\Xi_{22} = \tilde{C}^{\mathrm{T}}(W + \sum_{i=1}^{m} Z_i)\tilde{C} - W + \tilde{C}^{\mathrm{T}}\tilde{L}^{\mathrm{T}}\tilde{L}\tilde{C} + \varepsilon_1^{-1}\tilde{C}^{\mathrm{T}}\tilde{N}_1^{\mathrm{T}}\tilde{N}_1\tilde{C}.$$

According to Schur complement, Formula (22) is equivalent to next formula:

(23)
$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ * & \Psi_{22} & \Psi_{23} \\ * & * & \Psi_{33} \end{bmatrix} < 0,$$

where

$$\Psi_{11} = \begin{bmatrix} Q_{11} & Q_{12} & P\tilde{A}_1 & \cdots & P\tilde{A}_m \\ * & Q_{22} & 0 & \cdots & 0 \\ * & * & -Z_1 & \cdots & 0 \\ * & * & * & \ddots & \vdots \\ * & * & * & \cdots & -Z_m \end{bmatrix},$$

$$\Psi_{12} = \begin{bmatrix} P\tilde{D} & \tilde{L}^{\mathrm{T}} & P\tilde{M}_1 & P\tilde{M}_2 & \tilde{N}_1^{\mathrm{T}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{13} = \begin{bmatrix} 0 & 0 & 0 \\ \tilde{C}^{\mathrm{T}}\tilde{L}^{\mathrm{T}} & \tilde{C}^{\mathrm{T}}\tilde{N}_1^{\mathrm{T}} & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, \ \Psi_{23} = \begin{bmatrix} 0 & 0 & \tilde{N}_2^{\mathrm{T}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{22} = \mathrm{diag}\left\{-\gamma^2 I \quad -I \quad -\varepsilon_1^{-1}I \quad -\varepsilon_2^{-1}I \quad -\varepsilon_1 I\right\},$$

$$\Psi_{33} = \mathrm{diag}\{-I \quad -\varepsilon_1 I \quad -\varepsilon_2 I\},\ Q_{22} = \tilde{C}^{\mathrm{T}}(W + \sum_{i=1}^{m} Z_i)\tilde{C} - W,$$

$$Q_{11} = P\tilde{A}_0 + \tilde{A}_0^{\mathrm{T}} P + P\tilde{G}H + H^{\mathrm{T}}\tilde{G}^{\mathrm{T}} P + W + \sum_{i=1}^{m} Z_i,$$

$$Q_{12} = (\tilde{L}^{\mathrm{T}}\tilde{L} + P\tilde{A}_0 + P\tilde{G}H + W + \sum_{i=1}^{m} Z_i + \varepsilon_1^{-1}\tilde{N}_1^{\mathrm{T}}\tilde{N}_1)\tilde{C}.$$

Let

(24) $$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix},\ W = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix},\ Z_i = \begin{bmatrix} Z_{i1} & 0 \\ 0 & Z_{i2} \end{bmatrix},\ H = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}.$$

Putting the Formula (9), Formula (10) and formula (24) into Formula (23), and letting $Y = P_2 K$, then the theorem is proved.

3. Application of robust H_∞ filtering in flexible systems

3.1. The flexible system and its model

The flexible system is the control object in this paper, as shown in Fig. 1. The flexible system is mainly composed of three parts.

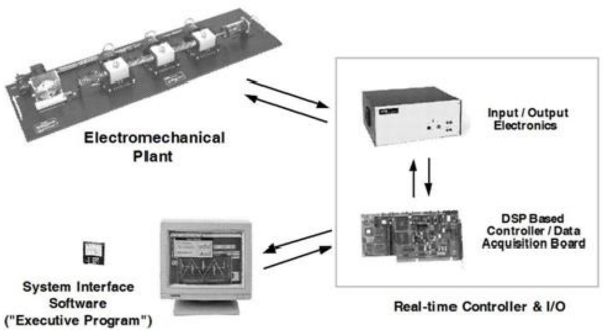

Fig. 1. The flexible system

The first part is the electromechanical device, including the spring mass module, the sensor behind the mass module, the damper with variable damping coefficient, the DC brushless servo motor with superior performance and the high-resolution incremental rotary encoder behind the motor. System configuration can be constructed according to the needs of actual operation. The second part is the real-time control box, which includes the DSP, servo and actuator interface circuits, servo amplifiers and auxiliary power supplies that enable real-time control. The third part is the self-contained software, providing a user interface, which can selectively add different input signals to the system. Users can write their own programs and implement control algorithms. The experimental equipment can also control the system in real time through Simulink modeling [11]. The system provides a configuration module corresponding to the experimental device, which can feed back the position information of the encoder in the actual system to the Simulink module, and then the feedback information is processed by the designed controller to realize real-time control of the experimental equipment.

In the case that the friction force is ignored, the configuration of the flexible system with 1 degree of freedom is considered. According to the reference [11], the linear equation of the flexible system can be obtained as

$$\ddot{x}_1 = \frac{-k_1}{m_1}x_1 + \frac{-c}{m_1}\dot{x}_1 + \frac{F}{m_1}.$$

This formula can be written as

$$\begin{bmatrix}\dot{x}_1\\ \ddot{x}_1\end{bmatrix} = \begin{bmatrix}0 & 1\\ \frac{-k_1}{m_1} & \frac{-c}{m_1}\end{bmatrix}\begin{bmatrix}x_1\\ \dot{x}_1\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m_1}\end{bmatrix}F, \tag{25}$$

where the device parameters are: $m_1 = 2.77$ kg, $k_1 = 200$ N/m, $c = 10.2$ N/m/s. Let $w = F$, Then the equation form of the flexible system can be transformed into the following equation of state:

$$\begin{cases}\dot{x}(t) = A_0x(t) + Dw(t),\\ y(t) = B_0x(t) + D_1w(t),\end{cases} \tag{26}$$

where the parameters of the equation of state are as follows:

$$A_0 = \begin{bmatrix}0 & 1\\ -72.2 & -3.68\end{bmatrix},\ D = \begin{bmatrix}0 & 4600\end{bmatrix},\ B_0 = \begin{bmatrix}1 & 0\end{bmatrix},$$

where D_1 is the zero matrix. Performance evaluation signal is as follows:

$$z(t) = L_0x(t),$$

where

$$L_0 = \begin{bmatrix}-0.5 & 0\\ 0 & -0.3\end{bmatrix}.$$

3.2. Application analysis

The experimental equipment of the flexible system is shown in Fig. 1. The disturbances of simulate external uncertainties under stable operation is added. Other parameters of the system (1) are as follows:

$$A_0 = \begin{bmatrix}0 & 1\\ -72.2 & -3.68\end{bmatrix},\ A_1 = \begin{bmatrix}-0.5 & -0.1\\ -0.2 & 0.4\end{bmatrix},\ G = \begin{bmatrix}0.3 & 0.4\\ 0.2 & 0.5\end{bmatrix},\ B_0 = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},$$

$$L_0 = \begin{bmatrix}-0.5 & 0\\ 0 & -0.3\end{bmatrix},\ M_1 = \begin{bmatrix}0.4 & -0.5\\ -0.1 & 0.3\end{bmatrix},\ M_2 = \begin{bmatrix}0.1 & -0.5\\ 0.1 & -0.3\end{bmatrix},\ N = \begin{bmatrix}0.1 & -0.3\\ 0.5 & 0.4\end{bmatrix},$$

$$H_1 = \begin{bmatrix}0.2 & 0.5\\ 0.1 & 0.4\end{bmatrix},\ H_2 = \begin{bmatrix}0.3 & 0.5\\ 0.2 & 0.1\end{bmatrix},\ C = \begin{bmatrix}0.01 & 0\end{bmatrix},\ D = \begin{bmatrix}0 & 4600\end{bmatrix},\ d = 0.5,\ h = 0.8.$$

The feasible solution is obtained by using MATLAB:

$$P_1 = \begin{bmatrix} 0.2809 & 0.0095 \\ 0.0095 & 0.0053 \end{bmatrix},\ P_2 = \begin{bmatrix} 0.4615 & -0.0418 \\ -0.0418 & 0.0160 \end{bmatrix},\ W_1 = \begin{bmatrix} 0.1532 & 0.0068 \\ 0.0068 & 0.0031 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 6.6628 & 0.3473 \\ 0.3473 & 0.0390 \end{bmatrix},\ Z_1 = \begin{bmatrix} 0.2195 & 0.0177 \\ 0.0177 & 0.0059 \end{bmatrix},\ Z_2 = \begin{bmatrix} 6.6947 & 0.3652 \\ 0.3652 & 0.0474 \end{bmatrix},$$

$$Y = \begin{bmatrix} 26.5700 & -39.9331 \\ 0.2513 & -0.5494 \end{bmatrix}.$$

Further, the parameters of the filter can be calculated as follows:

$$K = P_2^{-1}Y = \begin{bmatrix} 77.3691 & -117.5585 \\ 218.4376 & -342.3983 \end{bmatrix}.$$

The random signal is taken as disturbance input signal, and the nonlinear function is taken as $0.7\sin w(t)$. The simulation curves is as in Figs 2 and 3.

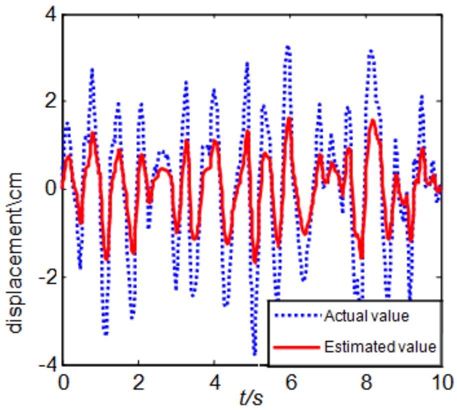

Fig. 2. Actual and estimated value of displacement curves

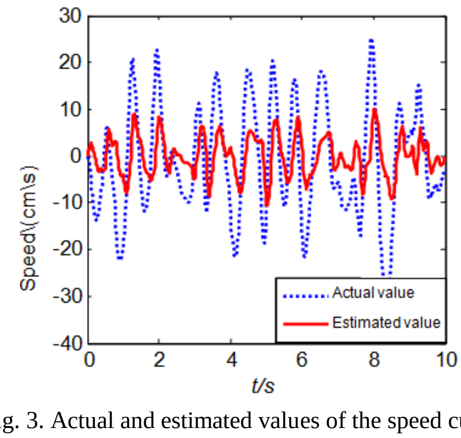

Fig. 3. Actual and estimated values of the speed curve

Fig. 2 is a displacement curve of the mass movement of the flexible system without adding the filter and adding the filter. Fig. 3 is the velocity curve of mass movement without adding filter and adding filter. It can be seen from the curve that the state of the system is greatly influenced by the vibration interference before the filter is added. After adding the filter, the vibration interference is largely reduced. It shows that the designed filter has obvious effect and is feasible.

4. Conclusion

In this paper, the robust $H\infty$ filter is designed for neutral multi-delay systems, and the sufficient conditions for the existence of the filter are given. For the vibration problem of flexible system, the robust $H\infty$ filter is applied to show that the filter has a certain inhibitory on the bounded disturbance input. It shows that the filter design algorithm studied in this paper has certain theoretical significance and practical application value.

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