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Dichotomized Incenter Fuzzy Triangular Ranking Approach to Optimize Interval Data Based Transportation Problem

Pankaj Kumar Srivastava, Dinesh C. S. Bisht

Department of Mathematics, Jaypee Institute of Information Technology, Noida, India
E-mail: drbisht.math@gmail.com

***Abstract*: *This research article discusses the problems having flexible demand, supply and cost in range referred as interval data based transportation problems and these cannot be solved directly using available methods. The uncertainty associated with these types of problems motivates authors to tackle it by converting interval to fuzzy numbers. This confront of conversion has been achieved by proposing a dichotomic fuzzification approach followed by a unique triangular incenter ranking approach to optimize interval data based transportation problems. A comparison with existing methods is made with the help of numerical illustrations. The algorithm proposed is found prompt in terms of the number of iteration involved and problem formation. This method is practical to handle the transportation problems not having a single valued data, but data in form of a range.*

***Keywords*: *Triangular fuzzy number, transportation problem, interval data based transportation problem, dichotomic, incenter.*

1. Introduction

Optimized transportation of manufactured goods is crucial for industries. Quantities of products to be supplied are also a major factor. When demand, supply and cost are in exact figures there are several methods to optimize the problem, but it is not always possible to quote an exact demand, supply and cost. In that case data may be given in a particular range. In the competitive world of today everyone wants to use best out of available resources. Transportation models play an important role to optimize these available resources [1]. The concept of transportation problem was introduced by Hitchcock in the year of 1941 [2]. In classical transportation problems supply, demand and cost to transport are fixed numbers, while in real life problems these parameters may be fuzzy quantities. If the supply and demand of goods are fuzzy in nature then this type of transportation problem referred to as Fuzzy Transportation Problem (FTP). This raises a need of new transportation model [3-5]. B e l l m a n and Z a d e h [6] introduced a new method on decision making in fuzzy environment, further used by C h a n a s, K o\l o d z i e j c z y k and M a c h a j [7] to solve Fuzzy Transportation Problems (FTPs). C h a n a s and K u c h t a [8] successfully solved fuzzy transportation with integer value problem. Extension principle has also been introduced for solution of fuzzy transportation problems [9]. The transportation cost is minimized for demands and supplies using trapezoidal fuzzy numbers by Gani and Razzak [10] and Dinagar, Palanivel [11]. The separation method based on the zero point method provides an optimal value of the objective function for the fully interval transportation problem [12, 13]. Unbalanced FTPs are also solved by representing all values as LR flat fuzzy numbers [14]. Chandran and Kandaswamy [15] proposed algorithm without converting the problem into a crisp transportation problem. For defuzzification Sudhagar score method is applied. Ebrahimnejad proposed a two-step method to solve such problems. The first step involves conversion of FTP to a linear programming having fuzzy costs with crisp constraints. The second step is application of a new decomposition technique to transform it into crisp bounded problem [16]. Bisht and Srivastava [17] introduced a unique triangular to trapezoidal fuzzy number conversion approach with a new ranking technique to solve FTP. Zheng and Ling [18] developed a cooperative optimization method to solve fuzzy optimization problem of emergency transportation planning requires in disaster relief supply chains. Thitipong Jamrus et al. [19] considered triangular fuzzy demands to minimize the total cost which also includes transportation costs.

In all methods mentioned above the given data takes the form of fuzzy numbers like triangular, trapezoidal fuzzy numbers, etc., but in real life the given data may be in intervals like the demand from any factory may be in range from *a* to *b*, i.e., [*a*, *b*], referred to as Interval data Based Transportation Problems (IBTPs). Such IBTP cannot be solved directly using available methods. In the present article a unique algorithm to solve IBTP is proposed. This is further justified by solving two real life examples. Results are also compared with existing trisectional trapezoidal method [20].

2. Interval based transportation problem

In our day to day life many decisions are not certain and are described with the help of linguistic terms. An attempt was made by Prof. L. A. Zadeh [21] to represent these vague or uncertain variables in terms of sets, this gave birth to concept of Fuzzy sets. In real world many problems are vague in nature where goals and constraints are not known. Thereafter different fuzzy approaches are applied to various field of optimization [22-26]. In transportation problem model such type of uncertain data may arise. For example the production of any company may vary between *a* to *b*. In this case classical methods cannot be applied due to their limitations to handle only crisp data. Let us take an example in which a company is not sure about its exact production in next year but it knows that it will lie in a particular range. Such type of transportation problems is given a name IBTP.

To differentiate between Fuzzy Transportation Problems (FTPs) and IBTPs two examples are described as follows:

Example 1 (Fuzzy transportation problems). Goods are to be shipped from three storehouses P_1, P_2 and P_3 to three different destinations L_1, L_2 and L_3. The availabilities at storehouses are $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) respectively. While the demands at destinations are $(\alpha_1, \beta_1, \gamma_1), (\alpha_2, \beta_2, \gamma_2)$ and $(\alpha_3, \beta_3, \gamma_3)$ respectively, and (u_{ij}, v_{ij}, w_{ij}) represents the cost of moving goods from P_i to L_j.

Example 2 (Interval based transportation problems). Goods are to be shipped from three storehouses P_1, P_2 and P_3 to three different destinations L_1, L_2 and L_3. The availabilities at storehouses are in the range $[p_1 - p'_1]$, $[p_2 - p'_2]$ and $[p_3 - p'_3]$, respectively, while the demands at destinations are in the range $[l_1 - l'_1]$, $[l_2 - l'_2]$ and $[l_3 - l'_3]$, respectively. The costs of moving goods from P_i to L_j are also given in interval $[k_{ij} - k'_{ij}]$.

The mathematical form of IBTP is as follows:

(1) $$\text{Minimize } W = \sum_{i=1}^{3}\sum_{j=1}^{3} k_{ij} x_{ij},$$

subject to

(2) $$\sum_{j=1}^{3} x_{ij} = [p_i - p'_i],\ i = 1, 2, 3,$$

(3) $$\sum_{i=1}^{3} x_{ij} = [l_j - l'_j],\ j = 1, 2, 3,$$

(4) $$x_{ij} \geq 0 \text{ for all } i \text{ and } j.$$

3. Proposed dichotomized fuzzification technique

In IBTP supply, demand and cost are in the form of interval. The technique proposed is based on dichotomization of given interval data, in which interval data are fuzzified to triangular fuzzy numbers. For example consider the supply from the source P_i in the interval form as $[p_i - p_i']$. This interval is fuzzified to a triangular fuzzy number (p_i, p_i*, p_i'), where $p_i* = \dfrac{p_i' + p_i}{2}$. Similarly demand $[l_j - l_j']$ of any destination L_j is fuzzified to a triangular fuzzy number (l_j, l_j*, l_j'), where $l_j* = \dfrac{l_j' + l_j}{2}$. Cost $[k_{ij} - k_{ij}']$ in transporting goods from source P_i to destination L_j converted to $(k_{ij}, k_{ij}*, k_{ij}')$ after fuzzification, where $k_{ij}* = \dfrac{k_{ij}' + k_{ij}}{2}$.

4. Proposed triangular incenter ranking technique

This study proposes a modified form of the ranking technique used by Bisht and Srivastava [20] and Akyar, E., H. Akyar, and S. Düzce [27]. This method is based on intersection point of angular bisector called incenter of a triangle, where the abscissas of vertices represent the elements of triangular fuzzy number. The ordinate represents their membership grades [28-31].

The abscissa of intersection point of the angular bisectors of a triangle PQR is given in Fig. 1 as

(6) $$X = \frac{aq + bp + cr}{a + b + c},$$

where $a = r - p$, $b = \sqrt{1 + (r - q)^2}$, $c = \sqrt{1 + (q - p)^2}$, and the ordinate of intersection point of the angular bisectors of a triangle PQR is given as

(7) $$Y = \frac{a}{a + b + c}.$$

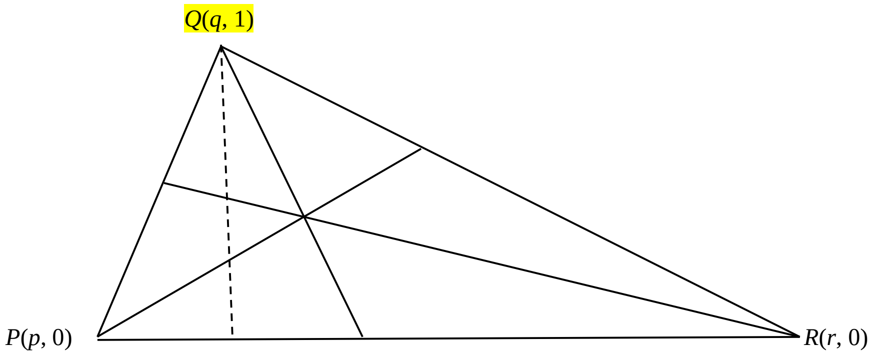

Fig. 1. Angular bisectors of a triangle PQR

Let $P = (p, q, r)$ be a normal triangular fuzzy number then ranking of P is defined as

(8) $$R(P) = X - 2Y.$$

5. Proposed method

The step by step mathematical formulation of proposed method is described below.

Step 1. Conversion of IBTP in tabular form as in Table 1.

Table 1. Tabular form of IBTP

Storehouse	L_1	L_2	L_3	Supply
P_1	$[k_{11} - k_{11}']$	$[k_{12} - k_{12}']$	$[k_{13} - k_{13}']$	$[p_1 - p_1']$
P_2	$[k_{21} - k_{21}']$	$[k_{22} - k_{22}']$	$[k_{23} - k_{23}']$	$[p_2 - p_2']$
P_3	$[k_{31} - k_{31}']$	$[k_{32} - k_{32}']$	$[k_{33} - k_{33}']$	$[p_3 - p_3']$
Demand	$[l_1 - l_1']$	$[l_2 - l_2']$	$[l_3 - l_3']$	

Step 2. Fuzzification of given data using dichotomic approach as discussed in Section 3.

Step 3. Formulation as linear programming problem using incenter ranking technique described in Section 4.

Step 4. Check the LPP for balance. If $\sum p_i = \sum l_j$, then problem is said to be balanced and proceed to Step 5. If not then dummy rows or dummy columns are introduced to make it balanced.

Step 5. Apply least cost method to find initial basic feasible solution [4]. Optimality test is to be performed through modified distribution technique.

6. Numerical problems

Problem 1. A company has three sources or deliverers A_1, A_2, A_3 with supply [1-9]; [4-10]; [4-11], respectively and it has three receivers R_1, R_2, R_3 with demand values [3-12]; [4-10]; [2-8], respectively. The cost in transporting goods from source A_i to receiver R_j is given in Table 2.

Table 2. Transportation cost for Problem 1

Deliverer	R_1	R_2	R_3
A_1	[1-19]	[1-9]	[2-18]
A_2	[8-26]	[3-12]	[7-28]
A_3	[11-27]	[0-15]	[4-11]

The solution is in fife steps.

Step 1. Conversion in tabular form.

Table 3. Tabular form of Problem 1

Deliverer	R_1	R_2	R_3	Supply
A_1	[1-19]	[1-9]	[2-18]	[1-9]
A_2	[8-26]	[3-12]	[7-28]	[4-10]
A_3	[11-27]	[0-15]	[4-11]	[4-11]
Demand	[3-12]	[4-10]	[2-8]	

Step 2. Fuzzification of given data using method proposed in Section 3.

Table 4. Fuzzified Interval data for Problem 1

Deliverer	R_1	R_2	R_3	Supply
A_1	(1, 10, 19)	(1, 5, 9)	(2, 10, 18)	(1, 5, 9)
A_2	(8, 17, 26)	(3, 7.5, 12)	(7, 17.5, 28)	(4, 7, 10)
A_3	(11, 19, 27)	(0, 7.5, 15)	(4, 7.5, 11)	(4, 7.5, 11)
Demand	(3, 7.5, 12)	(4, 7, 10)	(2, 5, 8)	

Step 3. Formulation as linear programming problem using ranking technique stated in Section 4.

Table 5. Defuzzified data for Problem 1

Deliverer	R_1	R_2	R_3	Supply
A_1	9.25	4.26	9.25	4.26
A_2	16.25	6.76	16.75	6.27
A_3	18.25	6.75	6.76	6.76
Demand	6.76	6.27	4.26	

Step 4. Here the demand and supply are equal so it's a balanced transportation problem.

Step 5. The initial basic feasible solution using least cost method is

$$x_{12} = 4.26; x_{21} = 6.27, x_{31} = 0.49, x_{32} = 2.01 x_{33} = 4.26.$$

The transportation cost is $Z = 171.34$.

Applying MODI method in the IBFS solution obtained, basic variables become

$$x_{11} = 0.49,\ x_{12} = 3.77,\ x_{21} = 6.27,\ x_{32} = 2.50,\ x_{33} = 4.26.$$

The total transportation cost $Z = 168.15$.

So the transportation cost for this interval based transportation problem will be optimum if the receivers R_1 receives product from suppliers A_1 and A_2, R_2 receives product from suppliers A_1 and A_3 and R_3 receives product from supplier A_3 only (Table 10).

Problem 2. A company has factories at three different locations L_1, L_2 and L_3 with production quantities [1-12]; [0-3] and [5-15.6] tons per month respectively and company has four warehouses H_1, H_2, H_3 and H_4 with demand [5-10]; [1-10]; [1-6] and [1-4]. The cost in transporting goods from location L_i to warehouse H_j is given in Table 6.

Table 6. Transportation cost for Problem 2

Location	H_1	H_2	H_3	H_4
L_1	[1,4]	[1-6]	[4-12]	[5-11]
L_2	[0-4]	[1-4]	[5-8]	[0-3]
L_3	[3-8]	[5-12]	[12-19]	[7-12]

The solution is in fife steps.

Step 1. Conversion in tabular form.

Table 7. Tabular form of Problem 2

Location	H_1	H_2	H_3	H_4	Supply
L_1	[1-4]	[1-6]	[4-12]	[5-11]	[1-12]
L_2	[0-4]	[1-4]	[5-8]	[0-3]	[0-3]
L_3	[3-8]	[5-12]	[12-19]	[7-12	[5-15.6]
Demand	[5-10]	[1-10]	[1-6]	[1-4]	

Step 2. Fuzzification of given data.

Table 8. Fuzzified data for Problem 2

Location	H_1	H_2	H_3	H_4	Supply
L_1	(1, 2.5, 4)	(1, 3.5, 6)	(4, 8, 12)	(5, 8, 11)	(1, 6.5, 12)
L_2	(0, 2, 4)	(1, 2.5, 4)	(5, 6.5, 8)	(0, 1.5, 3)	(0, 1.5, 3)
L_3	(3, 5.5, 8)	(5, 8.5, 12)	(12, 15.5, 19)	(7, 9.5, 12)	(5, 10.3, 15.6)
Demand	(5, 7.5, 10)	(1, 5.5, 10)	(1, 3.5, 6)	(1, 2.5, 4)	

Step 3. Formulation as Linear Programming Problem (LPP) using ranking technique.

Table 9. Defuzzified data for Problem 2

Location	H_1	H_2	H_3	H_4	Supply
L_1	1.82	2.78	7.26	7.27	5.76
L_2	1.29	1.82	5.82	0.82	0.82
L_3	4.78	7.76	14.76	8.78	9.56
Demand	6.78	4.76	2.78	1.82	

Step 4. Check for balance.

Here the demand and supply are equal so it's a balanced transportation problem.

Step 5. Application of Least Cost method to solve LPP.

The IBFS using least cost method is

$$x_{11} = 5.76,\ x_{24} = 0.82,\ x_{31} = 1.02,\ x_{32} = 4.76,\ x_{33} = 2.78,\ x_{34} = 1.$$

The total transportation cost is $Z = 102.78$.

Applying MODI method in the IBFS solution obtained by least cost method, the solution is

$$x_{12} = 2.98,\ x_{13} = 2.78,\ x_{24} = 0.82,\ x_{31} = 6.78,\ x_{32} = 1.78,\ x_{34} = 1.$$

The total transportation cost is $Z = 84.14$.

7. Comparative study

The Problems 1 and 2 under consideration are compared with solution obtained by trisectional fuzzy trapezoidal approach [20]. The computational cost in the proposed method is less as compared to trisectional approach not only in fuzzification and ranking but also in number of iterations involved to obtain optimal solution as depicted in Table 11 and Table 13. In the algorithm proposed transportation problem obtained is balanced after ranking for both problems, which makes it computationally faster and easier.

Table 10. Comparative table for Problem 1

A_i	R_1		R_2		R_3	
	Trisectional approach[20]	Proposed approach	Trisectional approach[20]	Proposed approach	Trisectional approach[20]	Proposed approach
A_1	√	√	X	√	X	X
A_2	√	√	√	X	X	X
A_3	X	X	√	√	√	√

Table 11. Comparison of iterations

Method	Initial basic feasible solution	Optimum solution	Post ranking balance status	No of iterations required
Trisectional approach [20]	173.20	154.93	No	4
Proposed approach	171.34	168.15	Yes	2

Table 12. Comparative table for Problem 2

L_i	H_1		H_2		H_3		H_4	
	Trisectional approach [20]	Proposed approach	Trisectional approach [20]	Proposed approach	Trisectional approach [20]	Proposed approach	Trisectional approach [20]	Proposed approach
L_1	X	X	√	√	√	√	X	X
L_2	X	X	X	X	X	X	√	√
L_3	√	√	√	√	X	X	√	√

Table 13. Comparison of iterations

Method	Initial basic feasible solution	Optimum solution	Post ranking balance status	No of iterations required
Trisectional approach[20]	119.69	93.39	No	4
Proposed approach	102.78	84.14	Yes	3

The transportation cost for this interval based transportation problem will be optimum if the warehouses H_1 receives product from suppliers L_3, H_2 receives product from suppliers L_1 and L_3, H_3 receives product from suppliers L_1 and H_3 receives product from suppliers L_2 and L_3 (Table 12).

8. Conclusion

Fuzzy transportation problems are discussed earlier in form of fuzzy numbers like triangular, trapezoidal etc. The previously proposed methods were applicable if data is given in a specific form or fuzzy numbers form. But present method can be applied even if data is vague or given in terms of a range. The present paper deals with interval data based transportation problem. The proposed ranking technique is based on triangular fuzzy model and the concept of incentre of a triangle. This research article also describes a unique fuzzification approach. This is an obvious but justified effort by considering a unique problem and given step by step methodology. The approach presented here consumes less time as this method is faster in terms of rate of convergence. The approach discussed here is long term beneficial in terms of a lump sum data. The concept used here will definitely open doors for future researchers.

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