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TOPSIS Modification with Interval Type-2 Fuzzy Numbers

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Abstract: *This paper proposes a new TOPSIS with interval type-2 fuzzy numbers. The extension applies graded mean integration to compare normal fuzzy trapezoidal sets. It is demonstrated by a numerical example for ranking business intelligence software that the TOPSIS modification can operate with qualitative and quantitative criteria, reduces evaluation uncertainty and provides a feasible solution.*

Keywords: *Multi-criteria decision making, TOPSIS, interval type-2 fuzzy sets, business intelligence.*

1. Introduction

Ranking alternatives based on experts' evaluations is an example of Multiple-Criteria Decision Making (MCDM) task. There are many diverse solutions to problems from this class in literature, some of which employ fuzzy numbers [1, 11, 16-20]. Nowadays, extended methods such as fuzzy AHP, fuzzy VIKOR, fuzzy TOPSIS, etc. are also developed and applied. Until recently, the subject of study were type-1 fuzzy numbers, however, currently growing in popularity are Multi-Attribute Decision Making (MADM) methods with sophisticated fuzzy numbers – interval-valued, intuitionistic etc. A comprehensive survey of fuzzy method applications is presented in [3, 4, 14].

The purpose of this work is developing and applying an interval type-2 (IT2) Fuzzy Sets (FSs) modification of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. The rest of the paper is organized as follows: The contemporary TOPSIS modifications are introduced in Section 2. Section 3 defines the basic operations for working with IT2 FSs and demonstrates the implementation steps used in fuzzy TOPSIS. Section 4 describes the peculiarities in this new TOPSIS modification. Section 5 presents a numerical example illustrating the application of the proposed new TOPSIS variation in Business Intelligence (BI) software selection. Finally, the results are compared with those obtained when applying an existing extension of fuzzy TOPSIS.

2. Short literature review

The subject of research is a form of TOPSIS which assesses alternatives and weighted coefficients, represented by a special type of fuzzy numbers. This method was preferred due to its algorithm simplicity and its intuitiveness – it does not require a lot of computations and the resulting ranking can be easily explained. In recent years, the extended TOPSIS methods underlying interval-valued fuzzy data are the focus of substantial research [5, 12]. The solutions of Chen [5] and Jahanshahloo et al. [12] are similar – extending the TOPSIS method with type-1 triangular numbers for decision making in a fuzzy environment.

Chu and Lin [7] are the first to convert weighted normalized decision matrix to crisp values by defuzzifying. In such a way, a fuzzy MCDM problem was changed into a crisp one. Ashtiani et al. [2] presented an interval-valued fuzzy TOPSIS method for solving MCDA problems with triangular numbers and normalized Euclidean distance. Chen and Lee [6] presented an interval type-2 fuzzy TOPSIS method for handling fuzzy multi-attribute group decision-making problems. It is based on defuzzification of interval type-2 fuzzy sets using concept of ranking values. Dymova, Sevastjanov and Tikhonenko [8] propose a new method for fuzzy MCDM based on the α-cuts of IT2 fuzzy values to avoid the restrictions concerned with the different shapes.

The modifications of fuzzy TOPSIS listed in the section apply two basic ranking techniques: defuzzification and comparing preference relations. Each of these techniques has its advantages and disadvantages. Defuzzification is simpler and easier than fuzzy pair-wise comparison on ranking fuzzy numbers. However, defuzzification loses uncertainty of messages. On the other hand, fuzzy pair-wise comparison is complex and difficult, but it preserves fuzziness in messages [17, 18]. The current work proposes a defuzzification based modification of IT2 FSs TOPSIS. The core of the proposed MCDM technique is the concept of selecting the solution with the shortest distance from the positive-ideal solution and the greatest distance from the negative-ideal solution by considering concepts of IT2 FSs.

3. Basic concepts

3.1. Interval type-2 fuzzy sets

The disadvantages of classic fuzzy sets such as difficulties in aggregation of alternative's evaluations, working with noisy data and natural language estimates, catalyzed the creation of type-2 fuzzy sets. A type-2 fuzzy set $\tilde{\tilde{A}}$ can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}(x,u)$, where $x \in X$ and $u \in J_X, J_X \subseteq [0,1]$ as follows:

(1) $$\tilde{\tilde{A}} = \left\{\left((x,u),\ \mu_{\tilde{\tilde{A}}}(x,u)\right) | \forall x \in X\ \ \forall u \in J_X\ \subseteq [0,1], 0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1\right\}.$$

The type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented as

(2) $$\tilde{\tilde{A}} = \int_{x\in X}\int_{u\in J_X} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u),$$

where $J_X \subseteq [0,1]$ and $\iint$ denote union over all admissible *x* and u.

With this definition, if all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, then $\tilde{\tilde{A}}$ is called an interval type-2 fuzzy set. An interval type-2 fuzzy set $\tilde{\tilde{A}}$ can be regarded as a special case of type-2 fuzzy set, represented as follows [15]:

(3) $$\tilde{\tilde{A}} = \int_{x\in X}\int_{u\in J_X} \frac{1}{(x,u)},$$

where $J_X \subseteq [0,1]$.

In accordance with the given definition, a trapezoidal interval type-2 fuzzy set can be represented as

(4) $$\tilde{\tilde{A}}_i = \left(\tilde{A}_i^U; \tilde{A}_i^L\right) =$$

$$=\left(\left(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1\left(\tilde{A}_i^U\right), H_2\left(\tilde{A}_i^U\right)\right), \left(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1\left(\tilde{A}_i^L\right), H_2\left(\tilde{A}_i^L\right)\right)\right),$$

where $\tilde{A}_i^U$ and $\tilde{A}_i^L$ are type-1 fuzzy sets; $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U$, and $a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L$ are the reference points of the interval type-2 fuzzy sets $\tilde{\tilde{A}}_i$; $H_j\left(\tilde{A}_i^U\right)$ shows the membership value of the element $a_{j(j+1)}^U$ in the upper trapezoidal membership function $\tilde{A}_i^U$, $1 \le j \le 2$; $H_j\left(\tilde{A}_i^L\right)$ denotes the membership value of the element $a_{j(j+1)}^L$ in the lower trapezoidal membership function $\tilde{A}_i^L, 1 \le j \le 2, H_1\left(\tilde{A}_i^U\right) \in [0,1], H_2\left(\tilde{A}_i^U\right) \in [0,1]$, $H_1\left(\tilde{A}_i^L\right) \in [0,1]$, $H_2\left(\tilde{A}_i^L\right) \in [0,1]$ and $1 \le i \le n$ [6]. Fig. 1a represents a trapezoidal interval-type 2 fuzzy set $\tilde{\tilde{A}}$ and its 3D view is shown in Fig. 1b.

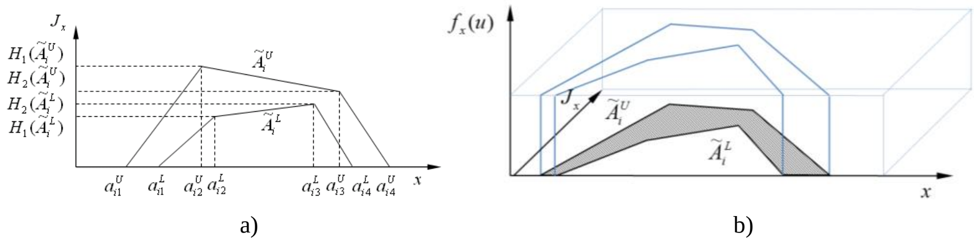

Fig. 1. An interval type-2 fuzzy set $\tilde{\tilde{A}}_i$ with Upper trapezoidal membership function $\tilde{A}_i^U$ and Lower trapezoidal membership function $\tilde{A}_i^L$ (where a) is based on [6])

3.2. A new formula for computing the distance between two IT2 fuzzy numbers

The important task in the TOPSIS method is finding the distances between each alternative and its respective positive and negative optimal solutions. The common approach is to defuzzify IT2 Fuzzy Numbers (FNs) to real numbers and then calculate the distance between two tuples of real values. Our approach first defuzzifies IT2 FNs into two crisp values and then computes their average value.

The proposed method for determining distance between IT2 FNs has two main steps. The first one is from the graded mean integration method [21]. Let $A = (a, b, c, d)$ is a normal fuzzy number with shape function

$$(5) \qquad \mu_A = \begin{cases} (\frac{x-a}{b-a})^n & \text{when } x \in [a,b), \\ w & \text{when } x \in [b,c], \\ (\frac{d-x}{d-c})^n & \text{when } x \in (c,d], \\ 0 & \text{otherwise.} \end{cases}$$

Let A be non-normal fuzzy number, denoted by $A = (a, b, c, d; w)_n$. If $n = 1$, we simply write $A = (a, b, c, d)$, $a < b \le c < d$, which is known as a normal trapezoidal fuzzy number (Fig. 2).

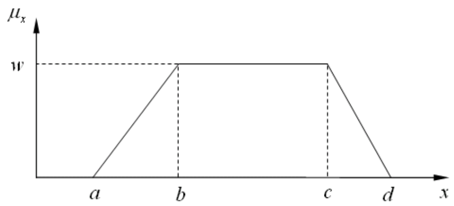

Fig. 2. A normal trapezoidal fuzzy number $A = (a, b, c, d)$

Let $A = (a_1, b_1, c_1, d_1; w_a)$ be a trapezoidal fuzzy number, then the graded mean integration representation of A is defined by:

$$(6) \qquad P(A) = \int_0^{w_a} h\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right) dh / \int_0^{w_a} h\, dh\,.$$

Let $A = (a, b, c, d)$ be a trapezoidal fuzzy number with normal shape function, where a, b, c, d are real numbers such that $a < b \le c < d$. Then the graded mean integration representation of A from [21] is

$$(7) \qquad P(A) = \frac{a+d}{2} + \frac{n}{2n+1}(b-a-d+c).$$

This holds, because

$$(8) \qquad L(x) = \left(\frac{x-a}{b-a}\right)^n \;;\text{ so } L^{-1}(h) = a + (b-a)h^{\frac{1}{n}},$$

$$(9) \qquad R(x) = \left(\frac{d-x}{d-c}\right)^n \;;\text{ so } R^{-1}(h) = d - (d-c)h^{\frac{1}{n}}.$$

Then,

$$(10) \qquad P(A) = \frac{1}{2}\int_0^1 h\left[(a+(b-a)h^{\frac{1}{n}} + (d-(d-c)h^{\frac{1}{n}})\right] dh / \int_0^1 h\, dh =$$

$$= \frac{1}{2}\int_0^1 h\left[(a+d)h + (b-a-d+c)h^{\frac{1}{n}}\right] dh / \int_0^1 h\, dh =$$

$$= \frac{1}{2}\int_0^1 \left[(a+d)h + (b-a-d+c)h^{\frac{n+1}{n}}\right] dh / \int_0^1 h\, dh =$$

$$= \frac{1}{2}\left[\frac{a+d}{2} + \frac{n}{2n+1}(b-a-d+c)\right] / \frac{1}{2} = \frac{a+d}{2} + \frac{n}{2n+1}(b-a-d+c).$$

As a result from one IT2 FN, we form a tuple of crisp values. First Equation (10) is applied to the low and the upper membership function to obtain two defuzzified values and then their average value forms a crisp representation of IT2 FS.

For the second step, we employ the Euclidean distance d_E. Let two intervals be $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$, then d_E can be respectively computed as

$$(11) \qquad d_E = \sqrt{(a_1-a_2)^2 + (b_1-b_2)^2}.$$

4. Proposed TOPSIS modification

Let a MCDM problem has n alternatives (A_1, …, A_n) and m decision criteria (C_1, …, C_m), each alternative is assessed according to the m criteria. Decision matrix $X = (x_{ij})_{n\times m}$ shows all values which are assigned to the alternatives for each criterion. The related weight of each criterion is shown as $W = (w_1, …, w_m)$.

Fig. 3 presents the stepwise modified procedure for implementing TOPSIS. After forming an initial decision matrix, the procedure starts by normalizing the decision matrix. This is followed by building the weighted normalized decision matrix in Step 2, determining the optimal and negative-optimal solutions in Step 3. In Step 4, for calculating the separation measures for each alternative we propose using graded mean integration (Equation (10)). The procedure ends by computing the relative closeness coefficients. The set of alternatives (or candidates) can be ranked according to the descending order of the closeness coefficient.

Start Construct the normalized decision matrix Construct the weighted normalized decision matrix Determine the optimal and negative-optimal solution Calculate the separation measurement based on *graded mean integration* Calculate closeness to the optimal solution Rank the alternatives to the closeness to the optimal solution Stop

Fig. 3. The new TOPSIS modification flow chart

The detailed steps of the TOPSIS modification are six.

1. Normalize the decision matrix $X = (x_{ij})_{n\times m}$ by using the equation below:

(12) $$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{n} x_{kj}^2}};\ i = 1,\dots, n;\ j = 1,\dots, m.$$

2. Calculate the weighted normalized decision matrix $V = (v_{ij})_{n\times m}$:

(13) $$v_{ij} = w_j r_{ij};\ i = 1,\dots, n;\ j = 1,\dots, m,$$

where w_j is the relative weight of the j-th criterion and $\sum_{j=1}^{m} w_j = 1$.

3. Determine the optimal and negative-optimal solutions:

(14) $$A^* = \{v_1^*, \dots,\ v_m^*\} = \left\{\max_j v_{ij}^* \mid j \in \Omega_\mathrm{b},\ \min_j v_{ij}^* \mid j \in \Omega_\mathrm{c}\right\},$$

(15) $$A^- = \{v_1^-, \dots,\ v_m^-\} = \left\{\min_j v_{ij}^* \mid j \in \Omega_\mathrm{b},\ \max_j v_{ij}^* \mid j \in \Omega_\mathrm{c}\right\},$$

where Ω_b are the sets of benefit criteria and Ω_c are the sets of cost criteria.

4. Calculate the separation measurement based on graded mean integration and the crisp assessments of alternatives using (10).

5. Determine the distances of each alternative from the ideal solution and the negative-ideal solution with Euclidean distance ((11), (16) and (17)):

(16) $$D_{\mathrm{E}i}^* = \sqrt{\sum_{j=1}^{m}\left(v_{ij} - v_j^*\right)^2};\ i = 1,\dots, n,$$

(17) $$D_{\mathrm{E}i}^- = \sqrt{\sum_{j=1}^{m}\left(v_{ij} - v_j^-\right)^2};\ i = 1,\dots, n.$$

6. Determine the relative closeness of each alternative to the optimal solution. The Relative Closeness (RC) of the alternative A_i concerning to D^* and D^- is

$$\text{RC}_i = \frac{D_i^-}{D_i^* + D_i^-};\ i = 1, \ldots, n. \tag{18}$$

5. BI platform selection example using the new TOPSIS modification

Business Intelligence (BI) and analytics platforms facilitate processing of evolving buyer and seller dynamics and act as an important tool in enhancing decision making in organizations [9]. Any BI architecture consists of two components – technical and analytical environment which integrate often inconsistent data gathered by a company's information systems. Therefore, selecting a proper BI technology in organizations exacts a detailed analysis of a growing number of requirements when only incomplete and inaccurate information is available. Despite the great variety of BI packages on the IT market, there are hardly any models for their assessment and selection in literature. The topic's relevance is determined by the lack of unambiguously defined comparison criteria for BI software.

In order for the compared BI platforms to be ranked, the following tasks need to be completed:

Step 1. Determine the main features of BI packages and how they could be described.

Step 2. Select the basic criteria for assessing BI packages.

Step 3. Apply a methodology for evaluation of the BI software based on the criteria from Step 2.

Step 4. Generate a ranking of the BI packages according to the proposed in Section 4 assessment model.

Step 5. Test the model's applicability.

Step 6. Specify the scope of applications of the proposed approach.

A literature review on business intelligence specifications was conducted as a part of fulfilling Steps 1 and 2. Let the first assessed feature be the capability of supporting group work. It is also important that BI platforms be able to simulate and model business processes (second and third criterion, respectively), discover hidden relationships and forecast (fourth criterion) and interact with other applications (fifth criterion). The next criterion considers offline and online reporting instruments. It comprises standard queries and reports for making tactical decisions, as well as real-time dashboards, alerts, etc., which show online how the business activities are performed. The qualities of OLAP systems for quick and interactive access to aggregated data will also be covered by this criterion. The Data Warehouses (DW) provide tools for integrating data that have been gathered from various systems for an extended time period. This is why we decided that DW will also be a main factor in BI packages selection. Finally, we defined an assessment model, based on the following seven criteria [10, 22, 23]:

1) groupware;
2) simulation;
3) modeling;

4) data mining and intelligent techniques;
5) interoperability;
6) reporting tools;
7) data Warehouse.

The characteristics are presented by normal fuzzy trapezoid numbers and Table 1 shows the correspondence between linguistic variables and IT2 FNs. The fuzzy numbers that have been used are depicted graphically in Fig. 4.

Table 1. Linguistic terms and their corresponding trapezoidal IT2 FNs

Linguistic term	Trapezoidal IT2 FNs
Very Low (VL)	((0, 0, 0.1, 0.2; 1, 1), (0, 0, 0.05, 0.15; 0.8, 0.8))
Low (L)	((0, 0.1, 0.3, 0.4; 1, 1), (0.05, 0.13, 0.27, 0.35; 0.8, 0.8))
Medium (M)	((0.2, 0.3, 0.5, 0.6; 1, 1), (0.25, 0.33, 0.47, 0.55; 0.8, 0.8))
High (H)	((0.4, 0.5, 0.7, 0.8; 1, 1), (0.45, 0.53, 0.67, 0.75; 0.8, 0.8))
Very High (VH)	((0.6, 0.7, 0.9, 1; 1, 1), (0.65, 0.73, 0.87, 0.95; 0.8, 0.8))
Absolutely High (AH)	((0.8, 0.9, 1, 1; 1, 1), (0.85, 0.93, 1, 1; 0.8, 0.8))

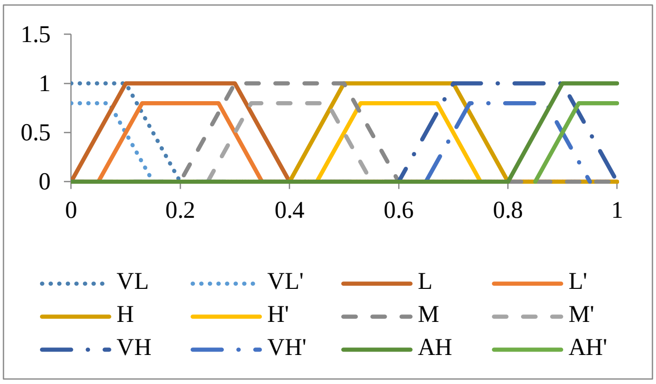

Fig. 4. Membership functions of the linguistic terms

We aim to rank top four BI and analytics products found at the Bulgarian BI platforms market. We chose one expert, an IT manager with experience working with BI alternatives, to create the initial decision matrix and weighted coefficients. The normalized solution matrix (Fig. 3, Step 2) can be found in Table 2.

Table 2. Decision matrix and weighted coefficients

Alternative	Criterion1	Criterion2	Crieterion3	Criterion4	Criterion5	Crietrion6	Criterion7
1	L	AH	VH	VH	H	AH	VH
2	L	M	L	L	VL	AH	VH
3	M	M	L	H	L	M	H
4	H	H	L	AH	H	H	L
W	H	M	L	VH	M	VH	AH

Table 3 contains defuzzyfied values, and Table 4 lists the distances to the optimal and negative-optimal solution and closeness coefficients (18).

Table 3. Defuzzified weighted decision matrix

Alternative	Criterion1	Criterion2	Criterion3	Criterion4	Criterion5	Criterion6	Criterion7
1	0.139	0.385	0.179	0.659	0.259	0.761	0.724
2	0.139	0.099	0.058	0.179	0.036	0.761	0.724
3	0.259	0.099	0.058	0.499	0.179	0.339	0.547
4	0.371	0.259	0.058	0.761	0.259	0.497	0.185

Table 4. Euclidean distance to the optimal and negative-optimal solution

Alternative	D_i^*	D_i^-	RC_i
1	0.2539	0.9219	0.7841
2	0.7362	0.6869	0.4827
3	0.6287	0.5187	0.4521
4	0.6283	0.7029	0.5281

As 0.7841 > 0.5281 > 0.4827 > 0.4521, the obtained ranking should be interpreted as follows: Alterntive1 > Alternative4 > Alternative2 > Alternative3. The same problem was solved in another way by performing defuzzyfication according to the DTraT formula, proposed by K a h r a m a n, O z t a y s i and T u r a n o g l u [13] (Table 5).

Table 5. Defuzzified weighted decision matrix and the closeness coefficients according to [13]

Alternative	Criterion1	Criterion2	Criterion3	Criterion4	Criterion5	Criterion6	Criterion7	RC
1	0.108	0.348	0.071	0.616	0.225	0.710	0.710	0.776
2	0.108	0.148	0.034	0.145	0.031	0.710	0.710	0.502
3	0.225	0.148	0.034	0.459	0.071	0.302	0.529	0.459
4	0.342	0.225	0.034	0.710	0.225	0.459	0.167	0.521

The ranking that it yields is identical:

Alterntive1 > Alternative4 > Alternative2 > Alternative3.

These results prove the applicability of the algorithm, since there is no difference between the rankings obtained by the two methods.

Conclusions

To summarize, the proposed TOPSIS modification can be successfully applied to multi-attribute decision making. The effectiveness and feasibility of the new TOPSIS variation are illustrated by a numerical example. The rankings obtained via two alternative TOPSIS extensions (the new modification and DTraT, used as a benchmark) are identical. The proposed modification does not require a complicated computations and it is beneficial to decision making procedures. A peculiarity of the new modification is that at the moment it works only with normal trapezoidal fuzzy numbers.

In order to overcome the dependence on fuzzy trapezoidal numbers' shapes, we plan to develop a general defuzzification formula in the future. It would be valid for arbitrary type-2 FSs and could be built in different multi-attribute decision making methods. Another part of our future work is to create a convertor from IT2 FNs to fuzzy relations for direct comparison with relations based decision making methods.

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