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A Report on Multilinear PCA Plus GTDA to Deal With Face Image

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Abstract: *Because face images are naturally two-dimensional data, there have been several 2D feature extraction methods to deal with facial images while there are few 2D effective classifiers. Meanwhile, there is an increasing interest in the multilinear subspace analysis and many methods have been proposed to operate directly on these tensorial data during the past several years. One of these popular unsupervised multilinear algorithms is Multilinear Principal Component Analysis (MPCA) while another of the supervised multilinear algorithm is Multilinear Discriminant Analysis (MDA). Then a MPCA+MDA method has been introduced to deal with the tensorial signal. However, due to the no convergence of MDA, it is difficult for MPCA+MDA to obtain a precise result. Hence, to overcome this limitation, a new MPCA plus General Tensor Discriminant Analysis (GTDA) solution with well convergence is presented for tensorial face images feature extraction in this paper. Several experiments are carried out to evaluate the performance of MPCA+GTDA on different databases and the results show that this method has the potential to achieve comparative effect as MPCA+MDA.*

Keywords: *Feature extraction, tensor objects, face recognition, multilinear principal component analysis, general tensor discriminant analysis.*

1. Introduction

On the account of face image being the most common visual pattern in our surrounding life, face recognition has attracted much attention from researchers over the past several years. It seems that facial image recognition is regarded as a fundamental field of biometrics which has been applied to various areas [1]. Meanwhile, most biometric signals have multi-dimensional representation. For example, two-dimensional biometric signals include grey-level images of fingerprint, palmprint, ear, face and multichannel ElectroEncephaloGraphy (EEG) signals in neuroscience. Three-dimensional data includes colour biometric images, Gabor faces, silhouette sequences in gait analysis and grey video sequences in action recognition [2].

Principal Component Analysis (PCA) [3] and Liner Discriminative Analysis (LDA) [4] are two of the most popular classical subspace learning methods in many existing dimensionality reduction algorithms. PCA seeks an optimal projection direction of maximal variation while it does not take the label information of samples into account. In contrast, LDA is a supervised method which searches for the optimal discriminative subspace by maximizing the ratio between the between-class scatter and the within-class scatter. However, applying these linear subspace learning methods described above on tensor data need to reshape tensors into vectors first. This may lead to overtraining, high computational complexity, and large memory requirements. To overcome this shortcoming, several multilinear algorithms which can operate directly on the original tensorial data without the limitation of order have been proposed recently.

Multilinear Principal Component Analysis (MPCA) [5], a tensor vision of PCA, performs dimensionality reduction in all tensor modes to capture most of the variation presented in the original tensors. By the same way, Multilinear Discriminant Analysis (MDA) [6], General Tensor Discriminant Analysis (GTDA) [7], Tensor Subspace Analysis (TSA) [8], apply LDA, Maximum Scatter Difference (MSD) [9] and LPP to transform each mode of the tensors respectively [10]. Moreover, to obtain better accuracy, a new method based on MPCA+MDA for face recognition has been proposed in reference [11-13]. In that paper, MPCA has been implemented in the tensorial data for dimensionality reduction, and then a new data set with a new dimension is generated. This new data set will be the inputs for the MDA algorithm to learn the most discriminative subspaces of the input samples. Due to the usage of MDA after applying the MPCA, not only is this method performed in a much lower-dimension feature space than the traditional methods, such as LDA and PCA, but it can also overcome the small sample size problem [14]. However, like 2DLDA, the MDA algorithm does not converge either [5], because the optimization algorithm applied to MDA fails to converge. On account of the instability, a stable and precise accuracy is difficult to achieve by MDA.

On the other hand, due to the converged alternating projection, GTDA can provide stable recognition accuracy whereas that of MDA cannot. By maximizing the between-class variance and minimizing the within-classes variance, GTDA decomposes tensors into core tensors and a series of discriminative matrices over every modality [15] just like MDA does. Moreover, from the empirical analysis on tensorial samples in [16], it is demonstrated that GTDA can achieve the comparative results of MDA. Consequently, GTDA can be used here to transform multilinear discriminative subspace from high dimensional and high order biometric signals. Motivated by the works briefly reviewed above, in this paper, we introduce a new

MPCA plus GTDA algorithm to deal with tensorial data instead of MPCA+MDA, and expect this novel method to be a better choice. As both MPCA and GTDA represent multilinear algorithm, lower dimensionality dilemma and better correct recognition rate can be captured. To the best of our knowledge, this is the first study that addresses MPCA+GTDA on feature extraction and dimensionality reduction for tensor object.

The remainder of the paper is organized as follows: Section 2 provides a brief introduction of multilinear algebra for dimensionality reduction. In Section 3, the algorithm of MPCA and GTDA are summarized and discussed in detail firstly. In Section 4, we analyse experiment results on several databases to verify the properties of the proposed method and compares performance against with the other algorithms. Finally, the major findings and conclusions are drawn in Section 5.

2. Tensor fundamentals

The notation in this chapter follows the conventions in multilinear algebra, such as the notation in [17]. Vectors are denoted by lowercase boldface letters, e.g., x; matrices by uppercase boldface, e.g., U; and tensors by calligraphic letters, e.g., $\mathcal{A}$. An *N*-th-order tensor is denoted as $\mathcal{A} \in \mathbf{R}^{I_1 \times I_2 \times \cdots \times I_N}$. Their elements are addressed by *N* indices i_n, $n = 1, \ldots, N$, and each i_n addresses the *n*-mode of $\mathcal{A}$.

The *n*-mode unfolding of $\mathcal{A}$ is defined as the I_n dimensional vectors are denoted as

(1) $$A_{(n)} \in \mathbf{R}^{I_n \times (I_1 \times \cdots \times I_{n-1} \times I_{n+1} \times \cdots \times I_N)},$$

where the column vectors of $A_{(n)}$ are gained from $\mathcal{A}$ by varying its index i_n while keeping all the other indices fixed.

The *n*-mode product of a tensor $\mathcal{A}$ by a matrix $\mathbf{U} \in \mathbf{R}^{J_n \times I_n}$, denoted by $\mathcal{A} \times_n \mathbf{U}$, is a tensor defined as

(2) $$(\mathcal{A} \times_n \mathbf{U})(i_1, \cdots, i_{n-1}, j_n, i_{n+1}, \cdots, i_N) = \sum_{i_n} \mathcal{A}(i_1, i_2, \cdots, i_N) \cdot \mathbf{U}(j_n, i_n).$$

One of the most commonly used tensor decompositions is Tucker, which can be regarded as higher-order generalization of the matrix Singular Value Decomposition (SVD). Let $\mathcal{A} \in \mathbf{R}^{I_1 \times I_2 \times \cdots \times I_N}$ denotes a *N*-th order tensor, then the Tucker decomposition is defined as

(3) $$\mathcal{A} = \mathcal{S} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \ldots \times_N \mathbf{U}^{(N)},$$

where $\mathcal{S} \in \mathbf{R}^{P_1 \times P_2 \times \cdots \times P_N}$ with $P_n < I_n$ denotes the core tensor and $\mathbf{U}^{(n)} = \left[\mathbf{u}_1^{(n)} \mathbf{u}_2^{(n)} \cdots \mathbf{u}_{P_n}^{(n)}\right]$ is an $I_n \times P_n$ matrix.

The scalar product of two tensors $\mathcal{A}, \mathcal{B} \in \mathbf{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is defined as

(4) $$\langle \mathcal{A}, \mathcal{B} \rangle = \sum_{i_1} \cdots \sum_{i_N} \mathcal{A}(i_1, i_2, \cdots, i_N) \cdot \mathcal{B}(i_1, i_2, \cdots, i_N).$$

The Frobenius norm of $\mathcal{A}$ is defined as

(5) $$\|\mathrm{A}\| = \sqrt{\langle \mathrm{A}, \mathrm{A} \rangle} = \left\| \mathrm{A}_{(n)} \right\|_{\mathrm{F}} = \sqrt{\sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \cdots \sum_{i_N=1}^{I_N} a_{i_1 i_2 \cdots i_N}^2} .$$

3. Multilinear principal component analysis and general tensor discriminant analysis

As a high-order extension of PCA, MPCA is an unsupervised Multilinear Subspace Learning (MSL) algorithm for general tensors targeting variation maximization by solving a Tensor-to-Tensor Projection (TTP). Meanwhile, as a multilinear extension of MSD, GTDA is a supervised MSL algorithm for performing discriminant analysis on general tensor inputs by solving a TTP too. Motivated by Fisherface [4] and MPCA+MDA algorithm, we use MPCA algorithm for face image feature extraction and dimension reduction and then applying GTDA on the low dimensionality features. Finally, nearest neighbor classifier is implemented to classify the computed GTDA features.

3.1. Multilinear principal component analysis

In this section, the MPCA algorithm is introduced in detail based on the analysis introduced in [5]. A set of *M* tensor object samples $\left\{ \mathrm{X}_1, \mathrm{X}_2, \cdots, \mathrm{X}_M \right\}$ are available for training and each tensor object $\mathrm{X}_m \in \mathbf{R}^{I_1 \times I_2 \times \cdots \times I_N}$. The MPCA objective is the determination of the *N* projection matrices $\left\{ \mathrm{U}^{(n)} \in \mathbf{R}^{I_n \times P_n},\ n = 1, \cdots, N \right\}$ that maximize the total tensor scatter Ψ_{Y},

(6) $$\left\{ \tilde{\mathrm{U}}^{(n)} \right\} = \arg \max_{\left\{ \mathrm{U}^{(n)} \right\}} \Psi_{\mathrm{Y}} = \arg \max_{\left\{ \mathrm{U}^{(n)} \right\}} \sum_{m=1}^{M} \left\| \mathrm{Y}_m - \bar{\mathrm{Y}} \right\|_{\mathrm{F}}^2 ,$$

where $\mathrm{Y}_m = \mathrm{X}_m \times \left\{ \mathrm{U}^{(n)^{\mathrm{T}}} \right\}_{n=1}^{N}$ according to Equation (6), $\bar{\mathrm{Y}} = \frac{1}{M} = \sum_{m=1}^{M} \mathrm{Y}_m$. The dimensionality P_n for each mode $\mathrm{U}^{(n)}$ should be known before.

The *N* optimization subproblems are solved by finding the mode-*n* projection matrix $\mathrm{U}^{(n)}$ that maximizes the mode-*n* total scatter conditioned on the projection matrices in all the other modes. Let $\left\{ \mathrm{U}^{(n)}, n = 1, \cdots, N \right\}$ be the answer to Equation (6), and $\left\{ \mathrm{U}^{(1)}, \cdots, \mathrm{U}^{(n-1)}, \mathrm{U}^{(n+1)}, \cdots, \mathrm{U}^{(N)} \right\}$ be all the other known projection matrices, the P_n eigenvectors reside in the matrix $\mathrm{U}^{(n)}$ corresponding to the largest P_n eigen values of the matrix $\Phi^{(n)}$.

(7) $$\Phi^{(n)} = \sum_{m=1}^{M} \left(\mathrm{X}_{m(n)} - \bar{\mathrm{X}}_{(n)} \right) \mathrm{U}_{\Phi^{(n)}} \mathrm{U}_{\Phi^{(n)}}^{\mathrm{T}} \left(\mathrm{X}_{m(n)} - \bar{\mathrm{X}}_{(n)} \right)^{\mathrm{T}} ,$$

where $\mathrm{X}_{m(n)}$ is the mode-*n* unfolding of X_m and

(8) $$\mathrm{U}_{\Phi^{(n)}} = \left(\mathrm{U}^{(n+1)} \otimes \mathrm{U}^{(n+2)} \otimes \cdots \otimes \mathrm{U}^{(N)} \otimes \mathrm{U}^{(1)} \otimes \mathrm{U}^{(2)} \otimes \cdots \mathrm{U}^{(n-1)}\right).$$

The projection matrices $\left\{\mathrm{U}^{(n)}\right\}$ are initialized through the Full Projection Truncation (FPT) described in [5] and updated one by one with all the others fixed.

3.2. General tensor discriminant analysis

GTDA aims to maximize a multilinear extension of the scatter-difference based discriminant criterion in [7]. GTDA intends to solve for a TTP $\left\{\mathrm{U}^{(n)} \in \mathbf{R}^{I_n \times P_n},\ P_n \le I_n,\ n = 1, \cdots, N\right\}$ that projects a tensor $\mathrm{X}_m \in \mathbf{R}^{I_1 \times I_2 \times \cdots \times I_N}$ to a low-dimensional tensor $\mathrm{Y}_m \in \mathbf{R}^{P_1 \times P_2 \times \cdots \times P_N}$.

A set of *M* labelled tensor data samples $\left\{\mathrm{X}_1, \mathrm{X}_2, \cdots, \mathrm{X}_M\right\}$ in $\mathbf{R}^{I_1} \otimes \mathbf{R}^{I_2} \cdots \otimes \mathbf{R}^{I_N}$ are available for training with class label $c \in \mathbf{R}^M$. The class label for the *m*-th samples X_m is $c_m = c(m)$ and there are *C* classes in total. The between-class scatter matrix of these tensors is defined as

(9) $$\Psi_{B_{\mathrm{Y}}} = \sum_{c=1}^{C} M_c \left\| \bar{\mathrm{Y}}_c - \bar{\mathrm{Y}} \right\|,$$

and the within-class scatter of these tensors is defined as

(10) $$\Psi_{W_{\mathrm{Y}}} = \sum_{m=1}^{M} \left\| \mathrm{Y}_m - \bar{\mathrm{Y}}_{c_m} \right\|,$$

where $\mathrm{Y}_m = \mathrm{X}_m \times \left\{\mathrm{U}^{(n)^{\mathrm{T}}}\right\}_{n=1}^{N}$, M_c is the number of samples for class *c*, c_m is the class label for the *m*-th sample X_m, the overall mean tensor $\bar{\mathrm{Y}} = \frac{1}{M}\sum_{m=1}^{M} \mathrm{Y}_m$ and the class mean $\bar{\mathrm{Y}}_c = \frac{1}{M_c}\sum_{m, c_m = c} \mathrm{Y}_m$. The objective function of GTDA can be written as

(11) $$\left\{\tilde{\mathrm{U}}^{(n)}\right\} = \arg\max_{\left\{\mathrm{U}^{(n)}\right\}} \Psi_{\mathrm{Y}_{dif}} = \arg\max_{\left\{\mathrm{U}^{(n)}\right\}} \Psi_{\mathrm{B}_{\mathrm{Y}}} - \zeta \cdot \Psi_{W_{\mathrm{Y}}},$$

where ζ is a tuning parameter and is automatically selected during the training procedure according to [7]. First, the input tensors (that are the outputs of MPCA) should be solved with the mode-*n* projection matrix conditioned on the projection matrices in all the other modes which defined in Equation (5), and then all the new tensors are unfolded into a matrix along the *n*-th-mode.

The mode-*n* between-class and within-class scatter matrices can be obtained from $\hat{\mathrm{Y}}_{m(n)}$ which is the mode-*n* unfolding of $\hat{\mathrm{Y}}_m^{(n)}$:

(12) $$S_{B_{\hat{Y}}}^{(n)}=\sum_{c=1}^{C}M_c\left(\bar{\hat{\bar{Y}}}_{c(n)}-\bar{\hat{\bar{Y}}}_{(n)}\right)\left(\bar{\hat{\bar{Y}}}_{c(n)}-\bar{\hat{\bar{Y}}}_{(n)}\right)^{\mathrm{T}},$$

(13) $$S_{W_{\hat{Y}}}^{(n)}=\sum_{m=1}^{M}\left(\hat{Y}_{m(n)}-\bar{\hat{\bar{Y}}}_{c_m(n)}\right)\left(\hat{Y}_{m(n)}-\bar{\hat{\bar{Y}}}_{c_m(n)}\right)^{\mathrm{T}},$$

respectively, where $\bar{\hat{\bar{Y}}}_{(n)}=\dfrac{1}{M}\sum_{m=1}^{M}\hat{Y}_{m(n)}$ and $\bar{\hat{\bar{Y}}}_{c(n)}=\dfrac{1}{M_c}\sum_{m=1,c_m=c}^{M}\hat{Y}_{m(n)}$.

Following Equation (11), the mode-*n* projection matrix $U^{(n)}$ in this conditional optimization problem are solved as

(14) $$\left\{\tilde{U}^{(n)}\right\}=\arg\max_{\left\{U^{(n)}\right\}}\operatorname{tr}\left(U^{(n)^{\mathrm{T}}}S_{B_{\hat{Y}}}^{(n)}U^{(n)}\right)-\zeta.\operatorname{tr}\left(U^{(n)^{\mathrm{T}}}S_{W_{\hat{Y}}}^{(n)}U^{(n)}\right)=$$
$$=\arg\max_{\left\{U^{(n)}\right\}}\operatorname{tr}\left(U^{(n)^{\mathrm{T}}}\left(S_{B_{\hat{Y}}}^{(n)}-\zeta.S_{W_{\hat{Y}}}^{(n)}\right)U^{(n)}\right).$$

This can be treated as an eigenvalue problem and the objective function above is maximized only if $\tilde{U}^{(n)}$ consists of the P_n eigenvectors of the total tensor scatter-difference $S_{B_{\hat{Y}}}^{(n)}-\zeta\cdot S_{W_{\hat{Y}}}^{(n)}$ associated with the P_n largest eigenvalues.

As described above, similar to MDA, GTDA aims to maximize the between-class variation while minimize the within-class variation to achieve the best class separation. The difference is that MDA maximizes the ratio of the between-class scatter over the within-class scatter while GTDA maximizes a tensor-based scatter difference criterion and the projection matrices in GTDA have orthonormal columns.

3.3. Feature extraction and classification using MPCA plus GTDA

In the problem of tensor samples recognition, the input training sample $\{X_m\}$ in $\mathbf{R}^{I_1}\otimes\mathbf{R}^{I_2}\cdots\otimes\mathbf{R}^{I_N}$ is projected through MPCA and GTDA for feature extraction. With the learned $\left\{U^{(n)}\in\mathbf{R}^{I_n\times P_n}\right\}$, a series of tensor Y_m in a lower-dimensional tensor space $\mathbf{R}^{P_1}\otimes\mathbf{R}^{P_2}\cdots\otimes\mathbf{R}^{P_N}$ can be obtained with $Y_m=X_m\times_1 U^{(1)^{\mathrm{T}}}\times_2 U^{(2)^{\mathrm{T}}}\cdots\times_N U^{(N)^{\mathrm{T}}}$. Then the *nearest-neighbor* method can be used for the final classification throughout all the experiments owing to its simplicity in computation. The algorithmic procedure for MPCA+GTDA is shown in Algorithm 1.

Algorithm 1. The procedure of MPCA+GTDA

Input: A series of *N*th order tensors $\left\{X_m\in\mathbf{R}^{I_1\times I_2\times\cdots\times I_N},\ m=1,\cdots,M\right\}$, the dimension of feature subspace $\left\{P_n,\ n=1,2,\cdots,N\right\}$, the tuning parameter in GTDA ζ, and the number of iteration *K*.

Output: Low-dimensional representations $\left\{Y_m\in\mathbf{R}^{P_1\times P_2\times\cdots\times P_N}\right\}$ of the input samples.

Step 1. *Pre-processing*: Center the input samples as

$$\left\{\tilde{\mathrm{X}}_m = \mathrm{X}_m - \bar{\mathrm{X}},\ m = 1, \cdots, M\right\},\ \text{where } \bar{\mathrm{X}} = \frac{1}{M}\sum_{m=1}^{M}\mathrm{X}_m .$$

Step 2. *Initialization*: Compute the eigen-decomposition of $\tilde{\mathbf{S}}_{T\mathrm{X}}^{(n)} = \sum_{m=1}^{M}\tilde{\mathbf{X}}_{m(n)} \cdot \tilde{\mathbf{X}}_{m(n)}^{\mathrm{T}}$, and initialize $\left\{\mathbf{U}^{(n)}\right\}$ with the eigenvectors corresponding to the most significant P_n eigenvalues of $\tilde{\mathbf{S}}_{T\mathrm{X}}^{(n)}$.

Step 3. *Local optimization:*

for $k = 1$ to K **do**

for $n = 1$ to N **do**

for $m = 1$ to M **do**

Calculate the mode-*n* partial multilinear projection of input tensors

$$\left\{\hat{\mathrm{Y}}_m^{(n)},\ m = 1, \cdots, M\right\}.$$

end for

(1) Calculate the mode-*n* unfolding scatter matrices $S_{T\hat{\mathrm{Y}}}^{(n)} = \sum_{m=1}^{M}\left(\hat{\mathbf{Y}}_{m(n)} - \bar{\hat{\mathbf{Y}}}_{(n)}\right)\left(\hat{\mathbf{Y}}_{m(n)} - \bar{\hat{\mathbf{Y}}}_{(n)}\right)^{\mathrm{T}}$,

(2) Solve for the $\left\{\mathbf{U}^{(n)}\right\}$ with the eigenvectors corresponding to the most significant P_n eigenvalues of $S_{T\hat{\mathrm{Y}}}^{(n)}$.

end for

If a maximum number of iterations K attains, break and output the current $\left\{\mathbf{U}^{(n)}\right\}$.

end for

Compute the feature tensor

$$\mathcal{A}_m = \mathcal{X}_m \times_1 \mathbf{U}^{(1)^{\mathrm{T}}} \times_2 \mathbf{U}^{(2)^{\mathrm{T}}} \times \cdots \times_N \mathbf{U}^{(N)^{\mathrm{T}}}.$$

for $k = 1$ to K **do**

for $n = 1$ to N **do**

for m=1 to M **do**

Calculate the mode-*n* unfolding local multilinear projection matrices:

$$\tilde{\mathrm{Y}}_m^{(n)} = \mathrm{X}_m \times_1 \mathbf{U}^{(1)^{\mathrm{T}}} \cdots \times_{(n-1)} \mathbf{U}^{(n-1)^{\mathrm{T}}} \times_{(n+1)} \mathbf{U}^{(n+1)^{\mathrm{T}}} \cdots \times_{(N)} \mathbf{U}^{(N)^{\mathrm{T}}} .$$

end for

(1) Calculate mode-*n* between-class and within-class scatter matrices of $\tilde{\mathrm{Y}}_m^{(n)}$ in Equation (12) and Equation (13),

(2) Set the $\tilde{\mathbf{U}}^{(n)}$ with the eigenvectors corresponding to the most significant P_n eigenvalues of $\mathbf{S}_{B\tilde{\mathrm{Y}}}^{(n)} - \zeta \cdot \mathbf{S}_{\mathrm{W}\tilde{\mathrm{Y}}}^{(n)}$.

end for

If a maximum number of iterations K attains, break and output the current $\left\{\tilde{\mathbf{U}}^{(n)}\right\}$.

end for

Output the feature tensor after projection

$$\mathrm{Y}_m = \mathrm{A}_m \times_1 \tilde{\mathbf{U}}^{(1)^{\mathrm{T}}} \times_2 \tilde{\mathbf{U}}^{(2)^{\mathrm{T}}} \times \cdots \times_N \tilde{\mathbf{U}}^{(N)^{\mathrm{T}}}.$$

4. Experiments and results

In this section, two experiments are designed to evaluate the performance of the proposed algorithm. The first one is using ORL face database which obtains 400 face images to discuss the effects of the convergence of GTDA. The following experiments, which are carried out on face and gait databases, illustrate the efficacy of proposed MPCA+GTDA in tensor object recognition and compare its performance against state-of-the-art algorithm. All experiments on the Microsoft Windows XP 64-bits version machine with 2.66 GHz Intel CPU and 32GB memory. For tensor operations, we used the tensor toolbox developed by B a d e r and K o l d a [21] in Matlab.

4.1. Convergence of GTDA on ORL database

The ORL database with 400 different face images is applied. The gray-level face images from the ORL database have a resolution of 112×92, and 40 subjects with 10 images each included in the database. Those images of individuals have been taken by different characteristics, such as with or without glasses, different facial expressions, and facial details.

In order to study the convergence of MDA and GTDA, the total scatter ratio of the between-class scatter over the within-class scatter Ψ_{Ydif} after MDA and the total scatter difference Ψ_{Ydif} after GTDA are plotted against the number of iterations, as a function of dimensionality reduction determined by Q [5]. As mentioned earlier, MDA algorithm does not converge during the iteration. This result can be seen from Fig. 1 that not only the ratio of the between-class scatter over the within-class scatter is not the same after every iteration (from 1 to 30), but also the alternating procedure for the ratio during the iteration is not monotonic. In contrast, as indicated in Fig. 2, applying GTDA on ORL database can obtain well convergence within four iterations. Furthermore, simulation results show that there is no observably difference in the obtained Ψ_{Ydif} when $Q \geq 0.5$, while there is a gap when a small Q (Q=0.1) is used. This observation demonstrates that the smaller of Q then the convergence of GTDA is poorer. Therefore, the number of iteration is set to be three in the following experiments.

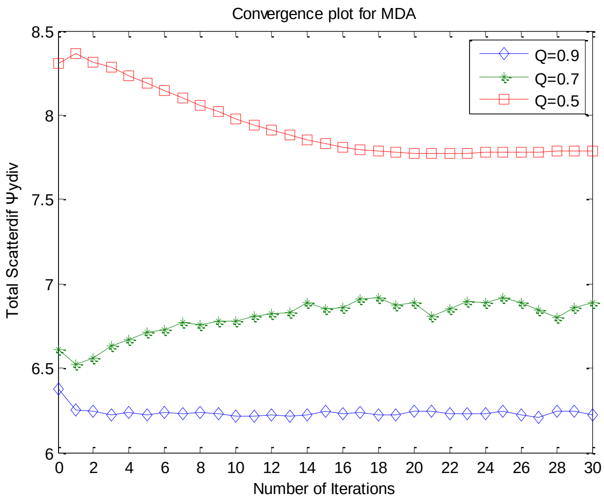

Fig. 1. Evolution of the total scatter ratio Ψ_{Ydif} over iterations when MDA is applied in ORL database with different Q

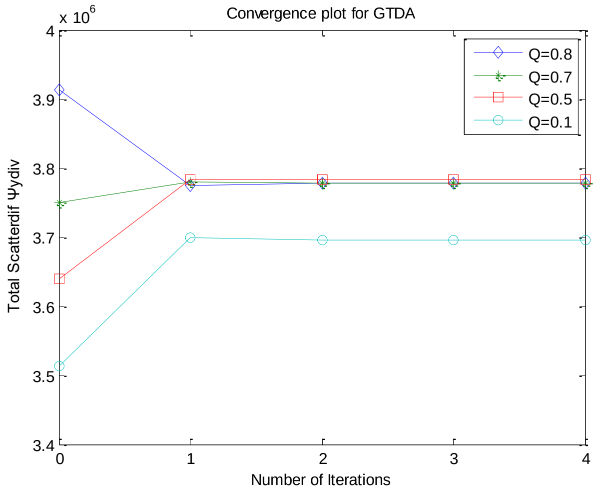

Fig. 2. Evolution of the total scatter difference Ψ_{Ydif} over iterations when GTDA is applied in ORL database with different Q

4.2. MPCA plus GTDA based face recognition on FERET database

In this section, the performance of the proposed MPCA+GTDA algorithm is tested to compare with that of other methods on the FERFT database. The Facial Recognition Technology (FERFT) database [18] which comprises 14126 gray-scale face images acquired from 1199 subjects is widely used for evaluating face recognition problem. In this experiment, a subset of 1400 faces from 200 subjects is selected from the FERET database, seven face images per subject with a resolution of 80×80 pixels. Fig. 3 depicts some face images from two subjects in FERET database.

Fig. 3. Sample images of two individual from the FERET database

A set of experiment are conducted to compare the Face Recognition (FR) performance of MPCA+GTDA against PCA+LDA, 2DPCA+2DLDA, MPCA, MDA, GTDA and MPCA+MDA. Because of gray-level images are natively second-order tensors, they are input directly to the multilinear algorithms as 80×80 tensors, while for PCA+LDA, they need to be vectorized first. For each subject in a FR experiment, *L* samples are selected randomly from each subject to form the training set and the remaining images are used for testing. In order to fairly evaluate the effectiveness of the proposed algorithm, we report the recognition accuracies averaged by over ten such random repetitions. Taking the fair comparison and computational issue in to consideration, all the iterative algorithms (MPCA, MDA and GTDA) are terminated by setting the maximum number of iterations *K* to be three. Meanwhile, the FPT initialization and the most descriptive *P* features are selected for recognition. Finally, the simple nearest neighbor classifier with Euclidean distance measures is applied for classification of these *P* extracted features.

It is easy to understand from Equation (9) and Equation (10), $\Psi_{B_{\mathrm{Y}}}$ and $\Psi_{W_{\mathrm{Y}}}$ can not be calculated with only one element, so the number of training sample *L* and the number of selected feature *P* cannot be set to one. In the first test, three images per subject were randomly selected for training and the remaining samples were used for testing. Fig. 4 shows the recognition accuracy for the PCA+LDA, 2DPCA+2DLDA, MPCA, MDA, MPCA+MDA, and MPCA+GTDA algorithm against *P* from 2 up to 40.

From the detailed results, it can be observed that MPCA+MDA and MPCA+GTDA methods outperform other algorithms across all dimensionality. Besides, when the number of selected feature is small ($P < 10$), the accuracy of MPCA+MDA is slightly higher than that of MPCA+GTDA, and they tend to obtain a comparative results as *P* increases. This is because the difference value has less discrimination than ratio. Therefore, the value *Q* which is used to determine the subspace dimension should be carefully selected to capture an appropriate *P*. Moreover, as supervised method, MDA and GTDA perform better than MPCA, and they also gain an analogous correct rate just like the conclusion in [16], while PCA+LDA performs the worst.

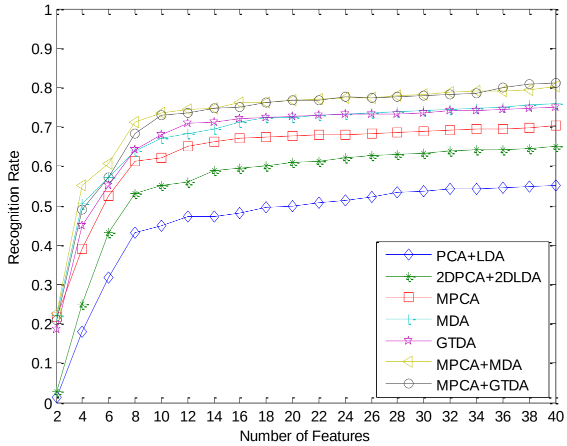

Fig. 4. The recognition rate over dimensions of feature on the FERET face database

5. Conclusion

In this paper, a new MPCA+GTDA framework has been proposed for supervised dimensionality reduction in face images. This algorithm overcomes the limitation of MPCA+MDA and a better convergence and recognition accuracy can be captured. The experimental results of comparing with the state-of-the-art algorithms indicated that the MPCA+GTDA method is a promising tool for face recognition in research and applications.

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