BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **15**, No 1

Sofia • 2015 Print ISSN: 1311-9702; Online ISSN: 1314-4081
DOI: 10.1515/cait-2015-0006

Public Opinion Evolution Based on Complex Networks

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Abstract: *The Sznajd model of sociophysics can describe the mechanism of making a decision in a closed community. The Complex Agent Networks (CAN) model is studied, based on the adaptability, autonomy and activity of the individuals, as well as the complex interactions of individuals in an open community for probing into evolution of the public opinion. With the help of the theory of complex adaptive systems and the methods of complex networks, the structure of agents, the dynamic networks scenarios and the evolutionary process of the agents are described. The simulation results of CAN model show that all individuals cannot reach a final consensus through mutual consultations when the small world networks rewiring probability p is less than a specified threshold. But when the rewiring probability p is larger than the given threshold, all individuals will eventually come to a finial consensus, and that the rewiring probability p increases, whereas the time of emergence of the public opinion will be significantly reduced. It is quite obvious that in real community the mass media and many other mechanisms have an effect on the evolutionary process of the public opinion.*

Keywords: *Complex networks, public opinion, complex adaptive systems, small world networks.*

1. Introduction

The public opinion presents the public comments on an event of particular society groups in specific environment that reach consensus at the end. Each individual in the society groups forms their own opinion in a certain social environment by interacting between each other. Due to the individual differences in handing the social events by the public opinion, especially the dynamic complexity of the interactive relationship between the individuals in social networks, causes complexity of the evolution of the public opinion. The important guiding significance of the public opinion is embodied in mediating the social contradictions, coordinating the social relationships, motivating and supervising the society members, promoting rapid economic growth, simulating and predicting the development trend of the future society.

The study of the public opinion has started in social sciences. In recent years, the researchers have constantly explored the complicated evolutionary mechanisms in the process of opinions formed by modelling in natural sciences and interdisciplinary perspectives, based on the modelling approaches of sociology, complex adaptive systems, statistics and system dynamics, and combined with artificial intelligence and psychology. The Ising spin system is one of the most frequently used models of statistical physics. Meanwhile, the Ising model has been frequently applied in other branches of science, including sociology [1, 2] and economy [3, 4]. Sznajd model [5] is a sociophysics model of opinion formation, which was based on the trade union maxim "United we stand, dived we fall". Since then, many scholars and researchers have continued to study the opinions by extending and refining Sznajd model. It is obvious that two dimensional models are much more realistic than the one dimensional model for the social systems. Hence, Stauffer et al. [6] have presented the model on a square $L\times L$ lattice, where again every spin can be up or down, six different rules were introduced in the paper, but only two of them were used in the complete consensus which is always reached as a steady state by both these rules. The generalization of Sznajd model to a triangular lattice with spreading of the mixed opinion was studied in Ochrombel [7] simplification, where a randomly chosen spin on the square lattice influences its four neighbours. People do not always rest on the site in the square lattice connected to their nearest neighbours. It is obvious that Ising-type models cannot explain the origin of very complicated phenomena observed in complex systems. In fact, it is most realistic to investigate the model on complex networks [8]. Up to now Sznajd model has been analyzed on small world networks [9, 10], on scale-free networks [11-14] and on complete graphs [15]. Small-world networks are found to be closer to the real social systems than both regular and random lattices. The Sznajd model for a two-state opinion formation problem was applied to small world networks and the simulation results usually display rich dynamics of the public opinion for society groups [10]. By simulating Sznajd model of sociophysics on more realistic networks with stronger clustering (scale-free networks), the results show that when simple and minor changes are considered in the model, the hyperbolic law observed in the real case can be reproduced quite well [14].

Computer simulations play an important role in the study of social systems and agent-based modelling is the most successful methodology for studying the social dynamics. The basic idea is to construct the computer object (known as agents with some properties) and then run them in parallel to simulate the real-world scenarios, and the final goal is to track the emergence of the research problem on the social systems from the micro level to the macro level. In these agent-based models [16, 17], the individual agents (homogeneity or heterogeneity) interact in the given environment according to procedural rules, adjusting their behaviours in response to the feature parameters, so we usually focus on the features of each individual instead of the whole population.

The system evolution model of the public opinion in a community is described by the information interaction between the individuals, namely the dynamic process of information exchange of the social networks which is composed of people and their relationships under the specific definition. According to the heterogeneity of the individual (vertices) and the dynamics of the networks topology in society groups, the system model of the public opinion was built based on the theory of complex adaptive systems and the modelling and analysis technology of complex networks, which can appropriately reflect the modelling ideas of the complex systems, meet the requirements of system simulation, and further research the essence and mechanism of evolution and communication in the community.

The rest of the paper is organized as follows. In Section 2 the algorithm of small world networks, based on Watts-Strogatz (WS) model is introduced. In Section 3 the public opinion system is defined and the complex agent networks are described as relationships between individuals of the society and nodes of complex networks. In Section 4 the model is initialized and analyzed. We offer concluding comments in Section 5.

2. Small world networks

The dynamic evolution of the public opinion was fulfilled by the mass media together with the social networks. In terms of the scope, the mass media acted as external conditions and the social networks acted as internal conditions in the dynamic process of the public opinion. Based on the viewpoint of networks, we can map the individuals (people) in the society as nodes of social networks, and denote the relationships of the individuals as edges. So the social networks are the important carrier for propagation and evolution of the public opinion.

At the beginning of the study many scholars have abstracted the social networks as single regular networks or random networks for analyzing the characteristics of the networks topology. But recent studies of complex networks have shown that the social networks are neither completely regular networks, nor completely random networks, but "small world" networks. The concept of small world networks was proposed to describe the social networks by Watts and Strogatz [18] in 1998. The social networks possess the characteristics of clustering and the small world effect. Small world effect means that the average path length between any two vertices is too short compared with the same size of the lattice. The Watts-Strogatz model has successfully generated the graphs with high clustering coefficients and a small average path length by interpolating between an ordered finite dimensional lattice and a random graph. The **Algorithm of Watts-Strogatz model** is as follows (Fig. 1):

Step 1. Start with a ring lattice with *N* nodes, in which every node is connected to its first *K* neighbours;

Step 2. Randomly rewire each edge of the lattice with probability *p*, such that self-connections and duplicate edges are excluded.

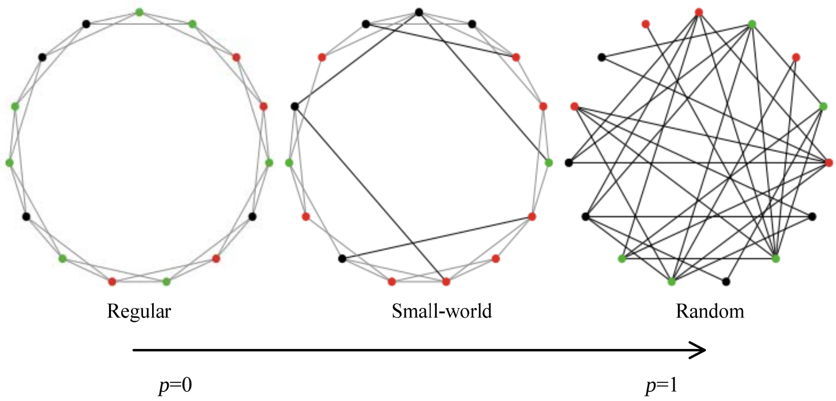

Fig. 1. Random rewiring process of Watts-Strogatz model

In Fig. 1 we start with N = 15 nodes, each connected to its four nearest neighbours. While p = 0, the original ring is unchanged, and then the networks become increasingly disordered until p = 1 when all edges are rewired randomly with *p* increasing.

3. Public opinion model

3.1. Public opinion system definition

Based on the analysis of the characteristics of the evolutionary process of the public opinion in the community, we can define the system of the public opinion as a complex dynamic networks system.

Definition 1. The system of the public opinion for the social networks in the open community is a tri-tuple $\mathrm{SNs} = (V(s), E(s), R_s)$, where:

$V(s) = \{v_{ij} \mid i, j \in N^+\}$ is the nonempty set of the vertices in the social networks;

$E(s) = \{e_k \mid k \in N\}$ is the set of edges in the social networks, and $E(s) \cap V(s) = \varnothing$;

$R_s = \{r_{xy}, x, y \in V(s); x \neq y\}$ is the mapping function of the relationships between the different vertices in the social networks; $r_{xy} = 1$ means there is a direct relationship between *x* and *y*, otherwise it is zero.

Definition 2. The individual (vertice) in the social networks is defined as an agent, and the agent is a six-tuple $\text{Agent=(Mark, Inp, Ma, Dm, Lb, Pe)}$, where:

Mark is the identity of the agent, because the ID number is unique, it has the accuracy of describing the individual;

Inp is the intelligent linking place, which is responsible for setting up the massages channel among agents;

Ma is the Massage channel of agents;

Dm is the Decision-making centre, which implements the information receiving, decomposition and allocation of the system task, updates the system and returns the task results;

Lb is the Local knowledge basis of an agent;

Pe is the characteristics of an agent, so we can define it as follows:

$\mathrm{Pe}=(\mathrm{Op}, \mathrm{Un}, \mathrm{Tr})$, where:

Op is the certain Opinion of the agent;

Un is the Uncertainty of the agent holding the certain opinion; $0 \le \mathrm{Un} \le 1$, the greater the value is, the more uncertain the agent holding the opinion is;

Tr is the Trust of the agent, it denotes the strength of the association between two agents.

3.2. Complex agent networks

The system model of the evolutionary process of the public opinion is built by combining the multi-agent systems with complex networks. The individual model of the public opinion is structured by agent-based modelling, and their complicated inter-relationships are described by complex networks. Thus, the system is a dynamic system, in which there are both fixed relationships and accidental connections among agents. So, the model fully reflects not only the dynamic evolutionary characteristics of the social networks but also the dynamic complexity of the whole system. To make full use of the advantages of the multi-agent systems and complex networks, we map the agents and their inter-relationships of the multi-agent systems to vertices and edges of complex networks respectively, as shown in Fig. 2. The forming and rewiring mechanism of the edges in complex networks with non-trivial topology can regulate the interactions among the agents in the multi-agent system. Furthermore, the development of the statistical methods for quantifying the characteristics of the complex networks is a good way to analyze the influence of the networks topology of the public opinion.

According to the mapping mechanism between multi-agent systems and complex networks given in Fig. 2, the evolutionary process of the public opinion can be achieved from different angles. From the individual perspective, we can utilize the agent to provide the individual behaviours, professional knowledge, cognitive attitude of special things and other personal properties; and from groups perspective, we can make use of the complex networks describing the complicated process of interaction among the agents, and further simulating the evolution and diffusion process of the public opinion by changing the networks scenarios with a simulation time step. Through the simulation of a system and analyzing the data by using statistical methods and tools of complex networks, we can reveal internal individual behaviours and emergence of social networks, so as to establish information feedback mechanisms between the group layer and the individual layer, and realize the simulation of the real world by changing the behaviour of the agents and the networks scenarios.

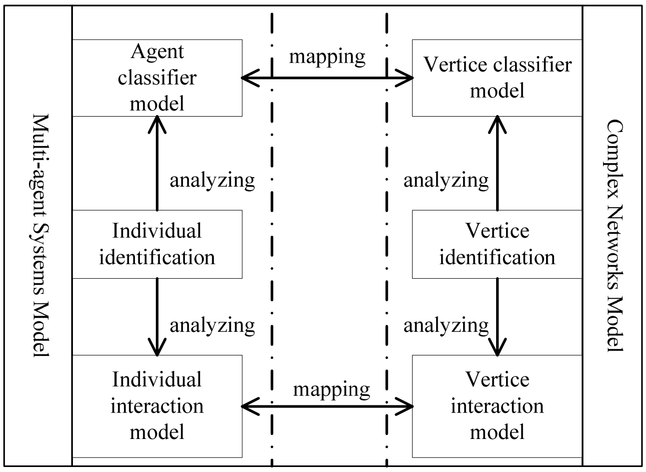

Fig. 2. Mapping mechanism between multi-agent systems and complex networks

According to the mapping mechanism between multi-agent systems and complex networks given in Fig. 2, the evolutionary process of the public opinion can be achieved from different angles. From the individual perspective, we can utilize the agent to provide the individual behaviours, professional knowledge, cognitive attitude of special things and other personal properties; and from groups perspective, we can make use of the complex networks describing the complicated process of interaction among the agents, and further simulating the evolution and diffusion process of the public opinion by changing the networks scenarios with a simulation time step. Through the simulation of a system and analyzing the data by using statistical methods and tools of complex networks, we can reveal internal individual behaviours and emergence of social networks, so as to establish information feedback mechanisms between the group layer and the individual layer, and realize the simulation of the real world by changing the behaviour of the agents and the networks scenarios.

Based on the micro level, the agent has the characteristics of a personal goal, self-determination, proactive behaviour and self-regulation in complex adaptive systems, so that the system model of the evolutionary process of the public opinion was built to realize individual behaviours, individual response to the environment and macro-control policies by agent-based modelling. Since the main carrier of the public opinion is the social networks which have the characteristics of complex networks, the model of complex networks was built to describe the interaction relationships of the individuals and dynamics of the networks scenarios. Such a system model which includes the individual (agent), the social networks and policies being built by combining the multi-agent systems with complex networks, is shown in Fig. 3. The agent perceives the environmental conditions and performs a series of actions by event correlation rules and inference rules in self consciousness and knowledge, meanwhile these actions will influence the surroundings and their relationships, and then the networks scenarios changes with the evolutionary process of the public opinion, and so on, the system will continue to evolve.

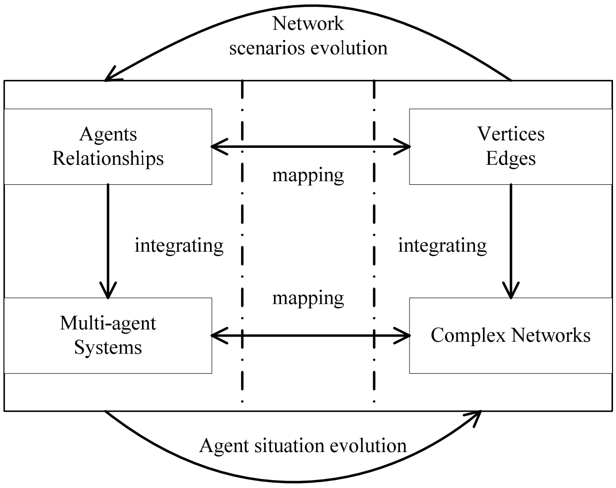

Fig. 3. Collaborative process of multi-agent systems and complex networks

4. Simulation

4.1. Initialization

The evolution of the public opinion is a complex dynamic process in time, which involves many uncertain factors influencing the information moving, exchanging and disseminating. The individual's social ideology, world outlook, outlook on life and values, benefit background, even state of mind in time and the interactions between individuals and the mass media are main influence factors. For convenience and manoeuvrability of the study, the model follows the next six assumptions.

1. Each individual's attitude for a special thing has three kinds of a situation, namely, Opinion A (Support), Opinion B (Neutrality) and Opinion C (Negate).

2. Each individual ideology is known, the individual attitude to the special thing is initialized by analyzing the historical data. Of course, the uniform distribution and random distribution are represented at initialization.

3. Since the external conditions of the number, attitude and influence degree of the media are known, the media attitude for special things has three kinds of a situation, namely, Opinion A (Support), Opinion B (Neutrality) and Opinion C (Negate), the influence of the media on individuals is random.

4. In the process of system evolution, the individual changes his attitude with probability p by interaction with his neighbourhood and the media, and insists on self-attitude towards the special thing with probability $1-p$.

5. The minority is subordinate to the majority, which is the basic principle for individual attitude in the system evolution.

6. An asymmetric influence function is used in the interaction rules of the individual opinion, namely [19]:

(1) $\mathrm{Op}'_i = \mathrm{Op}_i + f(j,i).(\mathrm{Op}_j - \mathrm{Op}_i)$, $u'_i = u_i + f(j,\ i).(u_j - u_i)$,

(2) $\mathrm{Op}'_j = \mathrm{Op}_j + f(i,\ j).(\mathrm{Op}_i - \mathrm{Op}_j)$, $u'_j = u_j + f(i,\ j).(u_i - u_j)$.

Based on the assumptions, the **Algorithm of the system modelling of the public opinion** is as follows:

Step 1. Initialize the parameters: total number of vertices (individuals), initial Opinion, Uncertainty and Trust of any of the individuals;

Step 2. According to WS model algorithm, generate a small world networks model with rewiring probability p;

Step 3. For individual i, look at the individual j in the neighbourhood. If there are relationships between the individual i and individual j, set Tr =10, otherwise, set the initial value of Trust;

Step 4. The opinion is exchanged between individual i and individual j by the assumption 6, and the attitude and uncertainty of opinion is updated simultaneously;

Step 5. Iterate through all individuals, synchronously update the attitude and uncertainty of the individuals plot curves according to the output parameter;

Step 6. Repeat Step 4 and Step 5 until meeting the termination conditions.

4.2. Simulation and results

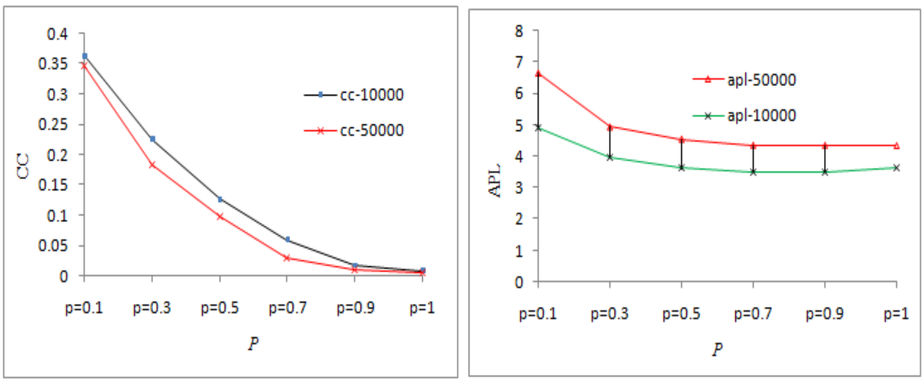

Fig. 4. Comparison of CC and APL for different networks

For the sake of analyzing the influence of the networks topology on the evolutionary process of the public opinion, we run the system model in different networks sizes (number of nodes). In Fig. 4 the comparison graph is shown of the Clustering Coefficient (CC) and the Average Path Length (APL) of the social networks in different networks sizes (number of nodes 10000, 50000) with different rewiring probability. Based on the comparison graph, we conclude that there are not obvious correlations between the networks sizes and networks parameters for the generation algorithm of the system model in the same rewiring probability. Thus the following simulation will be based on the same networks size ($N = 50000$).

4.1.1. Public opinion emergence in small world social networks

For fully exploring the influence of the networks topology structure on the evolutionary process of the public opinion in small world social networks, the system model was run in different networks scenarios with different rewiring probability p.

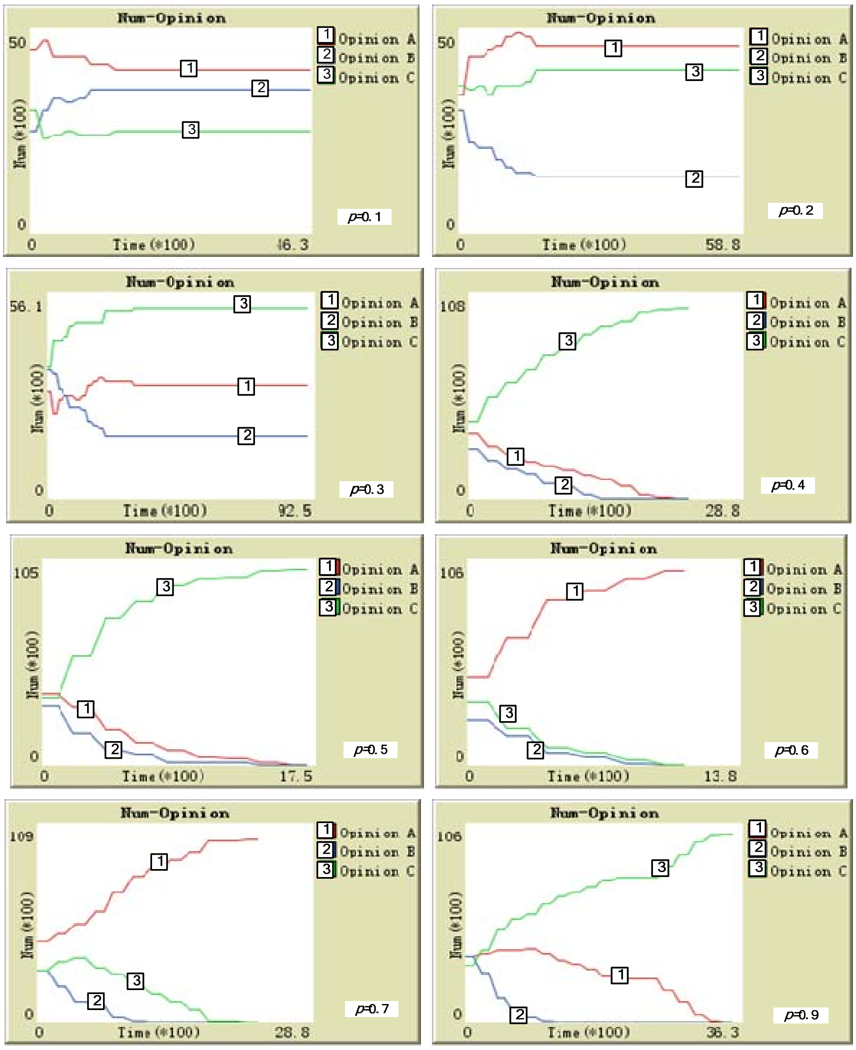

Fig. 5. The evolution in small world networks with different rewiring probability p (p = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.9)

The results in Fig. 5 show that when the rewiring probability *p* of the networks takes a smaller value ($p \leq 0.3$), ultimately, there are more than two kinds of results in the dynamic evolution, namely, all individuals cannot reach a final consensus through mutual consultations in the social networks. Once the rewiring probability *p* is greater than 0.3 ($p>0.3$) in the system networks model, the evolution of the public opinion will eventually emerge the uniqueness of the results, namely, all individuals can reach the consensus in a bounded time by exchanging ideas, i.e., the higher the rewiring probability *p* is, the shorter the average time of emergence of the public opinion is. Through analysis of the transformation process from regular networks to random networks, we have found that the changes of the rewiring probability *p* causes changes in the networks topology. Furthermore, the higher the rewiring probability *p* is, the higher the randomness of the networks topology is. So that the connections of different individuals are more flexible and random, the evolutionary process of the public opinion is more emerged, and it is less probable many kinds of opinion to appear as results of the local distribution of groups.

4.1.2. Mass media attitude influence on the public opinion

In order to intuitively analyze the effect of mass media on individuals, the system model was carried out under conditions of considering the mass media. In the simulation model we assume that the influence strength of the mass media on the individuals is plus or minus 50% by random distribution, namely, when the attitude of the individual is the same as in mass media, the certainty of the individual could be further enhanced; on the contrary, the certainty of the individual would be further weakened. According to the evolutionary process of the public opinion in the social networks with different rewiring probability *p*, shown in Fig. 6, there is a direct effect on the emergence of the system evolution, and the mass media attitude causes disturbances in the dynamic process of the system evolution. As can be seen from Figs 6 and 7, when the attitude of the mass media is the same as that of most of the individuals, the time of system evolution of the public opinion is significantly shortened if considering the attitude of the mass media than without considering the attitude of the mass media; and it shows a tendency of shortening with the increase of the rewiring probability *p* in social networks; when the attitude of the mass media is different compared to most of the individuals, the time of system evolution of the public opinion is significantly extended considering the attitude of the mass media than without considering the attitude of the mass media. It shows a tendency of extension with the increase of the rewiring probability *p* in social networks. In particular, when the attitude of the individual is of a more uniform distribution, the attitude of the mass media will determine the emergence of the evolution of the public opinion in social networks, and its effect on the emergence of system evolution is more obvious when increasing the rewiring probability *p* in social networks.

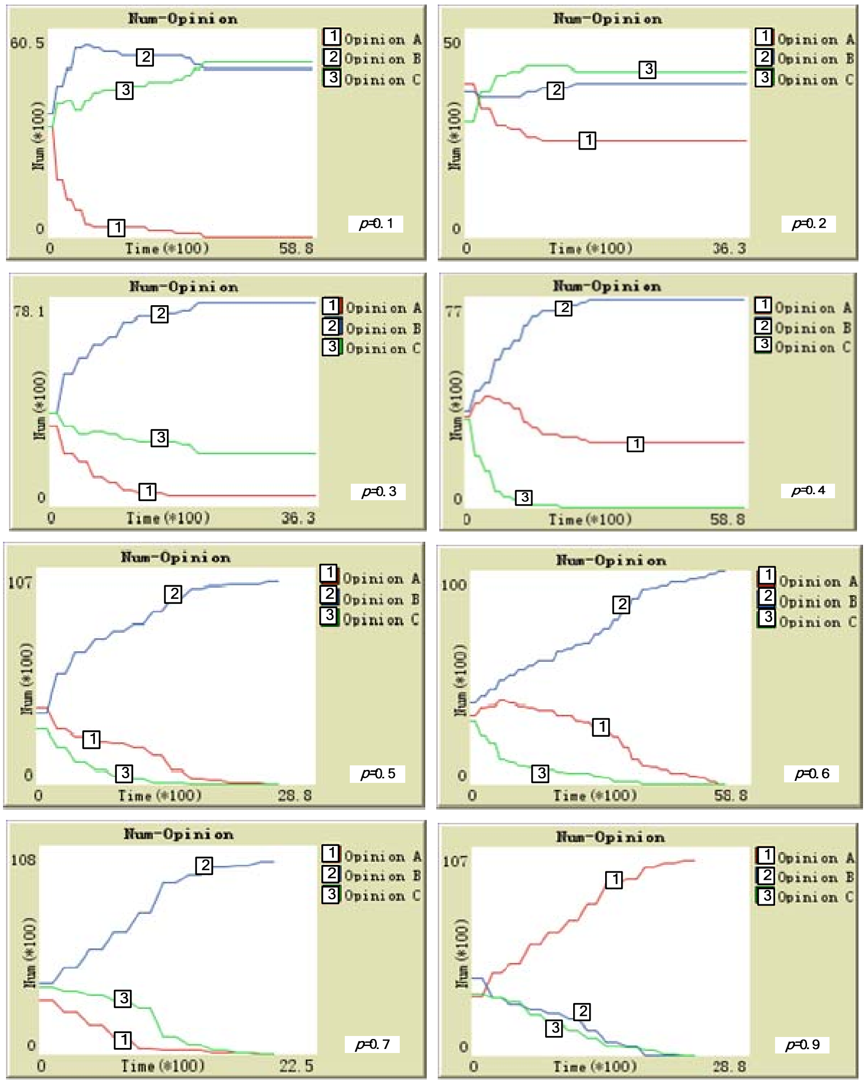

Fig. 6. The system evolution considering the mass media attitude
(*p*= 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.9)

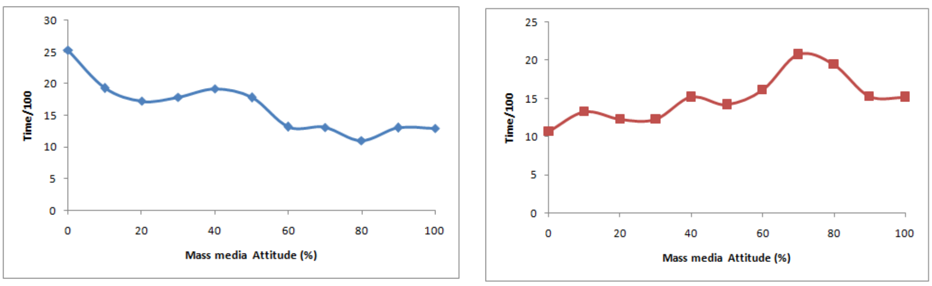

Fig. 7. The mass media effect on the system evolution time (mass media attitude is the same as in most of the individuals, see the left-hand side; otherwise, see the right-left side)

5. Conclusions

It can be well seen, that as in the traditional Sznajd model with random sequential updating, the neighbourhood is updated according to the states of the central spins. Thus the basic model, which is investigated on a square lattice, triangular lattice, dilute lattice in even complex networks, leads the system to a complete consensus (all sites have opinion +1 or −1), and the whole system will reach a fixed point after a certain time of simulation. Actually, the individuals are not always fixed in a site of a lattice connected to their nearest neighbours, or each of them does not always have the same four friends (neighbours). In the actual social environment, many factors influence the individual's fate to come across an acquaintance and to decide whether to become friends, as well as accept (share) an opinion. Shuguang Suo and Yu Chen [20] present a simple model to analyze the dynamics of the four types of networks (small world networks, random networks, scale-free networks and regular networks). The paper studies the problem of public opinion formation and concentrates on the interactions among three factors: individual attributes environmental influences and information flow; and gains some important insights by exploring the dynamic characteristics of the system equilibrium with different elasticity coefficients of the environmental influence, mechanisms of information updating, as well as networks topologies. Marco Toledo Bastos [21] discusses the propagation of opinions expressed in Internet (digital media matrix) in the view point of historic development of the public opinion. The relationship between the media and public opinion is described as forms of coding between the communication agents, such as senders and receivers (peer-to-peer), broadcasting (mass communication) and networks of nodes (digital communication). However, there are so far few agent-based models that formulate the public opinion dynamics by changing the networks scenarios of complex networks.

Herein we define the public opinion system as complex dynamic networks by analyzing the characteristics of the evolutionary process of the public opinion in real society. A multi-agent system was introduced to analyze the patterns and characteristic activities of the individuals from a microscopic level, and complex networks were introduced to describe the complicated relationships of the individuals and the specific scenarios of individuals in the time sequence at the middle level. In CAN model, we have mapped the agents and their inter-relationships of the multi-agent systems to vertices and edges of small-world networks, respectively. In open community we have explored the networks topology influence on the evolutionary process of the public opinion by constructing small world networks with high clustering coefficients and a small average path length. The simulations show that the changes of the networks rewiring probability p cause changes of the networks topology, and the networks topology randomness raises when the networks rewiring probability p grows. When the rewiring probability p of networks takes a smaller value ($p \leq 0.3$), there are three kinds of situation in the social networks, namely, the numbers of individuals (Opinion A, Opinion B and Opinion C) are not zero. Once the rewiring probability

p is more than 0.3 (p>0.3), the system will eventually emerge a single state. Moreover, for all the cases considered, one observes that the higher the rewiring probability p is, the shorter the average time of emergence of the public opinion is. The agent (individual) does not exist in isolation, but in a complex social environment. So that when the attitude of the agents (individuals) are of a more uniform distribution, the mass media plays an extreme role in forming and developing of the public opinion in social networks. Its effect on the emergence of the system evolution is more obvious with the increase of the rewiring probability p in small world networks. When the attitude of the individual is the same as in mass media, the certainty of the individual could be further enhanced; on the contrary, the certainty of individual would be further weakened. When the attitude of the mass media is the same as that of most of the individuals, the system of the public opinion is significantly shortened considering the attitude of the mass media than without considering the attitude of the mass media. The CAN model helps us to understand the characteristics and emergence from the interactions between the different individuals by mapping the real social environments into complex agent networks systems.

Acknowledgements: The author wishes to thank Anyang Normal University for collecting the simulation data. This work was supported partially by the National Natural Science Foundation of China (Grant No 61064010, 61364022).

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