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Absorptive Repetitive Filters with Operators of Generalized Fractional Calculus

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***Abstract**: An essentially new class of repetitive fractional disturbance absorptive filters in disturbances absorbing control systems is proposed in the paper. Systematization of the standard repetitive fractional disturbance absorptive filters of this class is suggested. They use rational approximations of the operators for fractional integration in the theory of fractional calculus. The paper discusses the possibilities for repetitive absorbing of the disturbances with integer order filters and with fractional order filters. The results from the comparative analysis of their frequency characteristics are given below.*

***Keywords**: Integer order repetitive disturbance absorptive filters, fractional order repetitive disturbance absorptive filters, analysis of frequency characteristics.*

1. Introduction

Control systems with absorbing disturbances of C. D. J o h n s o n [1-6] are well known. Unlike standard systems (Fig. 1a) with $\mathrm{R} = R*$, the control systems with absorbing disturbances' structure (Fig. 1b) are characterized by the fact that consistently in the main control algorithm $R*$, an absorptive filter (absorber) is included, absorbing the impact of the integral disturbance $v = [\ \upsilon \quad \varsigma \quad \ldots \quad f\ ]^{\mathrm{T}}$ of the tuning parameter in the system. In this case the main algorithm is $\mathrm{R} = AR*$ or $\mathrm{R} = \mathrm{A}R*$, where A is the absorber, based on integer order integration operators, and A − operators of non-integer, fractional order. The operating principles, the methods for their synthesis and rational approximations are discussed in [1-18, 31-34]. The analytical description of absorbers, using integer order operators *A* (1)-(13) and non-integer, fractional order operators A (14)-(18) are systemized in Table 1.

There are known [19-30, 33-40] repetitive control systems with a memory loop (Fig. 1c). They differ from the standard systems (Fig. 1a) by the fact that consistently in the main control algorithm a R* ML-memory loop filter (*Memory Loop*) and $\mathrm{R} = \mathrm{ML}\ R^*$ is included. Repetitive systems are designed for efficient filtration of the influence of external periodical signal disturbances $\vartheta_p = [\ f,\ \upsilon\ ]^{\mathrm{T}}$ over tuning *y* (if $T_p = 2\pi / \omega_p$ is preliminary known). Modifications of such class of systems, the operating principle, and the method for their synthesis are shown in [35-40]. The analytical descriptions of robust ML-memory loop filters (19)-(24) are systemized in Table 1. Among them with robust stability and quality criteria of control ML, the memory loop filter is shown to be superior (20).

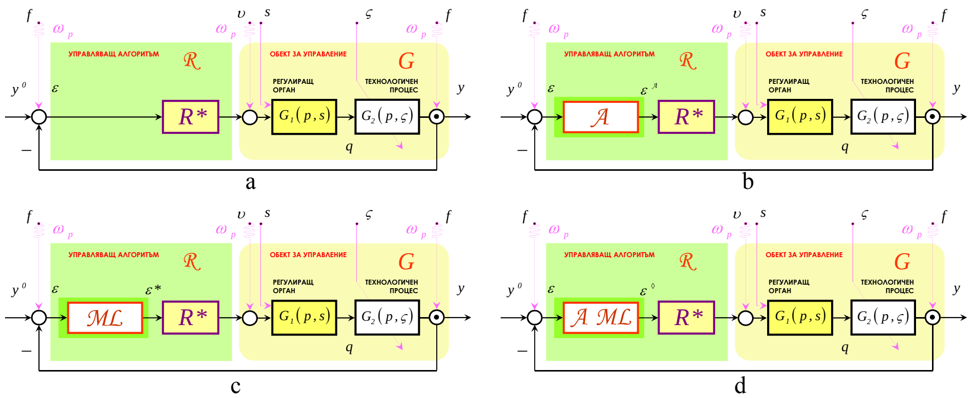

Fig. 1

2. Problems and goals

The main goals of the current paper are: ● a new class of absorptive repetitive $\mathcal{AML}, (\mathcal{AML})$-memory loop filters (from integer or non-integer order) to be synthesized; ● to design a new class of control systems, which combine the absorbing of the integral disturbance $v = [\ \upsilon \quad \varsigma \quad \ldots \quad f\ ]^{\mathrm{T}}$ and efficient filtering of the influence of the periodic disturbances $\vartheta_p = [\ f,\ \upsilon\ ]^{\mathrm{T}}$ over the tuning parameter *y* of the system with AML, (AML)-memory loop filters (from integer and non-integer order). The main problems in this paper are: ● to propose the functional structure and corresponding methods for synthesis of absorptive repetitive AML, (AML)-memory loop filters (of integer and fractal order); ● to model and analyze their dynamic time-frequency characteristics.

3. Structural solution and configuration of absorptive repetitive memory loop filters

The paper suggests a configuration for solving the problem with synthesis using a serial functional structure of a new class of AML, (AML)-absorptive repetitive memory loop filters. Their dynamics is determined in accordance with the general form relations (25) for filters with integer order operators and (26) – for filters with non-integer fractal operators (Figs 2 and 3). The properties of absorptive and repetitive forming in (25) and (26) are not functions of the base regulator R* in the system and the control object *G*, and they are not common functions. The absorptive component is a function of the analytical approximation of the mismatch trend ε (Fig. 1d), and the repetitive component is a function only of the preliminary known period $T_p = 2\pi / \omega_p$ of the external harmonic disturbances to the system. That is why in the current paper their analytical synthesis is offered to be independent and to be based on methods and algorithms for control, shown in [33, 34], and [35, 40], for both composing. Based on the above said, the decision of the task of designing a new class of control systems, combining the absorption of the impact of $v = [\ \upsilon \quad \varsigma \quad \dots \quad f\ ]^{\mathrm{T}}$ and efficient filtration of the influence of $\vartheta_p = [\ f,\ \upsilon\ ]^{\mathrm{T}}$ over the tuning parameter y in the system, is shown in Fig. 1d. This new class of systems differs from the standard systems (Fig. 1a) by the fact that in the serial control algorithm R^*, an absorptive repetitive AML, (AML)-memory loop filter (of integer or non-integer fractal order) is included.

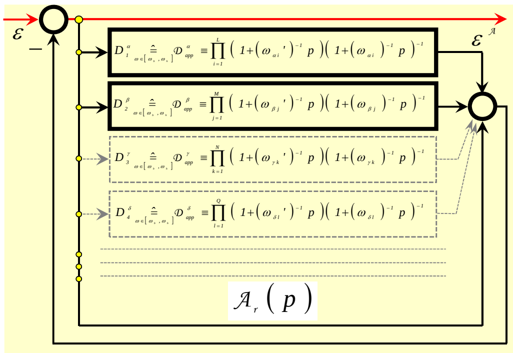

Fig. 2

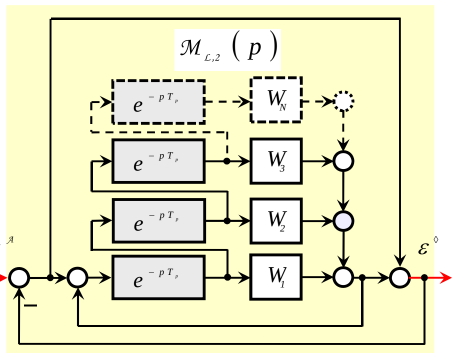

Fig. 3

4. Characteristics modeling and analysis of the absorptive repetitive memory loop filters

Analytical models of the configured absorptive repetitive memory loop filters of integer or non-integer order are shown in (25), (26) and (3)-(13), (22), respectively (14)-(18), (22). The rationality of the absorbers (3)-(13) causes no problems in the modeling of a new class of filters, configured with (25). For modeling the current class of filters, based on operators for fractal integration and differentiation (27)-(29), based on the theory of fractal calculus, in the paper frequency restricted (for $\omega \in \left[\omega_b, \omega_h\right]$ with lower ω_b and upper ω_h boundaries), rational approximations (30)-(31) are offered and used.

The analysis and characteristics of the proposed new class of filters (25), (26) is performed on the simulation results of their models as follows:

- In Figs 4-15 (Appendix 1), the frequency characteristics of the A_i ML - filters (25) are shown, including all shown in the paper absorptive repetitive memory loop filters (3-13) of integer nonhomogeneous order, used in configuration with (20); visualized characteristics are shown in parametrical dependence on the value of the time constants T_i in the absorptive components (3)-(13) of A_i ML -filters (25);
- In Figs 16-27 (Appendix 1), the frequency characteristics of the A_r ML - filters (26) are shown, including all monomial, binomial, trinomial and quadrinomial considered in the paper-absorptive repetitive memory loop filters of fractal order (14-18), serially involved with configuration (20).

(1) $$A_i(p) = \left(\sum_{i=1}^{M}\left(T_i p^i\right)\right)^{-1} = \varepsilon^A(p)/\varepsilon(p),$$

(2) $$A_r(p) = f_r\left({}_aD^r(p)\right)^{-1} = \varepsilon^A(p)/\varepsilon(p),$$

(3) $$A_1(p) = \left(T_1 p\right)^{-1},$$

(4) $$A_2(p) = \left(T_2 p^2\right)^{-1},$$

(5) $$A_3(p) = \left(T_3 p^3\right)^{-1},$$

(6) $$A_4(p) = \left(T_4 p^4\right)^{-1},$$

(7) $$A_{1\bullet 2}(p) = \left(\left(T_2 p^2\right) \pm \left(T_1 p\right)\right)^{-1},$$

(8) $$A_{2\bullet 3}(p) = \left(\left(T_3 p^3\right) \pm \left(T_2 p^2\right)\right)^{-1},$$

(9) $$A_{3\bullet 4}(p) = \left(\left(T_4 p^4\right) \pm \left(T_3 p^3\right)\right)^{-1},$$

(10) $$A_{4\bullet 1}(p) = \left(\left(T_4 p^4\right) \pm \left(T_1 p^1\right)\right)^{-1},$$

(11) $$A_{1\bullet 2\bullet 3}(p) = \left(\left(T_3 p^3\right) + \left(T_2 p^2\right) + \left(T_1 p\right)\right)^{-1},$$

(12) $$A_{2\bullet 3\bullet 4}(p)=\left(\left(T_4 p^4\right)+\left(T_3 p^3\right)+\left(T_2 p^2\right)\right)^{-1},$$

(13) $$A_{1\bullet 2\bullet 3\bullet 4}(p)=\left(\left(T_4 p^4\right)+\left(T_3 p^3\right)+\left(T_2 p^2\right)+\left(T_1 p\right)\right)^{-1},$$

(14) $$A_\alpha=\left(p^\alpha\right)^{-1} \underset{\omega\in\left[\omega_b,\ \omega_h\right]}{\widehat{=}} \left(D^\alpha_{\mathrm{app}\left(\omega_{u,\alpha}\right)}(p)\right)^{-1} \equiv \left(\prod_{i=1}^{L}\left(1+\frac{p}{\omega_{\alpha i}{}'}\right)\left(1+\frac{p}{\omega_{\alpha i}}\right)^{-1}\right)^{-1},$$

(15) $$A_{\alpha\bullet\beta} = \left(p^\alpha+p^\beta\right)^{-1} \underset{\omega\in\left[\omega_b,\ \omega_h\right]}{\widehat{=}} \left(D^\alpha_{\mathrm{app},\ \omega_{u,\alpha}}(p)+D^\beta_{\mathrm{app},\ \omega_{u,\beta}}(p)\right)^{-1} \equiv$$
$$\equiv\left(\prod_{i=1}^{L}\left(1+\frac{p}{\omega_{\alpha i}{}'}\right)\left(1+\frac{p}{\omega_{\alpha i}}\right)^{-1}+\prod_{j=1}^{M}\left(1+\frac{p}{\omega_{\beta j}{}'}\right)\left(1+\frac{p}{\omega_{\beta j}}\right)^{-1}\right)^{-1},$$

(16) $$A_{\alpha\bullet\beta\bullet\gamma}=\left(p^\alpha+p^\beta+p^\gamma\right)^{-1} \underset{\omega\in\left[\omega_b,\ \omega_h\right]}{\widehat{=}}$$
$$\underset{\omega\in\left[\omega_b,\ \omega_h\right]}{\widehat{=}}\left(D^\alpha_{\mathrm{app},\ \omega_{u,\alpha}}(p)+D^\beta_{\mathrm{app},\ \omega_{u,\beta}}(p)+D^\gamma_{\mathrm{app},\ \omega_{u,\gamma}}(p)\right)^{-1} \equiv$$
$$\equiv\left(\prod_{i=1}^{L}\left(1+\frac{p}{\omega_{\alpha i}{}'}\right)\left(1+\frac{p}{\omega_{\alpha i}}\right)^{-1}+\prod_{j=1}^{M}\left(1+\frac{p}{\omega_{\beta j}{}'}\right)\left(1+\frac{p}{\omega_{\beta j}}\right)^{-1}+\right.$$
$$\left.+\prod_{k=1}^{N}\left(1+\frac{p}{\omega_{\gamma k}{}'}\right)\left(1+\frac{p}{\omega_{\gamma k}}\right)^{-1}\right)^{-1},$$

(17) $$A_{\alpha\bullet\beta\bullet\gamma\bullet\delta}=\left(p^\alpha+p^\beta+p^\gamma+p^\delta\right)^{-1} \underset{\omega\in\left[\omega_b,\ \omega_h\right]}{\widehat{=}}$$
$$\underset{\omega\in\left[\omega_b,\ \omega_h\right]}{\widehat{=}}\left(D^\alpha_{\mathrm{app},\ \omega_{u,\alpha}}(p)+D^\beta_{\mathrm{app},\ \omega_{u,\beta}}(p)+D^\gamma_{\mathrm{app},\ \omega_{u,\gamma}}(p)+D^\delta_{\mathrm{app},\ \omega_{u,\delta}}(p)\right)^{-1} \equiv$$
$$\equiv\left(\prod_{i=1}^{L}\left(1+\frac{p}{\omega_{\alpha i}{}'}\right)\left(1+\frac{p}{\omega_{\alpha i}}\right)^{-1}+\prod_{j=1}^{M}\left(1+\frac{p}{\omega_{\beta j}{}'}\right)\left(1+\frac{p}{\omega_{\beta j}}\right)^{-1}+\right.$$
$$\left.+\prod_{k=1}^{N}\left(1+\frac{p}{\omega_{\gamma k}{}'}\right)\left(1+\frac{p}{\omega_{\gamma k}}\right)^{-1}+\prod_{l=1}^{Q}\left(1+\frac{p}{\omega_{\delta l}{}'}\right)\left(1+\frac{p}{\omega_{\delta l}}\right)^{-1}\right)^{-1},$$

$$\{1<\alpha<2\};\ \{1<\beta<2\};$$
$$\{2<\gamma<3\};\{3<\delta<4\};$$
(18) $$(\omega_{\alpha i}<\omega_{\alpha i}';\ \omega_{\beta j}<\omega_{\beta j}';\ \omega_{\gamma k}<\omega_{\gamma k}';\ \omega_{\delta l}<\omega_{\delta l}');$$
$$L=\{1,2,...\};\ M=\{1,2,...\};$$
$$N=\{1,2,...\};\ Q=\{1,2,...\},$$

(19) $$M_{L,1}(p)=\left(2-\prod_{q=2}^{n}\left(e^{-pT_p}\right)_q\right)^{-1}=\left(2-e^{-pqT_p}\right)^{-1},$$

(20) $$M_{L,2}(p)=\left(2-\sum_{k=1}^{m}W_k(p)\,e^{-pkT_p}\right)^{-1},$$

(21) $$M_{L,3}(p)=\left(2-\sum_{k=1}^{m}W_k(p)\prod_{q=2}^{n}\left(e^{-pT_p}\right)_q\right)^{-1}=\left(2-\sum_{k=1}^{m}W_k(p)\,e^{-pqT_p}\right)^{-1},$$

(22) $$M_{L,4}(p)=\left(2-\left(W_0(p)e^{-pT_p}+W_1(p)\prod_{q=2}^{n}\left(e^{-pT_p}\right)_q\right)\right)^{-1},\ (W_0(p)\equiv 1),$$

(23) $$M_{L,5}(p)=\left(2-\left(W_0(p)\,e^{-pT_p}+\sum_{k=1}^{m}W_k(p)\prod_{q=2}^{n}\left(e^{-pT_p}\right)_q\right)\right)^{-1},(W_0(p)\equiv 1),$$

(24) $$\sum_{k=1}^{m}\left|W_k(j\omega)\right|\equiv 1,\left(W_k(j\omega)=\kappa_k\,(j\,\omega\,T_k+1)^{-1}\right),$$

(25) $$A_i\,ML(p)=\left(\sum_{i=1}^{M}\left(T_i\,p^i\right)\right)^{-1}\left(2-\sum_{k=1}^{m}W_k(p)e^{-pkT_p}\right)^{-1}=\varepsilon^{\Diamond}(p)/\varepsilon(p),$$

(26) $$A_r\,ML(p)=f_r\left({}_aD^r(p)\right)^{-1}\left(2-\sum_{k=1}^{m}W_k(p)e^{-pkT_p}\right)^{-1}=\varepsilon^{\Diamond}(p)/\varepsilon(p),$$

(27) $${}_aI_t^{\alpha}\ f(t)=\frac{1}{\Gamma(-\alpha)}\int_a^t\frac{f(\tau)}{(t-\tau)^{\alpha+1}}d\tau,\ (\alpha,a\in\Re,0<\alpha<1),$$

(28) $${}_aD_t^{\beta}f(t)=\frac{1}{\Gamma(n-\beta)}\frac{d^n}{dt^n}\int_a^t\frac{f(\tau)}{(t-\tau)^{\beta-n+1}}\,d\tau,\ (n-1<\beta<n),$$

(29) $$\Gamma(x)=(x-1)\int_0^{\infty}t^{x-2}\,e^{-t}\,dt=(x-1)\Gamma(x-1),$$

(30) $$ {}_aI^{\alpha}(p) \underset{\omega\in[\omega_b,\,\omega_h]}{\hat{=}} {}_aI^{\alpha}_{\text{app}}(p) \equiv \prod_{i=1}^{N}\left(1+\frac{p}{\omega_i{}'}\right)\left(1+\frac{p}{\omega_i}\right)^{-1}; $$

$$ \left(\omega_i < \omega_i{}';\, N=\{1,2,\ldots\};\, n-1<\alpha<n\right), $$

(31) $$ D^{\alpha}(p) \underset{\omega\in[\omega_b,\,\omega_h]}{\hat{=}} D^{\alpha}_{\text{app}}(p) \equiv \prod_{i=1}^{N}\left(1+\frac{p}{\omega_i{}'}\right)\left(1+\frac{p}{\omega_i}\right)^{-1}; $$

$$ \left(\omega_i < \omega_i{}';\, N=\{1,2,\ldots\};\; n-1<\alpha<n\right). $$

Table 1

$\alpha_{\blacksquare}$	$\arg D^{\alpha\blacklozenge}$	$\alpha_{\blacksquare}$	$\arg D^{\alpha\blacklozenge}$	$\alpha_{\blacksquare}$	$\arg D^{\alpha\blacklozenge}$	$\alpha_{\blacksquare}$	$\arg D^{\alpha\blacklozenge}$
0.11155	10°	1.11155	100°	2.11150	190°	3.11150	280°
0.22225	20°	1.22255	110°	2.22222	200°	3.22250	290°
0.33355	30°	1.33355	120°	2.33333	210°	3.33350	300°
0.44455	40°	1.44445	130°	2.44454	220°	3.44450	310°
0.55555	50°	1.55555	140°	2.55555	230°	3.55550	320°
0.67555	60°	1.66755	150°	2.66750	240°	3.66750	330°
0.77835	70°	1.86755	160°	2.77850	250°	3.77850	340°
0.88955	80°	1.88955	170°	2.88930	260°	3.88950	350°

Their models are based on frequency-restricted rational approximations (30)-(31) of the fractal integration and differentiation operators (27)-(29), used in (14)-(18). The characteristics are visualized in parametrical dependence by the current $\alpha, \beta, \gamma, \delta$ fractional order of the members (14)-(18) in the absorptive component in $A_i\,\mathrm{ML}$ -filters (26) by the corresponding indicative combinations of the single frequency value ω_u and cutoff frequency value ω_c and their ratios of rational approximations for the frequency range from 0.1111 to 3.8895, as shown in Table 1.

5. Comparative analysis and conclusion

From the visualization of the characteristics it is obvious that: ● $\mathrm{A}_r\,\mathrm{ML}$ -absorptive repetitive memory loop filters of fractal order (14)-(18), (26) are significantly different in their frequency characteristics than $A_i\,\mathrm{ML}$ -filters, and this is most visible in ***Nyquist***-space**;** ●control systems absorbing disturbances (Fig. 1d) with $\mathrm{A}_r\,\mathrm{ML}$ (26) in their structure, performed absorption of $v = [\,\upsilon \;\; \varsigma \;\; \ldots \;\; f\,]^{\mathrm{T}}$ and filtration of $\vartheta_p = [\,f,\, \upsilon\,]^{\mathrm{T}}$ over y efficiency in the ***Nyquist***-space, the coordinate system in all its quadrants with some parametric sensitivity**:** ***the values*** of the fractal order of the used fractal operators (α or β, γ, δ)**,** ***the ratio of the values of the fractal order*** of the used fractal operators in the polynomial absorbers A_r ($\alpha, \beta, \gamma, \delta$)***, the number of the members in*** $A_r ML$ with integration of non-integer order: monomial (with an index α), binominal (with an index $\alpha \bullet \beta$), trinomial (with an index $\alpha \bullet \beta \bullet \gamma$), quadrinomial (with an index $\alpha \bullet \beta \bullet \gamma \bullet \delta$)***, the ratios of the values of the single frequencies*** of the approximations in polynomial absorbers $A_r ML$ $-$ $\omega_{u,\alpha}$, $\omega_{u,\beta}$, $\omega_{u,\gamma}$, $\omega_{u,\delta}$; ***the ratio of the values of fractal order*** of the used fractal operators in the polynomial filters $A_r ML$ ($\alpha, \beta, \gamma, \delta$); ***the number of the members*** in $A_r ML$; ***the ratio of the values of the single frequencies*** of the approximation in the polynomial absorbers $A_r ML$ **.**

The new and original aspect in the current paper is**:** ● a synthesized new class of absorptive repetitive $A_i ML$ $A_r ML$ -memory loop filters (from integer or non-integer fractal order)**;** ● a new class of control systems is designed, which combine absorption of the impact integral disturbance $v = [\ \upsilon \quad \varsigma \quad ... \quad f\ ]^{\mathrm{T}}$ and the efficiently filter the influence of the periodic $\vartheta_p = [f, \upsilon]^{\mathrm{T}}$ disturbances over the tuning parameter y of the system with the help of $A\mathrm{ML}, \left(\mathrm{AML} \right)$-filters**.** Dynamics is proposed and analyzed in this paper, a new class (25), (26) of absorptive repetitive $A\mathrm{ML}, \left(\mathrm{AML} \right)$ memory loop filters (from integer and non-integer order), functionally satisfactory to them. In comparison, $A_r ML$ -filters provide a significantly better ability in absorbing the disturbance $v = [\ \upsilon \quad \varsigma \quad ... \quad f\ ]^{\mathrm{T}}$ and filtration of $\vartheta_p = [\ f,\ \upsilon\ ]^{\mathrm{T}}$ over y, which is expressed not only in the space of the individual quadrants in the ***Nyquist***-space, but also in its four quadrants, and in the larger number of degrees of freedom for synthesis parameterization and tuning.

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Appendix 1

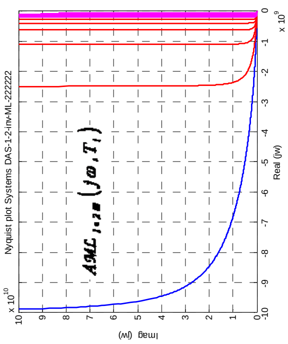
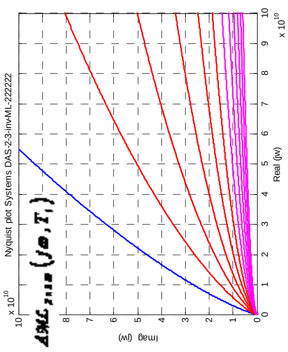


Fig.4



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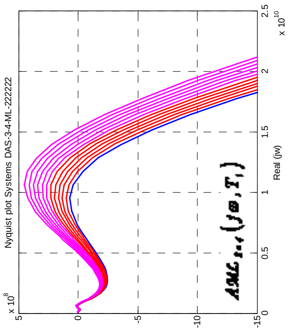

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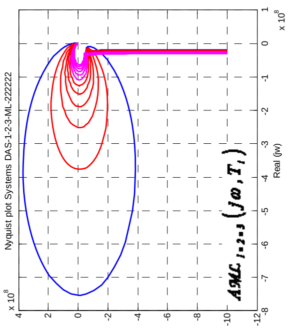

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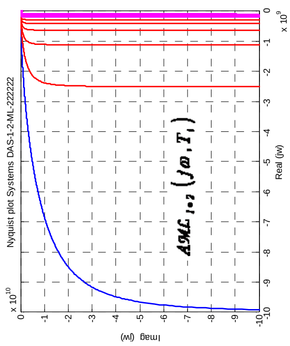

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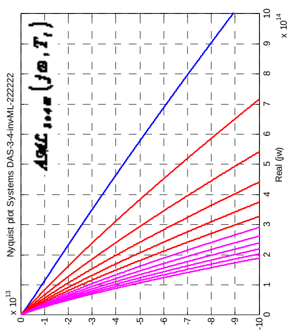

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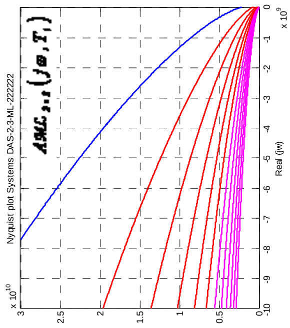

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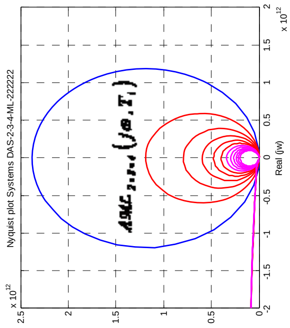

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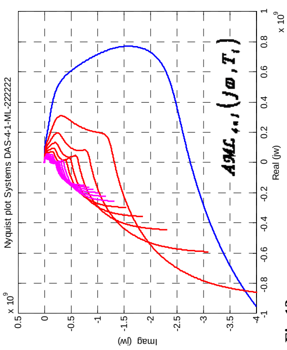

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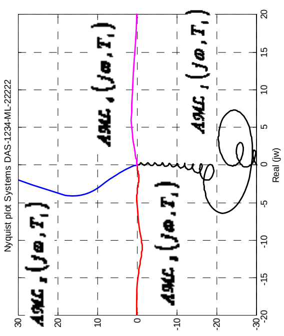

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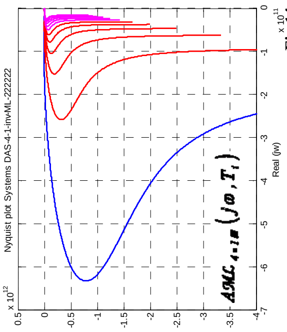

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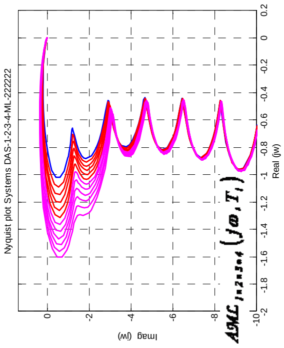

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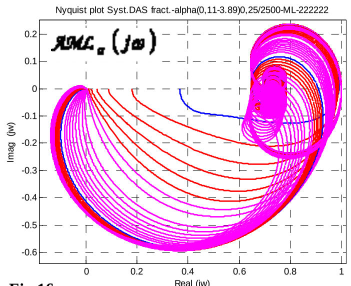

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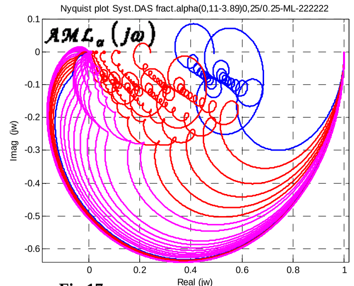

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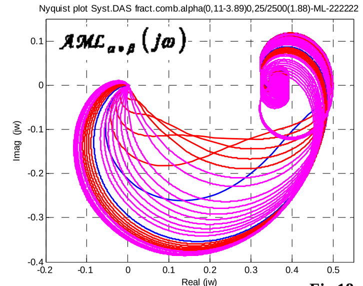

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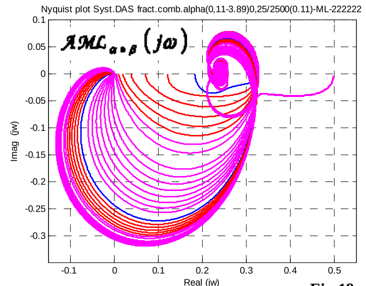

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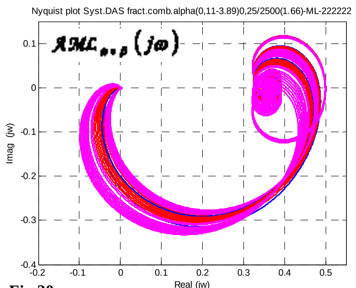

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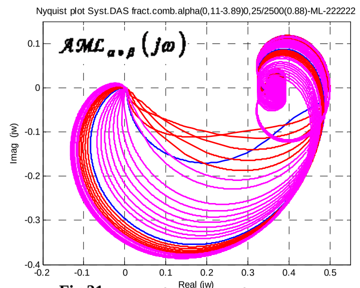

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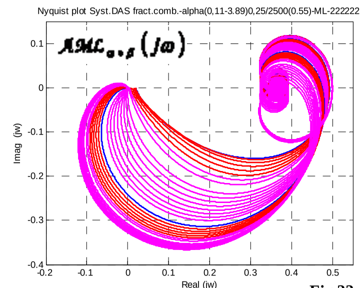

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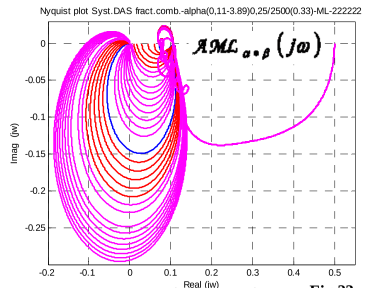

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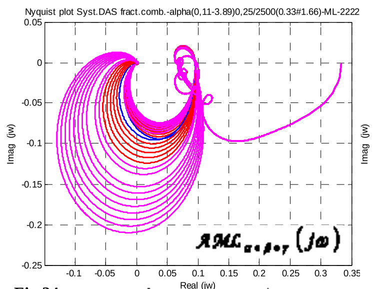

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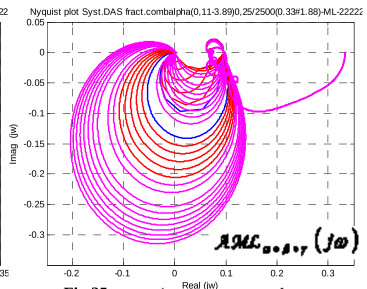

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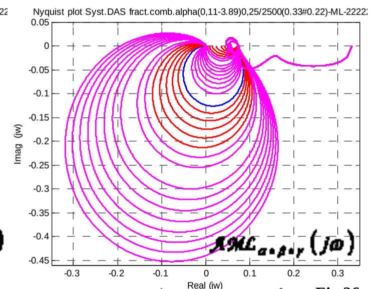

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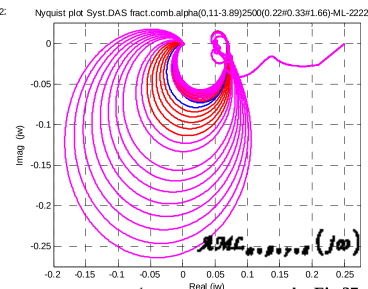

Fig.27