

Algebraic Methods for Traffic Flow Densities Estimation

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Abstract: *An estimation approach that allows recovering of the traffic state is proposed in this paper. The method used is based on numerical differentiation, which does not need any integration of differential equations and turns out to be quite robust with respect to perturbations and measurements noises. Numerical simulations, carried-out by using the so-called Cell Transmission Model (CTM) demonstrate the relevance of the proposed on-line estimation scheme.*

Keywords: *Traffic flow, cell transmission model, algebraic methods, state estimation.*

1. Introduction

Dynamic traffic management and control systems are becoming the most common solutions to alleviate the daily problem of congestion. Indeed, several studies have confirmed the efficiency of such systems in improving traffic flow networks and ensuring safe displacement of goods and people. They also contribute to pollution reduction. Traffic control in freeway networks consists of the use of several actions and measurements, such as dynamic speed limits, route guidance, ramp metering [16, 13].

Most of these strategies are based on the use of a macroscopic model. Nevertheless, the performance of any strategies requires accurate information about the traffic state¹. Such information is often provided by a set of loop detectors. As stated in [14], the loop detector data is frequently incomplete or contains bad samples. In most cases, the available sensors are faulty. Therefore, the use of online state estimation schemes becomes necessary in order to provide the whole traffic information needed. It is important to underline that the works devoted to traffic state estimation are few. In the frame of surveillance systems, [15] and [23] have developed a method based on time series of speed and flow data from a set of sensors in order to generate estimates of vehicles accounts. These crude estimates are then filtered using a Kalman filter. This method has two major drawbacks. Firstly, it requires large data storage and becomes then impracticable. Secondly, the use of a Kalman filter is mostly adapted for linear models, while the measurements are a nonlinear function of the state variables. [18] and [19] have considered the problem of processing data at a fixed spatial location to produce estimates of spatial average quantities. As stated in [21], with good initial conditions, Nahi's method shows the ability to estimate the density closely in homogeneous situations [20]. In [14], an estimation scheme was proposed based on a nonlinear switching model. In [15], Muñoz et al. have used a semi-automated method based on the least-squares techniques. In [22] Khan has introduced a robust sliding mode observer in order to predict the freeway traffic states, such as traffic density and velocity. Such a method, though robust, suffers of the chattering phenomena.

Notice that the widely used traffic state estimations methods are stemming from Kalman filtering techniques. In this context, [23] for example, has proposed an Extended Kalman filter using second order models (see also [24] for a general approach to real-time freeway estimation of both state and parametric estimations). See also, e.g. [16] as a comparison study of several filters configurations for freeway traffic state estimation). However, although such methods are more adapted when the set of the measurement and model uncertainties are assumed to be white noise with normal distributions, in practice this assumption is valid only for traffic measurements, and the uncertainties involved in the model equations, such as disturbances and modeling errors, cannot be simply regarded as white noise. Moreover, several difficulties are still persistent with respect to tuning (gain schedule), numerical analysis (Riccati's equation where the precise statistics of the noise has to be quite accurately known), and sensitivity to perturbations. Other works given in [25] have proposed an estimation scheme based on a particle filter. Such a method was formulated within a Bayesian recursive framework where the traffic state is modeled as a hybrid stochastic system (see, e.g., [7]).

The main objective of this paper is to deal with traffic state estimation using the recent advances on the algebraic methods in order to provide a real-time state estimation of the traffic density of freeway sections. Such an estimation method, which is of algebraic character, was developed by M. Fliess et al. (see [9]), and it differs from other approaches of this type because it involves the use of differential

¹ Traffic density (or occupancy) represents the basic state variable of the most available traffic flow models.

geometry. As mentioned in [4], the method is applied to obtain an estimate of the derivative from any signal, thus avoiding reliance on the system model at least in the estimation of states². The proposed approach assumes that the so-called Cell Transmission Model (CTM), which was developed by Daganzo [1], models the studied system.

The paper is organized as follows. Section 2 describes the principle of CTM. Section 3 recalls a short background of the proposed algebraic method and its application for density estimation. Some numerical simulations are provided in Section 4. Finally, the last section presents some conclusions and next step investigations.

2. Cell Transmission Model principles

Macroscopic traffic flow models are based on the conservation of the vehicles law that reads:

$$(1) \quad \frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0$$

where x is the longitudinal position along the freeway, t is the time; $\rho(x, t)$ and $q(x, t)$ are respectively, the vehicle density at position x and time t , and the traffic flow in vehicles per hour (veh/h).

In this paper the macroscopic cell-transmission traffic model was selected due to its analytical simplicity and ability to reproduce important traffic behavioral phenomena, such as the backward propagation of a congestion wave. Nevertheless, while CTM uses the cell occupancy, we use for simplicity, the cell density as state variables and acceptations, following [14] for nonuniform cell lengths. We also consider space discrete modeling.

Consider the following simple section depicted in Fig. 1.

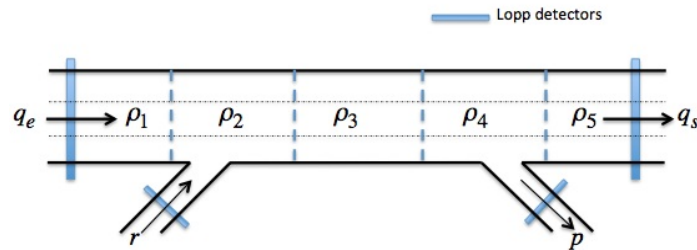


Fig. 1. Example of a freeway section

For space discrete representation, define the traffic density ρ_i as the number of vehicles in segment i at time t , divided by the segment length L_i . The traffic volume q_{i-1} in veh/h is defined as the number of vehicles entering segment i ; q_i is the number of vehicles leaving segment i .

² It should be emphasized that the proposed method does not use any probabilistic or optimization techniques [8].

The equation of the nonlinear model for each freeway segment is as follows:

$$(2) \quad \dot{\rho}_i(t) = \frac{1}{L_i} (q_{i-1}(t) - q_i(t) + a_i r_i(t) - b_i p_i(t))$$

where a_i and b_i are binary variables which indicate respectively the presence or the absence of an on-ramp $r_i(t)$ and an off-ramp $p_i(t)$. The model parameters include v , w , Q_{\max} , and ρ_j , which are depicted in the following fundamental diagram (Fig. 2).

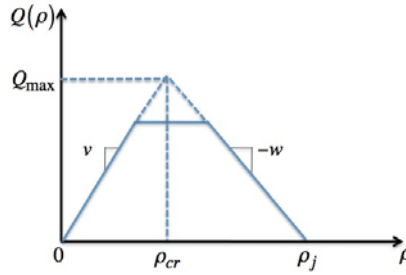


Fig.2. Fundamental diagram

They can be uniform over all cells or allowed to vary from a cell to a cell. The free-flow speed v is the average speed, at which the vehicles travel down the highway under uncongested (low density) conditions. w is the average speed at which congestion waves propagate upstream within congested (high density) regions of the highway. Q_{\max} is the maximum flow rate and ρ_j is the jam density. ρ_c , the critical density, is the density at which the free-flow curve $Q(\rho) = v\rho$ intersects the congestion curve $Q(\rho) = w(\rho_j - \rho)$. The congestion status of cell i is determined by comparing the cell density with critical density: if $\rho_i < \rho_{c,i}$ the cell has a free-flow status, otherwise $\rho_i \geq \rho_{c,i}$ and the cell is said to have a congested status. Three different types of intercell connections are allowed: simple connection, merge, and diverge. In this paper we will focus on the first one. As described in [1] $q_i(t)$, the flow entering cell i from the mainline is determined by taking the minimum of two quantities:

$$(3) \quad q_i(t) = \min(S_{i-1}(t), R_i(t)),$$

where $S_{i-1}(t) = \min(\rho_{i-1}v_{i-1}, Q_{\max,i-1})$, is the maximum flow that can be supplied by cell $i-1$ under free-flow conditions. $R_i(t) = \min((w_i\rho_{j,i} - \rho_i), Q_{\max,i})$, is the maximum flow that can be received by cell i under congested conditions.

According to the status of each cell, several modes can be depicted, which leads to the so-called Switching-Mode Model (SMM) [15]. Indeed, SMM is a hybrid model which switches between a set of linear differential equations, depending on the congestion status of the cells and the values of the mainline

boundary data (see, e.g. [15] for more explanations and discussions about the SMM). As stated in [15], the SMM is a modified version of CTM and can be obtained by expressing the inter-cellular q_i , as an explicit function of cell density, $q_i = v\rho_{i-1}$ or $q_i = w(\rho_j - \rho_i)$, or as a constant, $q_i = Q_{\max}$. According to these expressions, a cell i is called “congested-C”, if it is not able to accept the flow delivered by its upstream neighboring cell $i-1$, otherwise it is considered as Free-F.

In the general case, the densities vector gives the state of the system: $\rho = [\rho_1, \dots, \rho_N]$. Notice that the loop detectors, located upstream and downstream of the studied freeway segment give the measurements of the traffic flow denoted as q_e and q_s , respectively.

In the sequel, we consider that two adjacent cells can be in one of the following modes: Free-Flow (**FF**), Congested-Free (**FC**), and Congested-Congested (**CC**). The main objective then is to design an estimator for these different situations using the new algebraic methods of identification. The following section recalls a short background of this approach.

3. Estimation method

3.1. Background on numerical differentiation

Numerical differentiation is based on the algebraic setting started by F l i e s s [7], F l i e s s et al. [3-6] and provides a powerful tool for the estimation of derivatives of a noisy signal. In this section we just sketch the principle of the method, for more details and interesting discussions and comparisons the reader might refer to [8, 12].

Consider a real-valued signal $y(t) = \sum_{i=0}^{\infty} y^{(i)}(0) \frac{t^i}{i!}$, which, for sake of simplicity, is assumed to be analytic around $t=0$ and introduce its truncated Taylor expansion:

$$(4) \quad y(t) = \sum_{i=0}^N y^{(i)}(0) \frac{t^i}{i!} + O(t^{N+1}).$$

Approximate $y(t)$ in the interval $(0, \varepsilon)$, $0 < \varepsilon \leq \beta$ by its truncated Taylor expansion

$$y_N(t) = \sum_{i=0}^N y^{(i)}(0) \frac{t^i}{i!},$$

of degree N . The usual rules of symbolic calculus in Schwartz’s distributions theory yield

$$y_N^{(N+1)}(t) = y(0) \delta^N + \dots + y^{(N)} \delta,$$

where δ is the Dirac measure at 0. Multiply both sides by $(-t)^i$ and using the rules $t\delta=0, t\delta^{(i)}=-i\delta^{(i-1)}, i \geq 1$ leads to a triangular system of linear equations from which derivatives $y^{(i)}(0)$ can be obtained ($1 \leq i \leq N$),

$$(5) \quad (-t)^i y_N^{(N+1)}(t) = \frac{N}{(N-i)!} \delta^{(N-1)} y(0) + \dots + \delta y^{(N-i)}(0).$$

It means that those quantities are *linearly identifiable* [2]. The time derivative of $y_N(t)$, the Dirac measures and its derivatives are removed by integrating with respect to time both sides of equation (5) at least ν times ($\nu > N$):

$$\begin{aligned} & \int_0^t \int_0^{t_{v-1}} \dots \int_0^t (-\tau)^i y_N^{(N+1)} d\tau_{v-1} \dots dt_1 d\tau = \\ & = \frac{N}{(N-i)!} \frac{t^{v-N-i-1}}{(v-N-i-1)!} y(0) + \dots + \frac{t^{v-1}}{(v-1)!} y^{(N-i)}(0). \end{aligned}$$

Remark 1. These iterated integrals are low pass filters, which attenuate the noises, which are viewed as highly fluctuating or oscillatory phenomena (see [7] for more details).

Remark 2. An excellent estimated value, which is derived via iterated time integrals, may be obtained by utilizing quite short time windows. The above formulae may easily be extended to sliding time windows in order to obtain real times estimates [12].

Remark 3. The same calculations can be achieved using the operational calculus (see, e.g. [8, 17]).

3.2. Traffic densities estimation in a **FF** mode

In the case of a Free-Flow mode, all the cells $i \in \{2, \dots, N\}$, are able to accept the traffic flow coming from its upstream neighboring cell.

As demonstrated in [15], the observability results were derived using standard linear systems techniques. In the case of **FF** mode, the studied freeway segment is observable from the downstream measurements of q_s .

Consider for simplicity's sake, the following freeway section with three cells (Fig. 3). In the free-flow mode, the density evolution is³

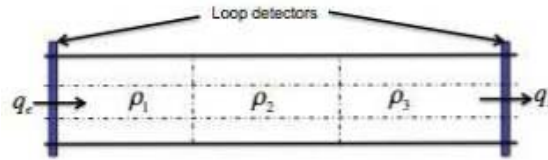


Fig. 3. A freeway section divided into three cells

³ We consider in this paper that all cells are with the same length L .

$$(6) \quad \begin{cases} L \dot{\rho}_1(t) = q_e - \rho_1 v_1, \\ L \dot{\rho}_2(t) = \rho_1 v_1 - \rho_2 v_2, \\ L \dot{\rho}_3(t) = \rho_2 v_2 - \rho_3 v_3. \end{cases}$$

Let $q_s = v_3 \rho_3$ and assume that $y(t) = \rho_3(t)$ is an output variable. Some simple manipulations allow us to express the missing densities values (ρ_2 and ρ_1) as a function of y , \dot{y} , and \ddot{y} ,

$$(7) \quad \begin{cases} \rho_3(t) = y, \\ \rho_2(t) = \frac{L}{v_2} \dot{y} + \frac{v_3}{v_2} y, \\ \rho_1(t) = \frac{L^2}{v_1 v_2} \ddot{y} + \frac{L}{v_1} \left(\frac{v_3}{v_2} + 1 \right) \dot{y} + \frac{v_3}{v_1} y. \end{cases}$$

3.3. Traffic densities estimation in **CF** mode

In this case all the cells $i \in \{2, \dots, N\}$ are not able to accept the traffic flow coming from its upstream neighboring cell. Nevertheless, the computation of the observability matrix allows concluding that the freeway section is observable from the downstream measurements of flow q_e . We assume then, that the measured variable is $y = \rho_1$. The traffic model in the **CC** mode is given as:

$$(8) \quad \begin{cases} L \dot{\rho}_1(t) = w_1 (\rho_{j,1} - \rho_1) - w_2 (\rho_{j,2} - \rho_2), \\ L \dot{\rho}_2(t) = w_2 (\rho_{j,2} - \rho_2) - w_3 (\rho_{j,3} - \rho_3), \\ L \dot{\rho}_3(t) = w_3 (\rho_{j,3} - \rho_3) - q_s. \end{cases}$$

where w_i , $i = 1, 2, 3$, represent the wave speeds.

The same calculations performed above allow us to express the variables ρ_2 and ρ_1 as follows:

$$(9) \quad \begin{cases} \rho_1(t) = y, \\ \rho_2(t) = \frac{L}{w_2} \dot{y} + \frac{w_1}{w_2} y - \frac{w_1}{w_2} \rho_{j,1} + \rho_{j,2}, \\ \rho_3(t) = \frac{L^2}{w_2 w_3} \ddot{y} + \frac{L}{w_3} \left(\frac{1}{w_3} + 1 \right) \dot{y} + \frac{w_1}{w_3} y - \frac{w_1}{w_3} \rho_{j,1} + \rho_{j,3}. \end{cases}$$

3.4. Traffic densities estimation in **CF** mode

We assume that the first segment is congested, while the second and the third are free. The traffic model describing this situation is:

$$(10) \quad \begin{cases} L \dot{\rho}_1(t) = w_1(\rho_{j,1} - \rho_1) - w_2(\rho_{j,2} - \rho_2), \\ L \dot{\rho}_2(t) = v_1 \rho_1 - v_2 \rho_2, \\ L \dot{\rho}_3(t) = v_3 \rho_2 - q_s. \end{cases}$$

The observability matrix allows confirming that in **CF** mode it is observable from both upstream and downstream measurements. Assuming that $y = \rho_3$, we obtain the same results as in equation (7).

4. Time derivative estimation

For the generation of time derivatives of the measured outputs y , consider a 4th order approximation of a smooth signal $y(t)$. Thus, it is not necessary to design the derivative estimator from a specific dynamic model of the traffic flow.

$$(11) \quad \frac{d^4 y(t)}{dt^4} = 0.$$

Rewriting expression (11) in the operational domain, we get [17]

$$(12) \quad s^4 y(s) - s^3 y(0) - s^2 \dot{y}(0) - s \ddot{y}(0) - y(0) = 0.$$

In order to eliminate the initial conditions, we take successive derivatives, with respect to the operational variable s , until the number of three is obtained:

$$(13) \quad \frac{d^4 (s^4 y)}{ds^4} = 0,$$

$$(14) \quad s^4 \frac{d^4 y}{ds^4} + 16s^3 \frac{d^3 y}{ds^3} + 72s^2 \frac{d^2 y}{ds^2} + 96s \frac{dy}{ds} + 24y(s) = 0.$$

Multiplying the above equation (14) by s^{-3} yields:

$$(15) \quad s \frac{d^4 y}{ds^4} + 16 \frac{d^3 y}{ds^3} + 72s^{-1} \frac{d^2 y}{ds^2} + 96s^{-2} \frac{dy}{ds} + 24s^{-3} y(s) = 0.$$

According to the equivalence:

$$s^{-1} \rightarrow \int_0^t ,$$

$$s^{-n} \rightarrow \int_0^{(n)} : \text{Multiple integrals.}$$

Derivative said ‘‘algebraic’’:

$$\frac{d}{ds} \rightarrow -t,$$

$$\frac{d^n}{d s^n} \rightarrow (-1)^n t^n \quad \text{and} \quad s^n = \frac{d^n}{d t^n}.$$

Equations (15) can be expressed in the time domain as follows:

$$(16) \quad \frac{d}{dt}(t^4 y(t)) - 16t^3 y(t) + 72\left(\int t^2 y(t)\right) - 96\left(\int^{(2)} t y(t)\right) + 24\left(\int^{(3)} y(t)\right) = 0.$$

For simplicity we have used here the following notation:

$$(17) \quad \left(\int^{(j)} t^k y\right) = \int_0^t \int_0^{\sigma_1} \dots \int_0^{\sigma_{j-1}} \sigma_j^k y(\sigma_j) d\sigma_j \dots d\sigma_1.$$

These expressions yield, after some algebraic manipulations, the approximations of the first and second order time derivatives of $y(t)$:

$$(18) \quad \begin{aligned} & \left[\frac{dy}{dt}\right]_e = \\ & = \frac{\left[-24\left(\int^{(3)} y\right) + 96\left(\int^{(2)} t y\right) - 72\left(\int t^2 y\right) + 12t^3 y\right]}{t^4}, \end{aligned}$$

$$(19) \quad \begin{aligned} & \left[\frac{d^2 y}{dt^2}\right]_e = \\ & = \frac{\left[-24\left(\int^{(2)} y\right) + 96\left(\int t y\right) - 36\left(t^2 y\right) + 8\left(t^3 [y]_e\right)\right]}{t^4}, \end{aligned}$$

where $[\bullet]_e$ is the estimation value. Notice that expression (19) for the second order time derivative estimate requires the outcome of the evaluation of the first derivative estimate. This is in complete agreement with the announced triangular structure of the generating system of equations.

The above formulas are valid for $t > 0$. Since (18) and (19) provide an approximated value of the first and second derivatives, these are only valid during a period of time. Thus the state estimation must be calculated periodically as follows:

$$(20) \quad \begin{aligned} & \left[\frac{dy}{dt}\right]_e = \\ & = \frac{1}{(t-t_i)^4} \left[-24 \left(\int_{t_i}^{(3)} y \right) + 96 \left(\int_{t_i}^{(2)} (t-t_i) y \right) \right] + \\ & \quad + \left[-72 \left(\int (t-t_i)^2 y \right) + 12 (t-t_i)^3 y \right], \end{aligned}$$

$$(21) \quad \left[\frac{d^2 y}{dt^2} \right]_e = \frac{1}{(t-t_i)^4} \left[-24 \left(\int_{t_i}^{(2)} y \right) + 96 \left(\int_{t_i} (t-t_i) y \right) \right] + \left[-36 \left((t-t_i)^2 y \right) + 8 (t-t_i)^3 [\dot{y}]_e \right],$$

where $t - t_i > 0$ is the estimation period.

5. Simulation results

In order to illustrate the relevance of the proposed approach, consider the freeway section depicted in Fig. 3. For the simulation purpose we consider the following traffic demand in vehicles per hours (Fig. 4).

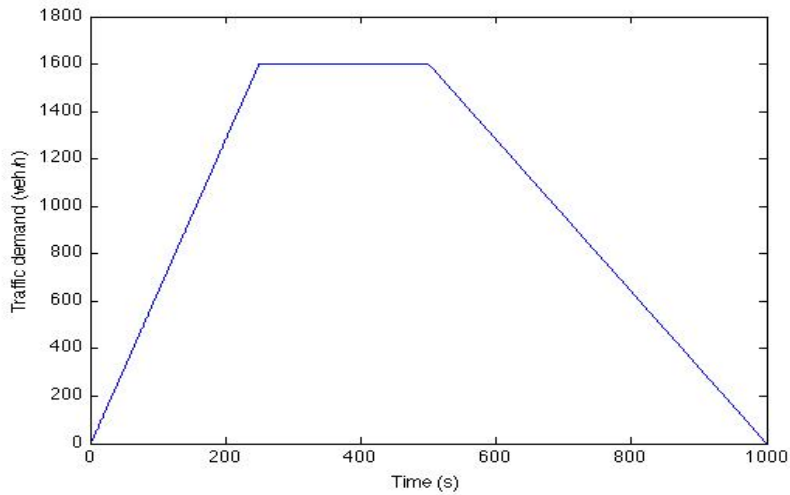


Fig. 4. Traffic demand

Table 1 summarizes the used model parameters.

Table 1. Model parameters

Parameters	Free-flow speed (m/s)	Wave speed (m/s)	Critical density (veh/m)	Jam density (veh/m)
Segment 1	26	6.4	0.05	0.1425
Segment 2	20	5.9	0.05	0.1425
Segment 3	18	6.3	0.05	0.1425

Figs 5 and 6 show the traffic densities time evolution when all the cells are in the free-flow situation. A very short time is needed for the developed estimator in order to reach the simulated traffic densities.

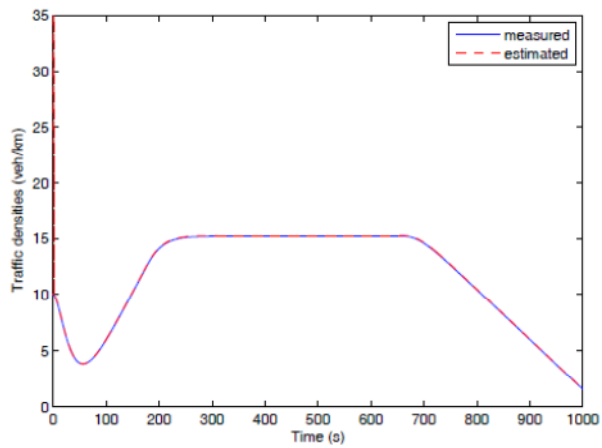


Fig. 5. Traffic densities evolution of the second cell: **FF** mode

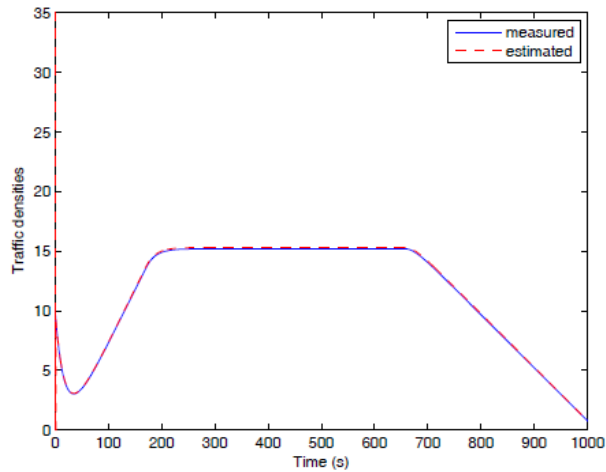


Fig. 6. Traffic densities evolution of the first cell: **FF** mode

Fig. 7 confirms the relevance of the algebraic estimation techniques for both **CC** and **CF** modes.

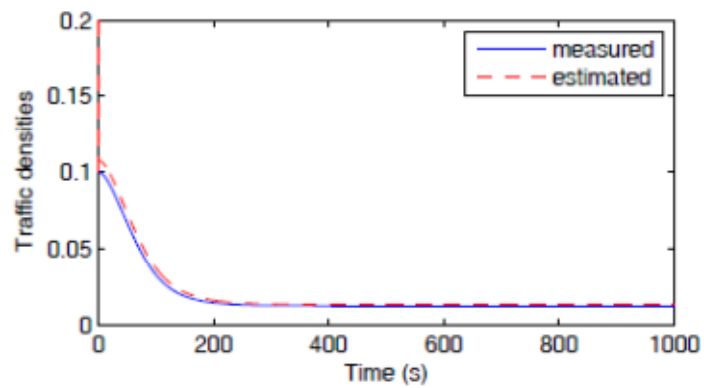


Fig. 7. Traffic densities evolution of the cells: **CC** and **CF** modes

6. Conclusion

The work presented in this paper demonstrates the relevance of the recently introduced identification methods for traffic state estimation. Such techniques, which are of algebraic character, do not use either optimization, nor probabilistic or asymptotic techniques, and lead to robust and fast traffic estimation. The proposed approach was successfully applied for the densities estimation of a freeway section modelled by using the so-called Switching Mode Model (SMM). It allows the estimation of the traffic densities according to the mode of the freeway, i.e., **FF**, **CF** and **CC** modes.

Further works will be focused on a comparative study between the proposed approach and the classical one, and the more frequently used in the traffic area state estimators, such as the Extended Kalman Filter (EKF). Other research works will be realized in order to exploit the approach proposed for dynamic traffic management and control measurements design, such as ramp metering, dynamic speed limits and dynamic traffic routing.

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