

## Discrete Time Sliding Mode Flow Controllers for Connection-Oriented Networks with Lossy Links

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**Abstract:** *In this paper we propose two discrete-time sliding mode flow controllers for multisource connection-oriented networks. Each connection in the considered networks is characterized by a time delay and a packet loss ratio. The proposed controllers take advantage of appropriately designed sliding hyperplanes, which ensure the closed-loop system stability and finite-time error convergence to zero. Moreover, the proposed controllers ensure full utilization of the bottleneck link available bandwidth, guarantee bounded transmission rates, and eliminate the risk of buffer overflow in the bottleneck node. Since the primary controller may lead to excessive values of the transmission rate at the beginning of the control process, the modified controller is designed using the concept of the reaching law, which helps the removing of the undesirable effect.*

**Keywords:** *Congestion control, sliding mode control, discrete time systems.*

### 1. Introduction

In connection-oriented communication networks the data units, sent by sources pass through a series of intermediate nodes before reaching their destinations. If an intermediate node – due to limited data flow rate of its outgoing link – cannot pass on all the data it receives, then a congestion occurs. Consequently, in order to maximize the throughput and minimize the queuing delays and jitter in modern communication networks, congestion control algorithms are applied. The main difficulty in appropriate congestion control algorithm design is caused by the large propagation delays in the networks. The delays are inevitable since information

about the congestion at a specific node must be dispatched to all sources transmitting data through this node, in order to enable adjustment of the source transmission rates, and this action does not affect the congested node immediately, but only with a delay usually called a Round Trip Time (RTT).

The problem of congestion control in the connection-oriented communication networks has been studied in many papers (see for example [2, 11, 15, 16]) and an extensive review of the papers can be found in the recent monograph [10]. Furthermore, due to the robustness of the Sliding Mode (SM) control [17], various types of SM congestion controllers have been proposed. In [13] a SM controller with a state predictor was used, and the maximum delay, necessary for the system stability has been established. A fuzzy controller combining the advantages of a linear and terminal SM was proposed in [14] for a simplified delay-free network model. For a DiffServ network an adaptive SM controller (using the backstepping procedure) for a model which neglects the feedback latency was presented in [22]. On the other hand, for a DiffServ network with a delay, the second order SM technique has been applied in [21] in order to reduce the chattering of the control signal. In [12] the problem of fair (in the max-min sense) data rate distribution among the sources is considered. A binary congestion signal is used to control the data output of the sources and the analysis of this algorithm is performed for a delay-free system. All controllers presented in the aforementioned publications are designed in the continuous time domain. However, it is evident that any flow control algorithm for a data transmission network must be implemented as a digital controller. Therefore, in the following works a discrete-time approach to the problem of the data flow control was used. A SM controller was presented in [18], but the result of this paper was derived without considering the system delays. In [19] it is shown that any max-min fair system with a stable symmetric Jacobian matrix maintains asymptotic stability under arbitrary directional delays. This means that if the controller is designed so that the system has a symmetric Jacobian matrix, its stability can be examined based on the corresponding undelayed system. A dead-beat SM controller for multi-source networks with a priori known round trip times is presented in [3] and in [7] an LQ optimal SM controller for single-source networks is proposed. The same approach is then extended for multi-source networks in [8], and in [9], a similar optimal flow controller is designed for multi-source networks with the round trip times which may change during the control process.

In most papers published up to now, only packet losses due to bottleneck link buffer overflows are considered, and the occurrence of lossy links in the network is neglected. Since in real networks transmission losses are inevitable, in this paper we present discrete-time sliding mode controllers [1, 4, 5, 6, 20] for multisource networks, in which packets are lost during the transmission process.

## 2. Network model

We consider a connection-oriented communication network which consists of persistent data sources, intermediate nodes and data destinations. We assume that

all connections pass through a single bottleneck node. The total amount of data to be sent (represented by  $u$ ) by all of the sources is determined by a controller placed at the bottleneck node. We assume that this amount is distributed among the connections by any higher-level algorithm. Each connection  $p$  receives  $\gamma_p$  of the total rate, where  $\gamma_p \in \langle 0, 1 \rangle$  and  $\sum_{p=1}^m \gamma_p = 1$  where  $m$  is the number of sources. In this way the flow control and the total rate distribution are decoupled. The block diagram of the network is shown in Fig. 1. Each signal  $\gamma_p u$  reaches source  $p$  after a backward delay  $T_{b,p}$ . The source then sends the requested amount of data, which is passed from node to node until it reaches the bottleneck queue after the forward delay  $T_{f,p}$ .

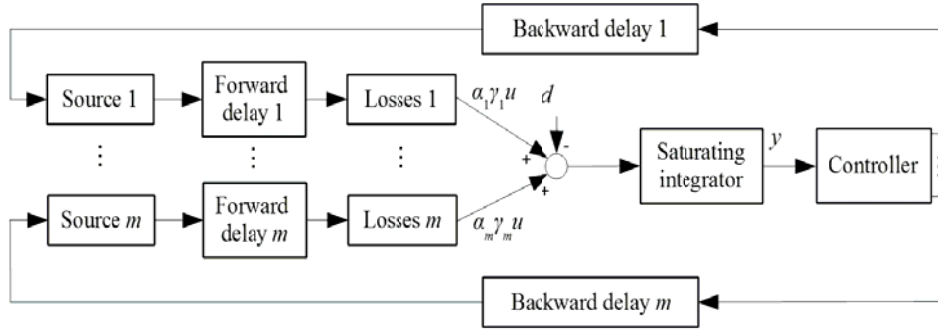


Fig. 1. The network model

The round trip time of each connection – the delay between generating the signal by the controller, and the moment when the requested data from source  $p$  arrives at the bottleneck node, can be expressed as a sum of the backward and forward propagation delays

$$(1) \quad \text{RTT}_p = T_{b,p} + T_{f,p}.$$

It is assumed, that during the transmission some percentage of data packets are lost, so that only  $\alpha_p \gamma_p u$  data packets from the source  $p$  arrive at the buffer, where  $\alpha_p \in (0, 1)$  for  $p = 1, 2, \dots, m$ .

We assume that each  $\text{RTT}_p$  is a multiple of the discretization period  $T$ , i.e.,  $\text{RTT}_p = m_{\text{RTT}_p} T$ . Furthermore,  $y(kT)$  denotes the bottleneck queue length at the time instant  $kT$ . The amount of the data which may leave the bottleneck node at time  $kT$  is modeled as an *a priori* unknown function of time  $d(kT)$ . The maximum value of  $d(kT)$  is represented by  $d_{\max}$ . The amount of data actually leaving the buffer at time  $kT$  is denoted by  $h(kT)$ . For any  $k \geq 0$

$$(2) \quad 0 \leq h(kT) \leq d(kT) \leq d_{\max}.$$

It is assumed that the buffer is empty prior to the data transmission process, i.e.,  $y(kT < 0) = 0$ , and that the first control signal is generated at  $kT = 0$ , i.e.,  $u(kT < 0) = 0$ . The queue length for any  $k \geq 0$  can be calculated from the following expression

$$(3) \quad y(kT) = \sum_{p=1}^m \sum_{j=0}^{k-1} \alpha_p \gamma_p u(jT - \text{RTT}_p) - \sum_{j=0}^{k-1} h(jT).$$

To simplify the model we can represent all connections with equal round trip times as a single connection. The amount of data that will arrive at the buffer from this connection is equal to  $a_i u$ , where  $a_i = \sum_{p: \text{mRTT}_p=i} \alpha_p \gamma_p$ ,  $i = 1, \dots, n-1$ , and

$n = \max(\text{mRTT}_p) + 1$ . Obviously, if no connection has the round trip time  $iT$ , then the corresponding coefficient  $a_i$  equals zero. Now we can rewrite (3) as follows:

$$(4) \quad y(kT) = \sum_{i=1}^{n-1} \sum_{j=0}^{k-1} a_i u[(j-i)T] - \sum_{j=0}^{k-1} h(jT).$$

The network can be described in the state space as

$$(5) \quad \begin{aligned} \mathbf{x}[(k+1)T] &= \mathbf{A}\mathbf{x}(kT) + \mathbf{b}u(kT) + \mathbf{o}h(kT), \\ y(kT) &= \mathbf{q}^\top \mathbf{x}(kT), \end{aligned}$$

where  $\mathbf{x}(kT) = [x_1(kT) \ x_2(kT) \ \dots \ x_n(kT)]^\top$  is the state vector and  $y(kT) = x_1(kT)$  is the queue length. The state variables except  $x_1$  are the delayed values of the control signal, i.e.

$$(6) \quad x_i(kT) = u[(k-n+i-1)T], \quad i = 2, \dots, n.$$

$\mathbf{A}$  is  $n \times n$  state matrix,

$$(7) \quad \mathbf{A} = \begin{bmatrix} 1 & a_{n-1} & a_{n-2} & \dots & a_1 \\ 0 & 0 & 1 & \dots & 0 \\ & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & & 0 \end{bmatrix},$$

and  $\mathbf{b}$ ,  $\mathbf{o}$ , and  $\mathbf{q}$  are  $n \times 1$  vectors

$$(8) \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{o} = \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}.$$

Alternatively, the state space equation can be written as follows:

$$(9) \quad \left\{ \begin{aligned} x_1[(k+1)T] &= x_1(kT) + a_{n-1}x_2(kT) + a_{n-2}x_3(kT) + \dots + a_1x_n(kT) - h(kT) \\ x_2[(k+1)T] &= x_3(kT) \\ &\vdots \\ x_{n-1}[(k+1)T] &= x_n(kT) \\ x_n[(k+1)T] &= u(kT) \end{aligned} \right. ,$$

with the output signal  $y(kT) = x_1(kT)$ . The desired state of the system is denoted by  $\mathbf{x}_d = [x_{d1} \ x_{d2} \ \dots \ x_{dn}]^T$ . The first state variable  $x_{d1}$  is the demand queue length, and further in the paper it is represented by  $x_d$ . It can be noticed from (9) that for  $h(kT) = 0$  all other components of the demand state vector are equal to zero.

### 3. Proposed control strategy

Further in the paper we apply the network model presented in the previous section to design two sliding mode congestion controllers: a classical dead-beat controller and its reaching law based extension.

#### 3.1. Primary sliding mode controller

In this section the flow control algorithms for the described network are designed and essential properties of the algorithms are proved. For the purpose of the controller design we assume  $h(kT) = 0$  and we introduce a sliding hyperplane described by equation

$$(10) \quad s(kT) = \mathbf{c}^T \mathbf{e}(kT) = 0,$$

where vector  $\mathbf{c}^T = [c_1 \ c_2 \ \dots \ c_n]$  satisfies  $\mathbf{c}^T \mathbf{b} \neq 0$ . The closed-loop system error is denoted by  $\mathbf{e}(kT) = \mathbf{x}_d - \mathbf{x}(kT)$ . Substituting (5) into  $\mathbf{c}^T \mathbf{e}[(k+1)T] = 0$  we obtain the following control law

$$(11) \quad u(kT) = (\mathbf{c}^T \mathbf{b})^{-1} \mathbf{c}^T [\mathbf{x}_d - \mathbf{A}\mathbf{x}(kT)],$$

with the application of this control signal, the closed-loop system state matrix has the form  $\mathbf{A}_c = [\mathbf{1} - \mathbf{b}(\mathbf{c}^T \mathbf{b})^{-1} \mathbf{c}^T] \mathbf{A}$ . The characteristic polynomial of this matrix can be found as

$$(12) \quad \det(z\mathbf{I}_n - \mathbf{A}_c) = z^n + \frac{c_1 a_1 + c_{n-1} - c_n}{c_n} z^{n-1} + \dots + \frac{c_1 a_{n-2} + c_2 - c_3}{c_n} z^2 + \frac{c_1 a_{n-1} - c_2}{c_n} z,$$

which leads to the condition  $c_n \neq 0$ . A discrete-time system is asymptotically stable if and only if all its eigenvalues are located inside a unit circle. Furthermore, in order to ensure error convergence to zero in finite time, the characteristic polynomial (12) has to satisfy

$$(13) \quad \det(z\mathbf{I}_n - \mathbf{A}_c) = z^n.$$

One can easily find that (13) is satisfied with the following vector  $\mathbf{c}$

$$(14) \quad \begin{cases} c_1 = 1, \\ c_i = \sum_{j=1}^{i-1} a_{n-j} \quad \text{for } i = 2, \dots, n. \end{cases}$$

Substituting (7) and (8) into (11) we obtain

$$\begin{aligned}
(15) \quad u(kT) &= \frac{x_d - x_1(kT) - \sum_{i=2}^n c_i x_i(kT)}{\sum_{j=1}^{n-1} a_j} = \\
&= \frac{x_d - x_1(kT) - \sum_{i=2}^n c_i u[(k-n+i-1)T]}{\sum_{j=1}^{n-1} a_j}.
\end{aligned}$$

This completes the design of the primary flow control algorithm for the considered network. Further in this section, important properties of the proposed algorithm will be discussed.

Let us first notice that the amount of data to be sent at the initial control instant is

$$(16) \quad u(0) = \frac{x_d}{\sum_{j=1}^{n-1} a_j}.$$

Next we determine the relation between the utilized bandwidth and the transmission rate generated by the controller. This relation is stated in the following lemma.

**Lemma 1.** If the proposed controller is applied, then its output signal for any  $k > 0$  is given by

$$(17) \quad u(kT) = \frac{1}{\sum_{j=1}^{n-1} a_j} h[(k-1)T].$$

*Proof:* From (14), for any  $i = 3, \dots, n$ , we obtain

$$(18) \quad c_i = c_{i-1} + a_{n-i+1}.$$

Then using (18) and (15) we get

$$\begin{aligned}
(19) \quad u[(k+1)T] &= \frac{1}{c_n} \left\{ x_d - x_1[(k+1)T] - \sum_{i=2}^{n-1} c_i x_i[(k+1)T] \right\} - u(kT) = \\
&= \frac{1}{c_n} \left\{ x_d - x_1[(k+1)T] - \sum_{i=2}^{n-1} c_i x_i[(k+1)T] \right\} - \\
&\quad - \frac{1}{c_n} \left[ x_d - x_1(kT) - \sum_{i=2}^n c_i x_i(kT) \right] = \frac{1}{c_n} h(kT) = \frac{1}{\sum_{j=1}^{n-1} a_j} h(kT).
\end{aligned}$$

This equation has been derived assuming  $k \geq 0$ . If we assume  $k > 0$  we can rewrite it to be identical to (17). This ends the proof.

**Remark 1.** Because  $h(kT) \in \langle 0, d_{\max} \rangle$  for any  $k \geq 0$ , and  $\sum_{j=1}^{n-1} a_j \in (0, 1)$  this

lemma with (16) imply that the value of the control signal is always nonnegative and upper-bounded.

Further important properties of the proposed controller are given in the following two theorems.

**Theorem 1.** With the application of the proposed control algorithm the queue length never exceeds its demand value.

*Proof:* Transforming (15) we obtain

$$(20) \quad x_d - x_1(kT) = c_n u(kT) + \sum_{i=2}^n c_i u[(k-n+i-1)T].$$

Since  $c_i \geq 0$  for  $i \in (1, n)$ , using Remark 1 we conclude that the right hand side of (20) is nonnegative, which implies that  $x_1(kT) \leq x_d$  for any  $k \geq 0$ . This ends the proof.

**Theorem 2.** If the proposed controller is applied and the demand queue length satisfies inequality

$$(21) \quad x_d > \frac{\sum_{i=1}^{n-1} [a_i(i+1)d_{\max}]}{\sum_{j=1}^{n-1} a_j},$$

then the queue length is strictly positive for any  $k > \max(m_{\text{RTT}p}) + 1$ .

*Proof:* We observe, that  $u(kT < 0) = 0$ . This allows us to rewrite (4) for  $k > \max(m_{\text{RTT}p}) + 1$  as

$$(22) \quad \begin{aligned} y(kT) &= \sum_{i=1}^{n-1} \left\{ a_i \sum_{j=0}^{k-1} u[(j-i)T] \right\} - \sum_{j=0}^{k-1} h(jT) = \\ &= \sum_{i=1}^{n-1} \left\{ a_i \sum_{j=0}^{i-1} u[(j-i)T] + a_i u(0) + a_i \sum_{j=i+1}^{k-1} u[(j-i)T] \right\} - \sum_{j=0}^{k-1} h(jT) = \\ &= \sum_{i=1}^{n-1} \left( a_i \frac{x_d}{\sum_{j=1}^{n-1} a_j} \right) + \sum_{i=1}^{n-1} \left\{ a_i \sum_{j=i+1}^{k-1} u[(j-i)T] \right\} - \sum_{j=0}^{k-1} h(jT) = \\ &= x_d + \frac{1}{\sum_{j=1}^{n-1} a_j} \sum_{i=1}^{n-1} a_i \left[ \sum_{j=0}^{k-2-i} h(jT) - \sum_{j=0}^{k-1} h(jT) \right] = \\ &= x_d + \frac{\sum_{i=1}^{n-1} \left[ -a_i \sum_{j=k-i-1}^{k-1} h(jT) \right]}{\sum_{j=1}^{n-1} a_j} \geq x_d - \frac{\sum_{i=1}^{n-1} [a_i(i+1)d_{\max}]}{\sum_{j=1}^{n-1} a_j} > 0. \end{aligned}$$

This ends the proof. This theorem shows that if condition (21) is satisfied, then the queue length is strictly greater than zero for any  $k > \max(m_{\text{RTT}\rho}) + 1$  which implies that the available bandwidth is fully used.

### 3.2. Reaching law based sliding mode controller

A disadvantage of the sliding mode controller presented in the previous chapter is the large value of the control signal at the initial time instant. Therefore, in this subsection we introduce a reaching law based control strategy in order to minimize this effect. The properties of the modified strategy are then formulated and proved.

In order to design the modified controller, we adapt the reaching law proposed in [6]. It can be formulated in the following way

$$(23) \quad s[(k+1)T] - s(kT) = -\phi[s(kT)],$$

where  $\phi[s(kT)] = \min[|s(kT)|, \delta] \text{sgn}[s(kT)]$  and  $\delta > 0$ . With this law applied to the dead-beat type controller – like the one proposed in the previous subsection – the system representative point is guaranteed to reach the hyperplane  $s(kT) = 0$  monotonically in a finite number of steps.

Equation (23) can be alternatively expressed as follows

$$(24) \quad s_1(kT) = \mathbf{c}^T \mathbf{e}(kT) + f(kT),$$

where  $s_1(kT)$  is a new sliding variable, vector  $\mathbf{c}$  is given by (14) and function  $f(kT)$  is defined as follows

$$(25) \quad \begin{cases} f(kT) = f[(k-1)T] + \delta \text{sgn}\{s[(k-1)T]\} & \text{for } k < k_0, \\ f(kT) = 0 & \text{for } k \geq k_0. \end{cases}$$

We assume, that  $s_1(0) = 0$ , which results in the following condition

$$(26) \quad f(0) = -\mathbf{c}^T \mathbf{e}(0) = -\mathbf{c}_1 x_d = -x_d.$$

In this way the representative point will move towards the original sliding hyperplane described by (10), attain it after  $k_0$ , and remain on it afterwards.

Now we will determine the relation between the design parameter  $\delta$  and the time instant  $k_0$ . From (25) we obtain

$$(27) \quad f[(k_0-1)T] = f(0) + \delta(k_0-1) \text{sgn}[s(0)] = -x_d + \delta(k_0-1).$$

If  $f(k_0T) = 0$ , then  $f[(k_0-1)T] \geq -\delta$ . From this relation it follows that

$$(28) \quad k_0 = \left\lceil \frac{x_d}{\delta} \right\rceil,$$

where function  $\lceil \xi \rceil$  denotes the smallest integer greater than its argument  $\xi$ . This shows how the choice of parameter  $\delta$  affects the time instant when the representative point reaches the sliding hyperplane.

With the application of the proposed reaching law, the control signal can be derived by substituting (5) into equation  $\mathbf{c}^T \mathbf{e}[(k+1)T] + f[(k+1)T] = 0$ . This leads to

$$(29) \quad u(kT) = (\mathbf{c}^T \mathbf{b})^{-1} \{ \mathbf{c}^T [\mathbf{x}_d - \mathbf{A}\mathbf{x}(kT)] + f[(k+1)T] \}.$$

Substituting (7), (8) and (14) into (29) we obtain



$$(30) \quad u(kT) = \frac{1}{c_n} \left\{ x_d - x_1(kT) - \sum_{i=2}^n c_i x_i(kT) + f[(k+1)T] \right\}.$$

This completes the design of the reaching law based control strategy. Further in this section, we state and prove important properties of the proposed strategy. Lemma 2 shows that the control signal is always nonnegative and upper bounded. Theorems 3 and 4 (analogous to Theorems 1 and 2) show that the queue length never exceeds its demand value and that after some initial time the queue length is always strictly positive, which means that the available bandwidth is fully used.

**Lemma 2.** If the designed sliding mode controller is applied, then its output for any  $k \geq 0$  satisfies

$$(31) \quad u(kT) = \frac{h[(k-1)T] + f[(k+1)T] - f(kT)}{\sum_{j=1}^{n-1} a_j}.$$

*Proof:* The proof is similar to the proof of Lemma 1. We can rewrite (30) as

$$(32) \quad \begin{aligned} u(kT) &= \frac{1}{c_n} \left\{ x_d - x_1(kT) - \sum_{i=2}^{n-1} c_i x_i(kT) + f[(k+1)T] \right\} - u[(k-1)T] = \\ &= \frac{1}{c_n} \left\{ x_d - x_1(kT) - \sum_{i=2}^{n-1} c_i x_i(kT) + f[(k+1)T] \right\} + \\ &= \frac{1}{c_n} \left\{ x_d - x_1[(k-1)T] - \sum_{i=2}^n c_i x_i[(k-1)T] + f(kT) \right\} = \\ &= \frac{h[(k-1)T] + f[(k+1)T] - f(kT)}{\sum_{j=1}^{n-1} a_j}. \end{aligned}$$

This ends the proof.

**Remark 2.** We notice that  $f(kT)$  is non-decreasing,  $h(kT) \geq 0$ , and  $\sum_{j=1}^{n-1} a_j \in (0, 1)$ . From this it follows, that the control signal is always nonnegative. Furthermore  $h(kT) \leq d_{\max}$ , and  $\max\{f[(k+1)T] - f(kT)\} = \delta$ . This means that the control signal is always upper-bounded, i.e.,

$$(33) \quad u(kT) \leq \frac{d_{\max} + \delta}{\sum_{j=1}^{n-1} a_j}$$

for any  $k \geq 0$ . By an appropriate choice of function  $f(kT)$  we obtained a constant upper bound on the control signal, which is practical for application in a real network.

**Theorem 3.** If the proposed control strategy is applied, then the queue length never exceeds its demand value.

*Proof:* Using (6) we transform (30) and obtain

$$(34) \quad \frac{x_d - x_1(kT)}{\sum_{j=1}^{n-1} a_j} = u(kT) + \frac{\sum_{i=2}^n c_i u[(k-n+i-1)T] - f[(k+1)T]}{\sum_{j=1}^{n-1} a_j}.$$

Function  $f(kT)$  is always smaller than or equal to zero,  $\sum_{j=1}^{n-1} a_j > 0$ , and from Remark 2 the control signal is always nonnegative. This means, that the right hand side of equation (34) is nonnegative, which gives  $x_1(kT) \leq x_d$  and ends the proof.

**Theorem 4.** If the proposed control algorithm is applied, and the demand queue length satisfies the following condition

$$(35) \quad x_d > \frac{\sum_{i=1}^{n-1} a_i (i+1) d_{\max}}{\sum_{j=1}^{n-1} a_j},$$

then for any  $k > k_0 + \max(m_{\text{RTT}_p}) + 1$  the queue length is strictly greater than zero.

*P r o o f:* Using (31) and relation  $u(kT < 0) = 0$ , we rewrite (4) for any  $k > k_0 + \max(m_{\text{RTT}_p}) + 1$  as follows:

$$(36) \quad \begin{aligned} y(kT) &= \sum_{j=0}^{k-1} \sum_{i=1}^{n-1} \{a_i u[(j-i)T]\} - \sum_{j=0}^{k-1} h(jT) = \\ &= \sum_{i=1}^{n-1} \left\{ a_i \sum_{j=i}^{k-1} u[(j-i)T] \right\} - \sum_{j=0}^{k-1} h(jT) = \\ &= \sum_{i=1}^{n-1} a_i \sum_{j=0}^{k-i-1} \left\{ \frac{h[(j-1)T]}{\sum_{r=1}^{n-1} a_r} + \frac{f[(j+1)T] - f(jT)}{\sum_{r=1}^{n-1} a_r} \right\} - \sum_{j=0}^{k-1} h(jT) = \\ &= \sum_{i=1}^{n-1} a_i \sum_{j=0}^{k-i-1} \left\{ \frac{h[(j-1)T]}{\sum_{r=1}^{n-1} a_r} \right\} - f(0) - \sum_{j=0}^{k-1} h(jT) = \\ &= x_d + \frac{1}{\sum_{r=1}^{n-1} a_r} \sum_{i=1}^{n-1} a_i \left\{ \sum_{j=0}^{k-i-2} h(jT) - \sum_{j=0}^{k-1} h(jT) \right\} = \\ &= x_d - \frac{\sum_{i=1}^{n-1} a_i \sum_{j=k-i-1}^{k-1} h(jT)}{\sum_{j=1}^{n-1} a_j} \geq x_d - \frac{\sum_{i=1}^{n-1} [a_i (i+1) d_{\max}]}{\sum_{j=1}^{n-1} a_j}. \end{aligned}$$

This ends the proof.

This theorem shows that if the demand queue length satisfies the same condition as for the basic version of the dead-beat sliding mode controller, then the queue length is strictly greater than zero for any  $k > k_0 + \max(m_{RTTp}) + 1$ , which implies that the available bandwidth is fully used.

#### 4. Simulation examples

In order to verify the properties of the proposed flow control strategy, computer simulations of the network with 4 data sources were performed. As stated in Section 2, we assume that the distribution of data rate among the connections is performed by some higher-level algorithm. The discretisation period  $T$  is selected as 1 ms. The parameters of the connections are shown in Table 1.

Table 1. Link parameters in the simulated network

$p$	1	2	3	4
$m_{RTTp}$	2	4	6	9
$T_{bp}$ , ms	1	2	3	5
$\gamma_p$	0.1	0.3	0.4	0.2
$\alpha_p$	0.98	0.99	0.95	0.93

Because  $\max(m_{RTTp}) = 9$ , we get  $n = 10$ . According to Section 2, the state space network model parameters are:  $a_1 = 0$ ,  $a_2 = 0.098$ ,  $a_3 = 0$ ,  $a_4 = 0.297$ ,  $a_5 = 0$ ,  $a_6 = 0.38$ ,  $a_7 = 0$ ,  $a_8 = 0$ ,  $a_9 = 0.186$ . The maximum available bandwidth of the bottleneck node is  $d_{\max} = 30$  kb. According to Theorems 2 and 4 in order to ensure full bandwidth utilization, the demand queue length must be greater than 197 kb for both proposed controllers. Therefore, the demand queue length has been chosen as 200 kb. The available bandwidth changes rapidly from low to high values, which reflects the most adverse possible conditions in the network. The available bandwidth is shown in Fig. 2. For the reaching law based sliding mode controller, parameter  $\delta$  has been chosen as 35 kb, which according to (28) gives  $k_0 = 6$ .

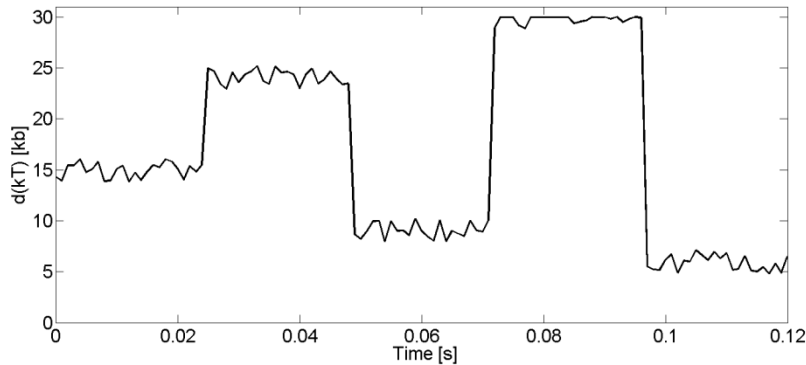


Fig. 2. Available bandwidth

Fig. 3 shows the bottleneck link queue length when the primary dead-beat controller is applied. We notice from the figure that the queue length never exceeds its demand value, and after the initial time derived in Theorem 2 it is always strictly positive. Therefore, there is no risk of a buffer overflow, and full bandwidth

utilization is ensured. The value of the control signal and the amount of data leaving the buffer are shown in Fig. 4. The value of the control signal at the first time instant is equal to 208.11 kb. As one can see from the figure, the control signal is always strictly positive and upper bounded. The transmission rates of individual sources with the application of the basic control strategy are shown in Fig. 5.

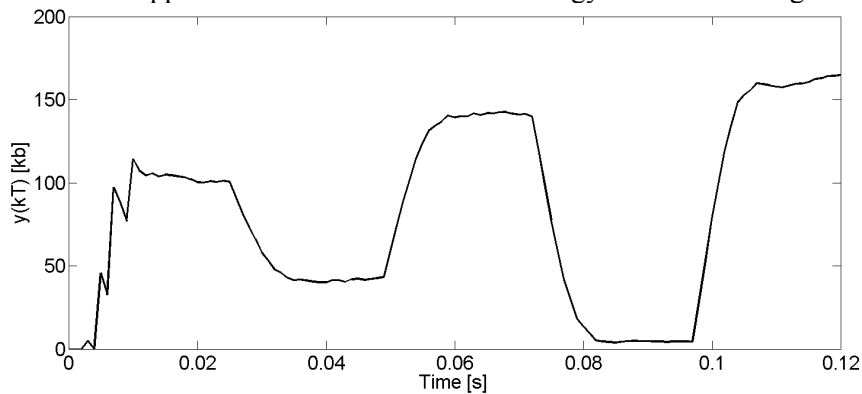


Fig. 3. Queue length with application of the primary sliding mode controller

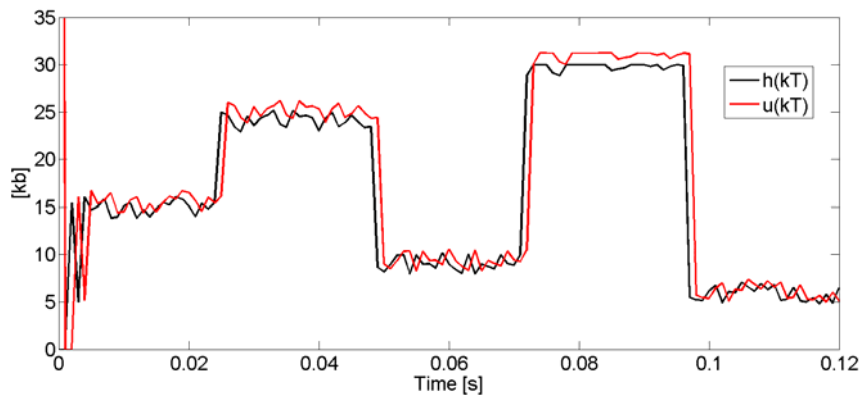


Fig. 4. Amount of data leaving the buffer and the control signal with application of the primary sliding mode controller

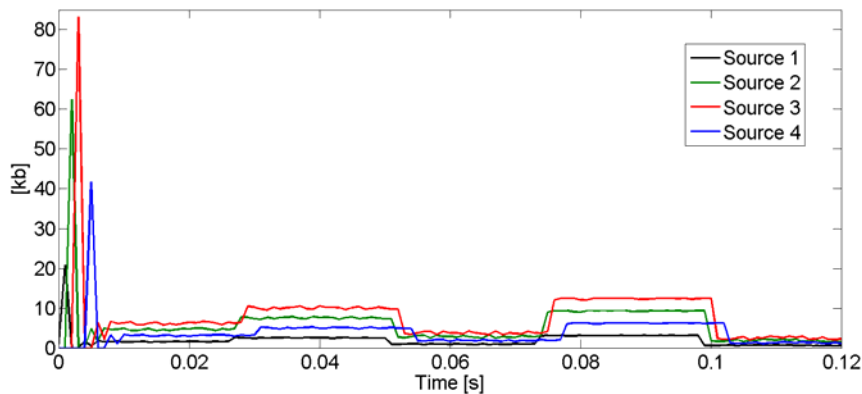


Fig. 5. Transmission rates of individual sources in the network with the primary sliding mode controller

Figs 6, 7 and 8 present the simulation results obtained for the reaching law based sliding mode controller. Fig. 6 shows the queue length in the bottleneck link buffer. It can be seen from the figure, that the queue never exceeds its demand value and after the initial time predicted by Theorem 4, it is always strictly positive. Fig. 7 shows the amount of data leaving the buffer and the value of the control signal. Comparing Figs 4 and 7 one can notice, that the introduction of the reaching law significantly reduces the maximum value of the control signal at the beginning of the data transmission process. The transmission rates of individual sources are shown in Fig. 8.

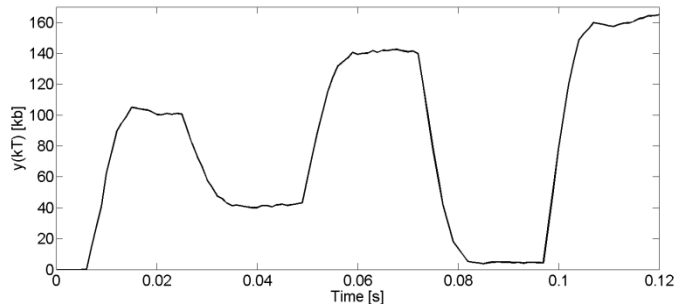


Fig. 6. Queue length with application of the reaching law based sliding mode controller

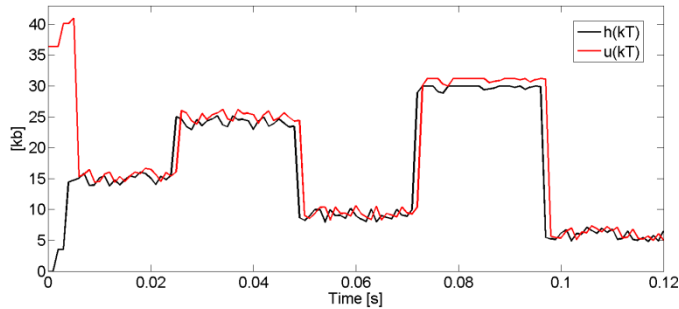


Fig. 7. The amount of data leaving the buffer and the control signal with application of the reaching law based controller

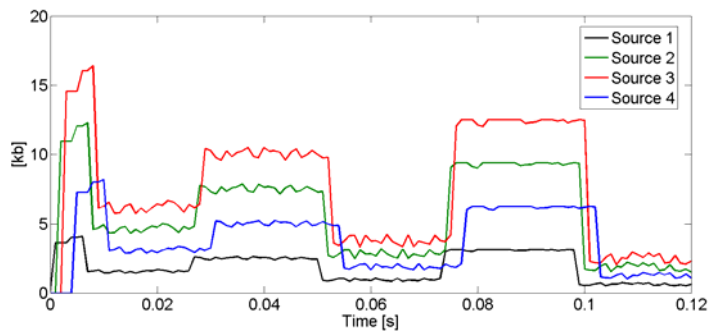


Fig. 8. Transmission rates of individual sources in the network with the reaching law based sliding mode controller

## 5. Conclusions

In this paper two discrete time sliding mode flow controllers for connection-oriented networks with multiple connections have been presented. The possible losses during the transmission have been explicitly taken into account in designing the sliding hyperplane. The first algorithm has been designed to ensure finite-time error convergence. Then it has been modified by introducing a reaching law to reduce the maximum value of the control signal at the beginning of the data transmission. It has been proved that the flow rates generated by both control algorithms are always nonnegative and upper-bounded. Furthermore, both proposed controllers ensure full bottleneck bandwidth consumption and eliminate the risk of buffer overflow.

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