

First Order Perturbation Bounds of the Discrete-Time LMI-Based H_∞ Quadratic Stability Problem for Descriptor Systems

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Abstract: *In this paper we propose an approach to obtain first order perturbation bounds for the discrete-time Linear Matrix Inequalities (LMI) based H_∞ quadratic stability problem for descriptor systems. Applying the considered approach we are able to compute tight first order perturbation bounds for the LMIs' solutions to the H_∞ quadratic stability problem for discrete-time descriptor systems. In the paper we present an approach to compute the estimates of the individual condition numbers of the considered LMIs. To illustrate the performance and applicability of the results obtained we present a numerical example.*

Keywords: *Discrete-time descriptor systems, first order perturbation bounds, H_∞ Quadratic Stability Problem, LMI based design.*

1. Introduction

Linear Matrix Inequalities (LMIs) are often used to find solutions in an efficient way to many modern and classical problems in control theory: H_∞ design, linear quadratic regulator problem, bounded energy problem, quadratic stability problem, model predictive control, etc. [1, 2, 6, 7, 12].

LMI design is of high performance and useful thanks to the existence of efficient convex optimization algorithms [3] and software [4] in addition to the MATLAB package Yalmip and SeDuMi solver [5].

Descriptor systems or singular systems present a wide class of systems, which are important from a theoretical and practical point of view. A great amount of investigations concerning linear descriptor systems has been performed in [10, 13, 14]. The concepts of controllability, observability, stability, model predictive control, linear quadratic optimal regulator, optimal state regulation, state feedback

and observer design have already been studied in [10, 15, 16, 17]. Different numerical methods for finding the solution of the singular systems are presented in [18, 19].

In the paper we consider an approach similar to the presented in [20] to compute the first order perturbation bounds of the LMI based H_∞ quadratic stability problem for discrete-time descriptor systems.

Throughout the paper we use the notations: $R^{m \times n}$ – the space of real $m \times n$ matrices; $R^n = R^{n \times 1}$; I_n – the identity $n \times n$ matrix; e_n – the unit $n \times 1$ vector; M^T – the transpose of M ; M^\perp – the pseudo inverse of M ; $\|M\|_2 = \sigma_{\max}(M)$ – the spectral norm of M , where $\sigma_{\max}(M)$ is the maximum singular value of M ; $\text{vec}(M) \in R^{mn}$ – the column-wise vector representation of $M \in R^{m \times n}$; $\Pi_{m,n} \in R^{m \times n \times m \times n}$ – the vec-permutation matrix, such that $\text{vec}(M^T) = \Pi_{m,n} \text{vec}(M)$; $M \otimes P$ – the Kronecker product of the matrices M and P . The notation “:=” stands for “equal by definition”.

The remaining part of the paper is as follows. Section 2 considers the problem set up and objective. Section 3 studies the performed linear perturbation analysis of the LMI-based discrete-time H_∞ quadratic stability problem for descriptor systems. In Section 4 we present a numerical example. At the end we finish in Section 5 with a conclusion.

2. Problem setup and objective

Linear discrete-time descriptor systems are described by the set of difference-algebraic equations given below:

$$(1) \quad \begin{aligned} Ex(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k), \quad k = 0, 1, \dots, L, \end{aligned}$$

where $x(k) \in R^n$, $u(k) \in R^m$ and $y(k) \in R^r$ are the system descriptor state, input and output, and A, B, C and E are constant matrices of corresponding size.

Definition 2.1 (System equivalence) [10]. Two systems (E, A, B, C) and $(\hat{E}, \hat{A}, \hat{B}, \hat{C})$ are said to be (system) equivalent, denoted by $(E, A, B, C) \approx (\hat{E}, \hat{A}, \hat{B}, \hat{C})$, if there exist nonsingular transformation matrices $L, R \in R^{n \times n}$ such that the equations

$$\hat{E} = LER, \hat{A} = LAR, \hat{B} = LB, \hat{C} = CR$$

hold true.

Definition 2.2 (Regularity) [10]. The system is termed regular, if the polynomial $\det(sE - A)$ satisfies $\det(sE - A) \neq 0$.

Definition 2.3 (Weierstrass normal form – WNF) [10]. For any regular system there exist two non-singular matrices $L, R \in R^{n \times n}$ such that by

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = P^{-1}x, x_1 \in R^r, x_2 \in R^{n-r}$$

the following decomposed representation can be obtained:

$$(2) \quad \begin{aligned} x_1(k+1) &= \hat{A}_r x_1(k) + \hat{B}_1 u(k), \\ Nx_2(k) &= x_2(k) + \hat{B}_2 u(k). \end{aligned}$$

Definition 2.4 (Index of nilpotence) [10]. The index of nilpotence ν , i.e., $\nu := \min\{q \mid N^q = 0\}$ is said to be an index of a linear descriptor system. The systems with $\nu \geq 2$ are called high index singular systems.

In expression (2), the first equation is a forward recurrent equation whose state is determined uniquely by the initial state $x_1(0)$ and $u(k) = 0, 1, \dots, L$. While the second equation is a backward recurrence with a state uniquely determined by the terminal state $x_2(L)$ and $u(k) = 0, 1, \dots, L$.

For the system described in Weierstrass normal, the state evolution can be described according to [10]:

$$(3a) \quad x_1(k) = \hat{A}_r x_1(0) + \sum_{i=0}^{k-1} \hat{A}_r^{k-i-1} \hat{B}_1 u(i),$$

$$(3b) \quad x_2(k) = N^{L-k} x_2(L) - \sum_{i=0}^{L-k-1} N^i \hat{B}_2 u(k+i).$$

Relation (3b) for the state evolution $x_2(k)$ suggests that index one descriptor systems $\nu = 1$ and $N = 0$ will have no infinite poles. In this case the system (1) is called causal and index one.

Let us investigate the linear discrete-time descriptor system (1), where there is no direct relation between the input and the output signal. For the rest of the paper we suppose the descriptor system (1) is an index one system.

The equivalent system

$$(\hat{E}, \hat{A}, \hat{B}, \hat{C}) = \left(\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} \hat{A}_r & 0 \\ 0 & I_{n-r} \end{bmatrix}, \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \end{bmatrix}, \begin{bmatrix} \hat{C}_1 & \hat{C}_2 \end{bmatrix} \right),$$

is given in Weierstrass canonical form where $A_r \in R^{r \times r}$ is a stable matrix. The transformed system is represented as follows:

$$(4) \quad \begin{aligned} x_1(k+1) &= \hat{A}_r x_1(k) + \hat{B}_1 u(k), \\ y(k) &= \hat{C}_1 x_1(k). \end{aligned}$$

The system (4) in WNF is obtained using the relation (3b) for state evolution $x_2(k)$.

An LMI approach is used to solve the H_∞ quadratic stability problem for descriptor systems as presented in [11]. For index one discrete-time descriptor systems we study the solution of the following system of inequalities

$$(5) \quad \begin{bmatrix} -P_1^{-1} & \hat{A}_r^T & \hat{B}_1^T & 0 \\ \hat{A}_r & -P_1 & 0 & \hat{C}_1^T \\ \hat{B}_1 & 0 & -\gamma I & 0 \\ 0 & \hat{C}_1 & 0 & -\gamma I \end{bmatrix} < 0, \quad P_1 > 0.$$

Actually this is an Eigenvalue Problem (EVP) with respect to the variables P_1 and γ . We suppose that the optimal closed-loop performance γ_{opt} of the system (4) is already computed.

To obtain quadratic H_∞ stability and to ensure closed-loop performance γ it is necessary to design a state-feedback control $u=K_1x_1$. Then it is necessary to apply Schur complement argument [8]:

$$(6) \quad \begin{bmatrix} -P_1^{-1} & (\hat{A}_r + \hat{B}_1 K_1)^T & 0 & 0 \\ (\hat{A}_r + \hat{B}_1 K_1) & -P_1 & 0 & \hat{C}_1^T \\ 0 & 0 & -\gamma I & 0 \\ 0 & \hat{C}_1 & 0 & -\gamma I \end{bmatrix} < 0, P_1 > 0.$$

Relation (6) is an inequality with respect to the variables K_1, P_1 and γ . We pre- and post-multiply inequality (6) by $\text{diag}\{I, P_1^{-1}, I, I\}$ and also change the variables $Q_1 = P_1^{-1}, Q_1 > 0$ and $Y_1 = K_1 P_1^{-1}$ to obtain the following system of LMIs:

$$(7) \quad \begin{bmatrix} -Q_1 & (\hat{A}_r Q_1 + \hat{B}_1 K_1)^T & 0 & 0 \\ (\hat{A}_r Q_1 + \hat{B}_1 K_1) & -Q_1 & 0 & Q_1 \hat{C}_1^T \\ 0 & 0 & -\gamma I & 0 \\ 0 & \hat{C}_1 Q_1 & 0 & -\gamma I \end{bmatrix} < 0, Q_1 > 0.$$

The aim of the paper is to compute first order perturbation bounds for the LMI system (7), which is necessary for the solution of the LMI based H_∞ quadratic stability problem for index one descriptor systems, near the optimal value of γ . We should have in mind that in this case we have to find out if the considered problem is really feasible in order that the applied LMIs have solutions. We should remind that the feasibility of an LMI depends strongly on the sensitivity of the interior point method, used to solve the considered LMI, rounding errors and the size of the investigated LMI system. The size of the LMI system in the LMI based H_∞ quadratic stability problem for discrete-time descriptor systems is bigger compared to the corresponding continuous-time case. Due to this fact the applied interior point method may not always lead to feasible solutions in the discrete-time case.

Let the system matrices $\hat{A}_r, \hat{B}_1, \hat{C}_1$, are subject to perturbations $\Delta \hat{A}_r, \Delta \hat{B}_1, \Delta \hat{C}_1$, and suppose that they do not change the sign of the LMI system (7).

3. First order perturbation bounds computation

We conduct perturbation analysis of the LMI (7) for the index one discrete-time descriptor system (1), given in Weierstrass normal form:

$$(8) \quad \begin{bmatrix} -(Q_1 + \Delta Q_1) & \hat{A}_r \hat{B}_1 Q_1 Y_1^T & 0 & 0 \\ \hat{A}_r \hat{B}_1 Q_1 Y_1 & -(Q_1 + \Delta Q_1) & 0 & Q_1 \hat{C}_1^T \\ 0 & 0 & -(\gamma I + \Delta \gamma I) & 0 \\ 0 & \hat{C}_1 Q_1 & 0 & -(\gamma I + \Delta \gamma I) \end{bmatrix} < 0, Q_1 > 0,$$

where $\hat{A}_r \hat{B}_1 Q_1 Y_1^T = (Q_1 + \Delta Q_1)(\hat{A}_r + \Delta \hat{A}_r)^T + (Y_1 + \Delta Y_1)^T (\hat{B}_1 + \Delta \hat{B}_1)^T$,
 $Q_1 \hat{C}_1^T = (Q_1 + \Delta Q_1)(\hat{C}_1 + \Delta \hat{C}_1)^T$, $\hat{C}_1 Q_1 = (\hat{C}_1 + \Delta \hat{C}_1)(Q_1 + \Delta Q_1)$.

It is necessary to analyze the impact of the perturbations $\Delta \hat{A}_r, \Delta \hat{B}_1, \Delta \hat{C}_1$ and $\Delta \gamma$ on the perturbed LMI solutions $Q_1^* + \Delta Q_1$ and $Y_1^* + \Delta Y_1$. With Q_1^*, Y_1^* and $\Delta Q_1, \Delta Y_1$ we denote the nominal solutions of the LMIs (8) and the perturbations, respectively. The essence of our approach is connected with introducing a slightly perturbed suitable right hand part, in order to ensure feasibility, then we can obtain

$$(9) \quad \begin{bmatrix} -(Q_1^* + \Delta Q_1) & \hat{A}_r \hat{B}_1 Q_1 Y_1^{*T} & 0 & 0 \\ \hat{A}_r \hat{B}_1 Q_1 Y_1^{*T} & -(Q_1^* + \Delta Q_1) & 0 & Q_1 \hat{C}_1^{*T} \\ 0 & 0 & -(\gamma_{\text{opt}} I + \Delta \gamma I) & 0 \\ 0 & \hat{C}_1 Q_1^* & 0 & -(\gamma_{\text{opt}} I + \Delta \gamma I) \end{bmatrix} = N_1^* + \Delta N_1 < 0,$$

where $\hat{A}_r \hat{B}_1 Q_1 Y_1^{*T} = (Q_1^* + \Delta Q_1)(\hat{A}_r + \Delta \hat{A}_r)^T + (Y_1^* + \Delta Y_1)^T (\hat{B}_1 + \Delta \hat{B}_1)^T$,
 $Q_1 \hat{C}_1^T = (Q_1^* + \Delta Q_1)(\hat{C}_1 + \Delta \hat{C}_1)^T$, $\hat{C}_1 Q_1 = (\hat{C}_1 + \Delta \hat{C}_1)(Q_1^* + \Delta Q_1)$, and N_1^* is calculated using the nominal LMI:

$$(10) \quad \begin{bmatrix} -Q_1^* & Q_1^* \hat{A}_r^T + Y_1^{*T} \hat{B}_1^T & 0 & 0 \\ \hat{A}_r Q_1^* + \hat{B}_1 Y_1^{*T} & -Q_1^* & 0 & Q_1 \hat{C}_1^{*T} \\ 0 & 0 & -\gamma_{\text{opt}} I & 0 \\ 0 & \hat{C}_1 Q_1^* & 0 & -\gamma_{\text{opt}} I \end{bmatrix} = N_1^* < 0.$$

We use the matrix ΔN_1 to take into account information with respect to perturbations introduced to data and closed-loop performance, the existence of rounding errors and sensitivity of the interior point method that is used to solve the investigated LMIs. Due to the high sensitivity of the applied solution method to the size of the used LMI system for discrete-time descriptor systems this is not as straightforward as in the continuous-time case [20].

Expression (10) allows us to rewrite the perturbed equation (9) in the following way

$$(11) \quad \Delta_{Q_1} + \Omega_{Q_1} = \Delta N_1,$$

where

$$\Delta_{Q_1} = \begin{bmatrix} -\Delta Q_1 & \Delta Q_1 \hat{A}_r^T & 0 & 0 \\ \hat{A}_r \Delta Q_1 & -\Delta Q_1 & 0 & \Delta Q_1 \hat{C}_1^T \\ 0 & 0 & 0 & 0 \\ 0 & \hat{C}_1 \Delta Q_1 & 0 & 0 \end{bmatrix},$$

$$\Omega_{Q_1} = \begin{bmatrix} 0 & Q^* \Delta \hat{A}_r^T + \Delta Y_1^T \hat{B}_1^T + Y_1^{*T} \Delta \hat{B}_1^T & 0 & 0 \\ \Delta \hat{A}_r Q^* + \hat{B}_1 \Delta Y_1 + \Delta \hat{B}_1 Y_1^* & 0 & 0 & Q^* \Delta \hat{C}_1^T \\ 0 & 0 & -\Delta \lambda_{\text{opt}} I & 0 \\ 0 & \Delta \hat{C}_1 Q^* & 0 & -\Delta \lambda_{\text{opt}} I \end{bmatrix}.$$

Further we perform similar mathematical transformations as in [20]. Since we have to compute first order perturbation bounds, the terms of second and higher order are annihilated. In this way the expression (11) in a vectorized form looks as

$$(12) \quad \text{vec}(\Delta_{Q_1}) + \text{vec}(\Omega_{Q_1}) = \text{vec}(\Delta N_1),$$

where

$$\text{vec}(\Delta_{Q_1}) = [-I, \hat{A}_r \otimes I, 0, 0, I \otimes \hat{A}_r, -I, 0, \hat{C}_1 \otimes I, 0, 0, 0, 0, 0, I \otimes \hat{C}_1^T, 0, 0]^T$$

$$* \text{vec}(\Delta Q) := L_1 \Delta q_1,$$

$$\text{vec}(\Omega_{Q_1}) =$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ (I \otimes Q^*) \Pi_r & (\hat{B}_1 \otimes I) \Pi_{r,m} & (I \otimes Y_1^{*T}) \Pi_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ (Q^* \otimes I) & (I \otimes \hat{B}_1) & (Y_1^* \otimes I) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (I \otimes Q^*) \Pi_r & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e_{m'} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (Q^* \otimes I) & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e_{m'} \end{bmatrix} \times$$

$$\times \begin{bmatrix} \text{vec}(\Delta \hat{A}_r) \\ \text{vec}(\Delta Y_1) \\ \text{vec}(\Delta \hat{B}_1) \\ \text{vec}(\Delta \hat{C}_1) \\ \Delta \gamma_{\text{opt}} \end{bmatrix} = [L_{t1} \quad L_{t2} \quad L_{t3} \quad L_{t4} \quad L_{t5}] \Delta_{AYBC\gamma} := L_t \Delta_{AYBC\gamma}.$$

The difference in size of the obtained matrices in a vectorized form for the discrete-time descriptor systems and the case considered in [20] is visible. The bigger size of the LMI system may lead to unfeasibility in the discrete-time case due to the inability of the interior point method to find feasible solutions. This inability has nothing to do with the presented method for computing the first order perturbation bounds of the LMI based H_∞ quadratic stability problem for discrete-time descriptor systems. We carry out some mathematical transformations to obtain

$$(13) L_1 \Delta q_1 + L_{t1} \text{vec}(\Delta \hat{A}_r) + L_{t2} \text{vec}(\Delta Y_1) + L_{t3} \text{vec}(\Delta \hat{B}_1) + L_{t4} \text{vec}(\Delta \hat{C}_1) + L_{t5} \Delta \gamma_{\text{opt}} = \text{vec}(\Delta N_1).$$

At the end the relative perturbation bound for the solution Q_1^* of the LMI (7) can be computed using the relation

$$(14) \quad \frac{\|\Delta q_1\|_2}{\|\text{vec}(Q_1^*)\|_2} \leq \frac{1}{\|\text{vec}(Q_1^*)\|_2} \left(L_{AYB1} \frac{\|\text{vec}(\Delta \hat{A}_r)\|_2}{\|\text{vec}(\hat{A}_r)\|_2} + L_{AYB2} \frac{\|\text{vec}(\Delta Y_1)\|_2}{\|\text{vec}(Y_1^*)\|_2} + L_{AYB3} \frac{\|\text{vec}(\Delta \hat{B}_1)\|_2}{\|\text{vec}(\hat{B}_1)\|_2} \right) \\ \leq \frac{1}{\|\text{vec}(Q_1^*)\|_2} \left(L_{AYB4} \frac{\|\text{vec}(\Delta \hat{C}_1)\|_2}{\|\text{vec}(\hat{C}_1)\|_2} + L_{AYB5} \frac{|\Delta \gamma_{\text{opt}}|}{|\gamma_{\text{opt}}|} + N_1 \frac{\|\text{vec}(\Delta N_1)\|_2}{\|\text{vec}(N^*)\|_2} \right)$$

here

$$\frac{L_{AYB1}}{\|\text{vec}(Q_1^*)\|_2} := \frac{\|L_1^\perp\|_2 \|L_{t1}\|_2 \|\text{vec}(\hat{A}_r)\|_2}{\|\text{vec}(Q_1^*)\|_2}, \quad \frac{L_{AYB2}}{\|\text{vec}(Q_1^*)\|_2} := \frac{\|L_1^\perp\|_2 \|L_{t2}\|_2 \|\text{vec}(Y_1^*)\|_2}{\|\text{vec}(Q_1^*)\|_2}, \\ \frac{L_{AYB3}}{\|\text{vec}(Q_1^*)\|_2} := \frac{\|L_1^\perp\|_2 \|L_{t3}\|_2 \|\text{vec}(\hat{B}_1)\|_2}{\|\text{vec}(Q_1^*)\|_2}, \quad \frac{N_1}{\|\text{vec}(Q_1^*)\|_2} := \frac{\|L_1^\perp\|_2 \|\text{vec}(N^*)\|_2}{\|\text{vec}(Q_1^*)\|_2}, \\ \frac{L_{AYB4}}{\|\text{vec}(Q_1^*)\|_2} := \frac{\|L_1^\perp\|_2 \|L_{t4}\|_2 \|\text{vec}(\hat{C}_1)\|_2}{\|\text{vec}(Q_1^*)\|_2}, \quad \frac{L_{AYB5}}{\|\text{vec}(Q_1^*)\|_2} := \frac{\|L_1^\perp\|_2 \|L_{t5}\|_2 |\gamma_{\text{opt}}|}{\|\text{vec}(Q_1^*)\|_2}$$

are considered as estimates of the individual relative condition numbers of the LMI (7) with respect to the perturbations $\Delta \hat{A}_r$, $\Delta \hat{B}_1$, $\Delta \hat{C}_1$, $\Delta \gamma$ and ΔY_1 .

We apply a similar procedure as the already presented to obtain the relative perturbation bound for the solution Y_1^* of the LMI (7)

$$(15) \quad \frac{\|\Delta y_1\|_2}{\|\text{vec}(Y_1^*)\|_2} \leq \frac{1}{\|\text{vec}(Y_1^*)\|_2} \left(M_{AYB1} \frac{\|\text{vec}(\Delta \hat{A}_r)\|_2}{\|\text{vec}(\hat{A}_r)\|_2} + M_{AYB2} \frac{\|\text{vec}(\Delta Q_1)\|_2}{\|\text{vec}(Q_1^*)\|_2} + M_{AYB3} \frac{\|\text{vec}(\Delta \hat{B}_1)\|_2}{\|\text{vec}(\hat{B}_1)\|_2} \right) \\ \leq \frac{1}{\|\text{vec}(Y_1^*)\|_2} \left(M_{AYB4} \frac{\|\text{vec}(\Delta \hat{C}_1)\|_2}{\|\text{vec}(\hat{C}_1)\|_2} + M_{AYB5} \frac{|\Delta \gamma_{\text{opt}}|}{|\gamma_{\text{opt}}|} + N_2 \frac{\|\text{vec}(\Delta N_2)\|_2}{\|\text{vec}(N^*)\|_2} \right)$$

here

$$\begin{aligned} \frac{M_{AQB1}}{\|\text{vec}(Y_1^*)\|_2} &:= \frac{\|M^\perp\|_2 \|M_{t1}\|_2 \|\text{vec}(\hat{A}_r)\|_2}{\|\text{vec}(Y_1^*)\|_2}, & \frac{M_{AQB2}}{\|\text{vec}(Y_1^*)\|_2} &:= \frac{\|M^\perp\|_2 \|M_{t2}\|_2 \|\text{vec}(Q_1^*)\|_2}{\|\text{vec}(Y_1^*)\|_2}, \\ \frac{M_{AQB3}}{\|\text{vec}(Y_1^*)\|_2} &:= \frac{\|M^\perp\|_2 \|M_{t3}\|_2 \|\text{vec}(\hat{B}_1)\|_2}{\|\text{vec}(Y_1^*)\|_2}, & \frac{N_2}{\|\text{vec}(Y_1^*)\|_2} &:= \frac{\|M^\perp\|_2 \|\text{vec}(N^*)\|_2}{\|\text{vec}(Y_1^*)\|_2}, \\ \frac{M_{AQB4}}{\|\text{vec}(Y_1^*)\|_2} &:= \frac{\|M^\perp\|_2 \|M_{t4}\|_2 \|\text{vec}(\hat{C}_1)\|_2}{\|\text{vec}(Y_1^*)\|_2}, & \frac{M_{AQB5}}{\|\text{vec}(Y_1^*)\|_2} &:= \frac{\|M^\perp\|_2 \|M_{t5}\|_2 |\gamma_{\text{opt}}|}{\|\text{vec}(Y_1^*)\|_2} \end{aligned}$$

are considered as estimates of the individual relative condition numbers of the LMI (7) with respect to the perturbations $\Delta\hat{A}_r$, $\Delta\hat{B}_1$, $\Delta\hat{C}_1$, ΔQ_1 and $\Delta\gamma$.

In this case the size of the obtained matrices in a vectorized form for the discrete-time descriptor systems is again bigger compared to the case considered in [20]. This may lead to unfeasibility in the discrete-time case due to the inability of the interior point method to find feasible solutions. This inability has nothing to do with the presented method for computing the first order perturbation bounds of the LMI based H_∞ quadratic stability problem for discrete-time descriptor systems.

4. A numerical example [10]

Consider the discrete-time index one descriptor system (1) given in Weierstrass normal form, i.e.,

$$\hat{E} = \begin{bmatrix} 1 & 0 & \vdots & 0 & 0 \\ 0 & 1 & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \vdots & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 \end{bmatrix}, \hat{A} = \begin{bmatrix} 0.5 & 0 & \vdots & 0 & 0 \\ 0 & 0.7 & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \vdots & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} 0 \\ 1 \\ \cdots \\ 1 \\ -1 \end{bmatrix}, \hat{C} = [0 \quad 1 \quad \vdots \quad 1 \quad 0]$$

In this paper we would like to compute the first order bounds, that is why the perturbations in the system matrices are chosen in such a way as to annihilate the second and higher order terms in the mathematical transformations, shown above, i.e.,

$$\begin{aligned}\Delta\hat{A}_r &= A_r \times 10^{-i}, \Delta\hat{B}_1 = \hat{B}_1 \times 10^{-i}, \\ \Delta\hat{C}_1 &= \hat{C}_1 \times 10^{-i}, \Delta\gamma_{\text{opt}} = \gamma_{\text{opt}} \times 10^{-i}, \\ \Delta N_1 &= N_1^* \times 10^{-i}, \Delta N_2 = N_2^* \times 10^{-i}, \\ \Delta Q_1 &= Q_1^* \times 10^{-i}, \Delta Y_1 = Y_1^* \times 10^{-i} \text{ for } i = 8, 7, \dots, 4.\end{aligned}$$

The solutions of the perturbed LMI (8) $Q_1^* + \Delta Q_1$ and $Y_1^* + \Delta Y_1$ are computed following the method presented in [9] and with the help of the software [4]. After the application of the presented approach the first order relative perturbation bounds for the solutions Q_1^* and Y_1^* of the LMI system (7) are obtained using expressions (14) and (15), respectively.

For various size of perturbations we compute the first order perturbation bounds and illustrate the corresponding results in the table below

Table 1

I	$\frac{\ \Delta q_1\ _2}{\ \text{vec}(Q_1^*)\ _2}$	Bound (14)	$\frac{\ \Delta y_1\ _2}{\ \text{vec}(Y_1^*)\ _2}$	Bound (15)
8	7.2245×10^{-8}	1.1328×10^{-7}	5.8113×10^{-8}	0.9134×10^{-7}
7	7.2245×10^{-7}	1.1328×10^{-6}	5.8113×10^{-7}	0.9134×10^{-6}
6	7.2245×10^{-6}	1.1328×10^{-5}	5.8113×10^{-6}	0.9134×10^{-5}
5	7.2245×10^{-5}	1.1328×10^{-4}	5.8113×10^{-5}	0.9134×10^{-4}
4	7.2245×10^{-4}	1.1328×10^{-3}	5.8113×10^{-4}	0.9134×10^{-3}

To perform sensitivity analysis of the discrete-time LMI based H_∞ quadratic stability problem for descriptor systems we use the presented and investigated solution approach, which helps us come up with the perturbation bounds (14) and (15). The obtained first order bounds are tight and close to the real relative perturbation bounds $\frac{\|\Delta q_1\|_2}{\|\text{vec}(Q_1^*)\|_2}$ and $\frac{\|\Delta y_1\|_2}{\|\text{vec}(Y_1^*)\|_2}$. Based on the obtained experimental results we can conclude that the investigated method is suitable for computing the first order perturbation bounds of the discrete-time LMI based H_∞ quadratic stability problem for descriptor systems even if the sensitivity of the interior point method is higher compared to the continuous-time case. Let us remind that this high sensitivity may lead to unfeasibility when considering discrete-time descriptor systems.

5. Conclusion

In this paper we present and study an approach to compute the first order perturbation bounds of the discrete-time LMI based H_∞ quadratic stability problem for descriptor systems. We also demonstrate how the estimates of the individual

condition numbers of the considered LMIs can be computed. We obtain tight first order perturbation bounds for the matrix inequalities depicting the problem solution. Based on the presented mathematical transformations we have derived some theoretical results. Then a numerical example was considered in order to illustrate the applicability of the obtained results.

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