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A Measure of Inaccuracy between Two Fuzzy Sets

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Abstract: *Decision making requires a strong, as well as flexible approach. Flexibility, in mathematical language, can be termed as fuzziness. A firm or corporation deals with several parties and groups. The nature of negotiations or resulting flexibilities will change from party to party. For example if a firm is dealing with a supplier or a consumer, the nature of negotiations will be different, while the nature of its own interests remains the same. For the purpose of such and many other similar dealings and negotiations an intelligent decision making mathematical measure has to be defined, based on two fuzzy criteria between the two negotiating parties, with a prime concern for one of these.*

In this paper a way of measuring the inaccuracy between two fuzzy sets propose is proposed. This "Inaccuracy" measure has to be based on fuzziness rather than on randomness. Some interesting mathematical properties of this measure are analyzed and the relations between fuzzy entropy, fuzzy inaccuracy and measure of fuzzy divergence are also established. Finally an example is presented to illustrate the application of the measure proposed.

Keywords: *Fuzzy sets, entropy, divergence measure, inaccuracy measure, fuzzy entropy, fuzzy divergence measure.*

1. Introduction

The theory of fuzzy sets, proposed by Zadeh [9] in 1965 has gained quite considerable importance in various fields of signal and image processing in recent times [1, 3]. Fuzziness, a feature of uncertainty, results from the lack of sharp distinction of being or not being a member of the set. A measure of fuzziness used and cited in literature is the fuzzy entropy, also first mentioned by Zadeh [10] in 1968. The name entropy was chosen due to intrinsic similarity with Shannon's entropy [8]. Fuzzy entropy is the measurement of fuzziness in a fuzzy set and thus it has an important position in fuzzy systems, such as fuzzy knowledge based systems, fuzzy decision making systems, fuzzy control systems, fuzzy neural network systems, fuzzy pattern recognition systems and fuzzy management information systems.

While Shannon's entropy revolutionized the communication theory with extensive applications in several branches, statistical studies found greater use of Kullback-Leibler's measure of divergence [6, 7], being a measure of the distance between two distributions (observed and theoretical, say) of a random variable

Yet another information type of measure, proposed by Karridge [5] measures the inaccuracy of the distribution of a random variable with respect to another distribution under reference. This is the so called "measure of inaccuracy". Here the information contents of a distribution under study is averaged over a pre-assigned (known) distribution. This measure has close connections and potential role in statistical studies in emerging situations, when negotiations and deals are struck to a successful settlement in two parties to achieve meaningful information.

There are probabilistic studies on the "measure of inaccuracy and its generalizations", but for fuzzy phenomena similar research has not been done. This is the aim of this paper.

Let us first examine situations where the "fuzzy inaccuracy measure", studied here, may play a significant role. Consider an example – the case of a corporation having clients. It is commonly observed that a corporation dealing with its clients investigates (in fact, looks for) the manner, in which the client considers, of course in a rather fuzzy way, the set of issues between them. Thus in general the corporation and the client have different fuzzy functions on the set of their common issues. The knowledge of the client's function provides an element of "information" for the corporation, whose averaging with weights as its own values of the elements give an idea of the underlying inaccuracy in dealing with its client. It is natural for a dynamic corporation, in this competitive world, to have a measure of inaccuracy and then to make an attempt to find the lacking part of information that is the real measure of their negotiations.

Some basic definitions related to probabilistic and fuzzy set theory are presented in Section 2. In Section 3 we introduce a measure of inaccuracy between two fuzzy sets. In Section 4 some properties of fuzzy inaccuracy are considered. The relations between fuzzy entropy, fuzzy inaccuracy and fuzzy divergence are established in Section 5. In Section 6 an example is presented to illustrate the application of the inaccuracy measure proposed and our conclusions are presented in Section 7.

2. Preliminaries

First, let us cover the probabilistic part of the preliminaries.

Let $\Delta_n = \left\{ P = (p_1, p_2, ..., p_n) : p_i \geq 0, \sum_{i=1}^{n} p_i = 1 \right\}$, $n \geq 2$, be a set of n-complete probability distributions.

For any probability distribution $P = (p_1, p_2, ..., p_n) \in \Delta_n$, Shannon's entropy [8], is defined as

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i . \tag{1}$$

Kullback-Leibler's [6, 7] measure of divergence of "true" probability distribution $P = (p_1, p_2, ..., p_n) \in \Delta_n$ to an arbitrary probability distribution $Q = (q_1, q_2, ..., q_n) \in \Delta_n$ is given by

$$D(P \,\|\, Q) = \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i} . \tag{2}$$

And Kerridge's inaccuracy [5] of distribution Q with respect to distribution P is given by

$$I(P;Q) = -\sum_{i=1}^{n} p_i \log q_i .$$

This obviously can be seen as the average of information elements, namely, $-\log q_i$ distribution Q over a distribution P, in a sense generalizing Shannon's entropy.

Next for fuzzy theory: Let $X = \{x_1, x_2, ..., x_n\}$ be a discrete universe of discourse. A *fuzzy set* A on X is characterized by a membership function $\mu_A : X \to [0,1]$. The value $\mu_A(x)$ of A at $x \in X$ stands for the degree of membership of $x \in X$ in A.

The set of all fuzzy sets on X will be denoted by FS(X).

Further on, the set-theoretic operations on fuzzy sets, by Zadeh, are defined as follows. Let A and B be two fuzzy sets on X.

The ***union*** $A \bigcup B$ of A and B is defined by

$$\forall x \in X \quad (\mu_A \cup \mu_B)(x) = \max(\mu_A(x), \mu_B(x)) \ \text{(or simply } \mu_A(x) \vee \mu_B(x)\text{)}.$$

The ***intersection*** $A \bigcap B$ of A and B is defined by

$$\forall x \in X \quad (\mu_A \cap \mu_B)(x) = \min(\mu_A(x), \mu_B(x)) \ \text{(or simply } \mu_A(x) \wedge \mu_B(x)\text{)}.$$

The ***complement*** $\overline{A}$ of A is defined by

$$\forall \ x \in X \quad \mu_{\overline{A}}(x) = 1 - \mu_A(x) .$$

The first attempt to quantify the uncertainty associated with a fuzzy event in the context of a discrete probabilistic framework appears to have been made by Zadeh [10], who defined the (weighted) entropy of a fuzzy set A with respect to (X, P) as

$$H(A,P) = -\sum_{i=1}^{n} \mu_A(x_i)\, p_i \log p_i \,.$$

De Luca and Termini [4] defined fuzzy entropy for a fuzzy set A corresponding to (1) as

$$H(A) = -\frac{1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)\log(\mu_A(x_i)) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))\right]. \tag{3}$$

Bhandari and Pal [2] proposed fuzzy divergence for two fuzzy sets A and B, corresponding to (2) given by

$$D(A\,|\,B) = \frac{1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)\log\frac{(\mu_A(x_i))}{(\mu_B(x_i))} + (1-\mu_A(x_i))\log\frac{(1-\mu_A(x_i))}{(1-\mu_B(x_i))}\right].$$

3. Inaccuracy measure of a fuzzy set

We proceed with the following formal definition

Definition. Let A be the fuzzy set and B − another fuzzy set defined on a discrete universe of discourse $X = \{x_1, x_2, ..., x_n\}$ having membership values $\mu_A(x_i),\ i=1, 2, ..., n,$ and $\mu_B(x_i),\ i=1, 2, ..., n,$ respectively.

We define the measure of inaccuracy of a fuzzy set B with respect to fuzzy set A, as

$$I(A;\ B) = -\frac{1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)\log(\mu_B(x_i)) + (1-\mu_A(x_i))\log(1-\mu_B(x_i))\right]. \tag{4}$$

This can also be written in the following form:

$$I(A;\ B) = \frac{1}{n}\sum_{i=1}^{n} S(f(x_i),\ f(y_i)),$$

where $S(x,y) = -x\log y - (1-x)\log(1-y)$, is Karridge's inaccuracy [5] function for two events.

It is interesting to note that when $A = B$, then (4) becomes (3), the measure of fuzziness given by De Luca and Termini [4].

In the next section we study some properties of $I(A;\ B)$, the fuzzy inaccuracy.

4. Properties of the fuzziness measure of inaccuracy

The measure of fuzzy inaccuracy defined in (4) has the following properties.

Theorem 1 (for a crisp set). $I(A;\ B) = 0$ if and only if either $\mu_A(x_i) = \mu_B(x_i) = 0$ or $\mu_A(x_i) = \mu_B(x_i) = 1$ $\forall i = 1, 2, ..., n.$

P r o o f: First, let $I(A;\ B)=0$, then

$$\sum_{i=1}^{n}\left[\mu_A(x_i)\log(\mu_B(x_i))+(1-\mu_A(x_i))\log(1-\mu_B(x_i))\right]=0,$$

$$\left[\mu_A(x_i)\log(\mu_B(x_i))+(1-\mu_A(x_i))\log(1-\mu_B(x_i))\right]=0 \qquad \forall i=1,\ 2,\ ...,\ n.$$

The above relation holds only when either $\mu_A(x_i)=\mu_B(x_i)=0$ or $\mu_A(x_i)=\mu_B(x_i)=1$ for all $i=1,\ 2,\ ...,\ n.$

Conversely, let either $\mu_A(x_i)=\mu_B(x_i)=0$ or $\mu_A(x_i)=\mu_B(x_i)=1$, this implies

$$\mu_A(x_i)\log(\mu_B(x_i))+(1-\mu_A(x_i))\log(1-\mu_B(x_i))=0 \qquad \forall i=1,\ 2,\ ...,\ n,$$

$$I(A;\ B)=0.$$

This proves the theorem. ■

Note. This means that zero inaccuracy implies a correct statement made with complete certainty.

Theorem 2. For any $A\in \mathrm{FS}(X)$, and A_{F} the most fuzzy set, i.e., $\mu_{A_{\mathrm{F}}}(x)=0.5$ for all x,

$$I(A;\ A_{\mathrm{F}})=1.$$

P r o o f: Let $A\in\mathrm{FS}(X)$, from definition

$$I(A;\ A_{\mathrm{F}})=-\frac{1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)\log(\mu_{A_{\mathrm{F}}}(x_i))+(1-\mu_A(x_i))\log(1-\mu_{A_{\mathrm{F}}}(x_i))\right]=$$

$$=-\frac{1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)\log(0.5)+(1-\mu_A(x_i))\log(0.5)\right]=$$

$$=\frac{1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)+(1-\mu_A(x_i))\right]=1.$$

This proves the theorem. ■

Theorem 3. For $A, B, C\in \mathrm{FS}(X)$,

$$I(A;\ B\cup C)+I(A;\ B\cap C)=I(A;B)+I(A;C).$$

P r o o f: Let

$$X^{\div}=\{x\mid x\in X,\ \mu_B(x)\geq\mu_C(x_{\mathrm{i}})\},$$

$$X^{-}=\{x\mid x\in X,\ \mu_B(x)<\mu_C(x_i)\},$$

where $\mu_A(x)$, $\mu_B(x)$ and $\mu_C(x)$ are the fuzzy membership functions of A, B and C respectively.

Then we have

(5) $$I(A;\ B\cup C)=$$

$$=-\sum_{i=1}^{n}\left[\mu_A(x_i)\log(\mu_B(x_i)\vee\mu_C(x_i))+(1-\mu_A(x_i))\log(1-(\mu_B(x_i)\vee\mu_C(x_i)))\right]=$$

$$= -\left[\left(\sum_{x_i \in X^+} \mu_A(x_i)\log \mu_B(x_i) + (1-\mu_A(x_i))\log(1-\mu_B(x_i))\right) + \right.$$

$$\left. + \left(\sum_{x_i \in X^-} \mu_A(x_i)\log \mu_C(x_i) + (1-\mu_A(x_i))\log(1-\mu_C(x_i))\right)\right]$$

and

(6) $$I(A;\ B \cap C) =$$

$$= -\sum_{i=1}^{n} \left[\mu_A(x_i)\log(\mu_B(x_i) \wedge \mu_C(x_i)) + (1-\mu_A(x_i))\log(1-(\mu_B(x_i) \wedge \mu_C(x_i)))\right] =$$

$$= -\left[\left(\sum_{x_i \in X^+} \mu_A(x_i)\log \mu_C(x_i) + (1-\mu_A(x_i))\log(1-\mu_C(x_i))\right) + \right.$$

$$\left. + \left(\sum_{x_i \in X^-} \mu_A(x_i)\log \mu_B(x_i) + (1-\mu_A(x_i))\log(1-\mu_B(x_i))\right)\right].$$

Adding (5) and (6) we obtain,

$$I(A;\ B \cup C) + I(A;\ B \cap C) = I(A; B) + I(A; C).$$

This proves the theorem. ■

Theorem 4. For $A, B, C \in \mathrm{FS}(X)$

$$I(A \cup B;\ C) + I(A \cap B;\ C) = I(A; B) + I(A; C).$$

P r o o f: Let us here take

$$X^+ = \{x \mid x \in X, \mu_A(x) \geq \mu_B(x)\},$$

$$X^- = \{x \mid x \in X, \mu_A(x) < \mu_B(x)\},$$

where $\mu_A(x)$, $\mu_B(x)$ and $\mu_c(x)$ are the fuzzy membership functions of A, B and C respectively.

Then we have

(7) $$I(A \cup B; C) =$$

$$= -\sum_{i=1}^{n} \left[(\mu_A(x_i) \vee \mu_B(x_i))\log \mu_C(x_i) + (1-(\mu_A(x_i) \vee \mu_B(x_i)))\log(1-\mu_C(x_i))\right] =$$

$$= -\left[\left(\sum_{x_i \in X^+} \mu_A(x_i)\log \mu_C(x_i) + (1-\mu_A(x_i))\log(1-\mu_C(x_i))\right) + \right.$$

$$\left. + \left(\sum_{x_i \in X^-} \mu_B(x_i)\log \mu_C(x_i) + (1-\mu_B(x_i))\log(1-\mu_C(x_i))\right)\right]$$

and

(8) $$I(A \cup B; C) =$$

$$= -\sum_{i=1}^{n} \left[(\mu_A(x_i) \wedge \mu_B(x_i)) \log \mu_C(x_i) + (1 - (\mu_A(x_i) \wedge \mu_B(x_i))) \log(1 - \mu_C(x_i)) \right] =$$

$$= -\left[\left(\sum_{x_i \in X^{\div}} \mu_B(x_i) \log \mu_C(x_i) + (1 - \mu_B(x_i)) \log(1 - \mu_C(x_i)) \right) + \right.$$

$$\left. + \left(\sum_{x_i \in X^{-}} \mu_A(x_i) \log \mu_C(x_i) + (1 - \mu_A(x_i)) \log(1 - \mu_C(x_i)) \right) \right].$$

Adding (7) and (8) we get the result. ■

Corollary 4.1. Let $A,\ B,\ C \in \mathrm{FS}(X)$, then

$$I(A;\ B \cup C) + I(A\ ; B \cap C) = I(A \cup B;\ C) + I(A \cap B;\ C).$$

P r o o f: It obvious follows Theorems 2 and 3. ■

Theorem 5. For $A, B \in \mathrm{F}(X)$,

$$I(A \cup B;\ A \cap B) + I(A \cap B;\ A \cup B) = I(A; B) + I(B; A).$$

P r o o f: Let

$$X^{\div} = \{x \mid x \in X,\ \mu_A(x) \geq \mu_B(\mathrm{x})\},$$
$$X^{-} = \{x \mid x \in X,\ \mu_A(x) < \mu_B(x)\},$$

where $\mu_A(x)$ and $\mu_B(x)$ are the fuzzy membership functions of A and B respectively.

The result is as follows:

(9) $$I(A \cup B;\ A \cap B) =$$

$$= -\left[\left(\sum_{x_i \in X^{\div}} \mu_A(x_i) \log \mu_B(x_i) + (1 - \mu_A(x_i)) \log(1 - \mu_B(x_i)) \right) + \right.$$

$$\left. + \left(\sum_{x_i \in X^{-}} \mu_B(x_i) \log \mu_A(x_i) + (1 - \mu_B(x_i)) \log(1 - \mu_A(x_i)) \right) \right]$$

and

(10) $$I(A \cap B;\ A \cup B) =$$

$$= -\left[\left(\sum_{x_i \in X^{\div}} \mu_B(x_i) \log \mu_A(x_i) + (1 - \mu_B(x_i)) \log(1 - \mu_A(x_i)) \right) + \right.$$

$$\left. + \left(\sum_{x_i \in X^{-}} \mu_A(x_i) \log \mu_B(x_i) + (1 - \mu_A(x_i)) \log(1 - \mu_B(x_i)) \right) \right].$$

Adding (9) and (10) we get the result. ■

Theorem 6. For $A, B \in \mathrm{F}(X)$:

(a) $I(A; B) = I(\overline{A};\ \overline{B})$,

(b) $I(A; \overline{A}) = I(\overline{A};\ A)$,

(c) $I(A; \overline{B}) = I(\overline{A};\ B)$,

(d) $I(A;\ B) + I(\overline{A};\ B) = I(\overline{A};\ \overline{B}) + I(A; \overline{B})$,

where $\overline{A}$ and $\overline{B}$ represent the complements of fuzzy sets A and B respectively.

P r o o f: **(a)** It follows evidently from the relation of the membership of an element in a set and its complement.

(b) Let us consider the expression

$$I(A; \overline{A}) - I(\overline{A};\ A) =$$

$$= -\sum_{i=1}^{n}\left[\mu_A(x_i)\log(1-\mu_A(x_i)) + (1-\mu_A(x_i))\log\mu_A(x_i)\right] +$$

$$+\sum_{i=1}^{n}\left[(1-\mu_A(x_i))\log\mu_A(x_i) + \mu_A(x_i)\log(1-\mu_A(x_i))\right] = 0.$$

This completes the proof.

(c) Let us consider the expression

$$I(A; \overline{B}) - I(\overline{A};\ B) =$$

$$= -\sum_{i=1}^{n}\left[\mu_A(x_i)\log(1-\mu_B(x_i)) + (1-\mu_A(x_i))\log\mu_B(x_i)\right] +$$

$$+\sum_{i=1}^{n}\left[(1-\mu_A(x_i))\log\mu_B(x_i) + \mu_A(x_i)\log(1-\mu_B(x_i))\right] = 0.$$

This completes the proof.

(d) It obviously follows from **(a)** and **(c)**. ■

In the next section we propose a relation between fuzzy entropy, fuzzy inaccuracy and fuzzy divergence measure.

5. Relation between fuzzy entropy, fuzzy inaccuracy and fuzzy measure of divergence

Theorem 7. Let A and B are two fuzzy sets, then

$$H(A) \le I(A; B)$$

with equality if and only if $A = B$, i.e., $\mu_A(x_i) = \mu_B(x_i)\ \ \forall\ i$.

P r o o f: Without loss of generality, we use natural logarithms.

Using the well-known inequality,

(11) $\qquad \log_e(x) \le x - 1$ with equality if and only if $x = 1$.

Putting $x = \dfrac{\mu_B(x_i)}{\mu_A(x_i)}$ in equation (11), we get

(12) $\log_e\left(\dfrac{\mu_B(x_i)}{\mu_A(x_i)}\right) \le \dfrac{\mu_B(x_i)}{\mu_A(x_i)} - 1$ with equality if and only if $\mu_A(x_i) = \mu_B(x_i)\ \forall\, i$.

Again putting $x = \left(\dfrac{1-\mu_B(x_i)}{1-\mu_A(x_i)}\right)$ in equation (11) yields

$$\log_e\left(\frac{1-\mu_B(x_i)}{1-\mu_A(x_i)}\right) \le \left(\frac{1-\mu_B(x_i)}{1-\mu_A(x_i)}\right) - 1 \tag{13}$$

with equality if and only if $\mu_A(x_i) = \mu_B(x_i)\ \forall\ i$.

Multiplying (12) by $\mu_A(x_i)$ and (13) by $(1-\mu_A(x_i))$, and summing over i we obtain

$$\sum_{i=1}^{n}\left[\mu_A(x_i)\log_e\left(\frac{\mu_B(x_i)}{\mu_A(x_i)}\right) + (1-\mu_A(x_i))\log_e\left(\frac{1-\mu_B(x_i)}{1-\mu_A(x_i)}\right)\right] \le$$

$$\le \sum_{i=1}^{n}\left[\left(\mu_A(x_i)\left(\frac{\mu_B(x_i)}{\mu_A(x_i)} - 1\right)\right) + (1-\mu_A(x_i))\left(\left(\frac{1-\mu_B(x_i)}{1-\mu_A(x_i)}\right) - 1\right)\right],$$

$$\sum_{i=1}^{n}\left[\mu_A(x_i)\log_e\left(\frac{\mu_B(x_i)}{\mu_A(x_i)}\right) + (1-\mu_A(x_i))\log_e\left(\frac{1-\mu_B(x_i)}{1-\mu_A(x_i)}\right)\right] \le$$

$$\le \sum_{i=1}^{n}\left[(\mu_B(x_i) - \mu_A(x_i)) + (\mu_A(x_i) - \mu_B(x_i))\right],$$

$$\sum_{i=1}^{n}\left[\mu_A(x_i)\log_e\left(\frac{\mu_B(x_i)}{\mu_A(x_i)}\right) + (1-\mu_A(x_i))\log_e\left(\frac{1-\mu_B(x_i)}{1-\mu_A(x_i)}\right)\right] \le 0,$$

with equality if and only if $\mu_A(x_i) = \mu_B(x_i)\ \forall\ i$.

Thus

$$\sum_{\mathrm{i}=1}^{\mathrm{n}}\left[\mu_{\mathrm{A}}(x_i)\log(\mu_{\mathrm{B}}(x_i)) + (1-\mu_{\mathrm{A}}(x_i))\log(1-\mu_{\mathrm{B}}(x_i))\right] -$$

$$-\sum_{\mathrm{i}=1}^{\mathrm{n}}\left[\mu_{\mathrm{A}}(x_i)\log(\mu_{\mathrm{A}}(x_i)) + (1-\mu_{\mathrm{A}}(x_i))\log(1-\mu_{\mathrm{A}}(x_i))\right] \le 0,$$

$$-\sum_{\mathrm{i}=1}^{\mathrm{n}}\mu_{\mathrm{A}}(x_i)\log(\mu_{\mathrm{A}}(x_i)) + (1-\mu_{\mathrm{A}}(x_i))\log(1-\mu_{\mathrm{A}}(x_i)) \le$$

$$\le -\sum_{\mathrm{i}=1}^{\mathrm{n}}\mu_{\mathrm{A}}(x_i)\log(\mu_{\mathrm{B}}(x_i)) + (1-\mu_{\mathrm{A}}(x_i))\log(1-\mu_{\mathrm{B}}(x_i)).$$

This proves the theorem. ■

Theorem 8. For two fuzzy sets *A* and *B*,

$$I(A;B) - H(A) = D(A\,|\,B).$$

P r o o f: From (3) and (4) we have:

$$H(A) = -\frac{1}{n}\sum_{\mathrm{i}=1}^{\mathrm{n}}\left[\mu_{\mathrm{A}}(x_i)\log(\mu_{\mathrm{A}}(x_i)) + (1-\mu_{\mathrm{A}}(x_i))\log(1-\mu_{\mathrm{A}}(x_i))\right], \tag{14}$$

(15) $$I(A;\ B) = -\frac{1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)\log(\mu_B(x_i)) + (1-\mu_A(x_i))\log(1-\mu_B(x_i))\right].$$

Subtracting (14) from (15) we get

$$\frac{1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)\log\frac{(\mu_A(x_i))}{(\mu_B(x_i))} + (1-\mu_A(x_i))\log\frac{(1-\mu_A(x_i))}{(1-\mu_B(x_i))}\right] = D(A|B).$$

This proves the theorem. ■

Theorem 9. If the membership function $\mu_A(x_i)$ is kept fixed and variation in $\mu_B(x_i)$ is allowed, $I(A;\ B)$ attains its minimum value when $\mu_A(x_i) = \mu_B(x_i)\ \forall i$.

P r o o f: We recall that

$$I(A; B) = H(A) + D(A\,|\,B).$$

Next the result directly follows from the fact that

$H(A) \geq 0$, $D(A\,|\,B) \geq 0$, and $D(A\,|\,B) = 0$ if and only if $A = B$.

This proves the theorem. ■

6. A numerical example

Let us consider a very pertinent corporate problem in which corporation X deals with several clients and negotiates on a certain number of issues.

Let the set of clients and issues be $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$, $I = \{Z_1, Z_2, Z_3, Z_4, Z_5\}$.

Table 1 represents the weights of issues of the corporation X in terms of fuzzy memberships $\mu_X(Z_i)$ where $i = 1, 2, 3, 4, 5$.

Table 1. Weights of issues of the corporation

$\mu_X(Z_1)$	$\mu_X(Z_2)$	$\mu_X(Z_3)$	$\mu_X(Z_4)$	$\mu_X(Z_5)$
0.8	0.9	0.6	0.5	0.7

It may be noted that $\mu_X(Z_i)$ indicates the degree of importance of issue Z_i to the corporation in dealing.

Table 2 represents the degree of importance of clients $\mu_{C_j}(Z_i)$ on issue Z_i in this dealing, where $j = 1, 2, 3, 4, 5$.

Table 2. Weights of issues of the clients

C_1	$\mu_{C_1}(Z_1)$=0.6	$\mu_{C_1}(Z_2)$=0.7	$\mu_{C_1}(Z_3)$=0.5	$\mu_{C_1}(Z_4)$=0.8	$\mu_{C_1}(Z_5)$=0.7
C_2	$\mu_{C_2}(Z_1)$=0.9	$\mu_{C_2}(Z_2)$=0.8	$\mu_{C_2}(Z_3)$=0.6	$\mu_{C_2}(Z_4)$=0.4	$\mu_{C_2}(Z_5)$=0.8
C_3	$\mu_{C_3}(Z_1)$=0.7	$\mu_{C_3}(Z_2)$=0.6	$\mu_{C_3}(Z_3)$=0.8	$\mu_{C_3}(Z_4)$=0.4	$\mu_{C_3}(Z_5)$=0.6
C_4	$\mu_{C_4}(Z_1)$=0.6	$\mu_{C_4}(Z_2)$=0.8	$\mu_{C_4}(Z_3)$=0.6	$\mu_{C_4}(Z_4)$=0.7	$\mu_{C_4}(Z_5)$=0.9
C_5	$\mu_{C_5}(Z_1)$=0.3	$\mu_{C_5}(Z_2)$=0.7	$\mu_{C_5}(Z_3)$=0.4	$\mu_{C_5}(Z_4)$=0.6	$\mu_{C_5}(Z_5)$=0.5

In a deal the objective of the corporation being to choose the best client, which with the degree of negotiation on common issues should be minimum (equal to the entropy value of the corporation).

So, using formula (10) we obtain the fuzzy inaccuracy measures $I(X;\ C_i)$ where $i = 1, 2, 3, 4, 5$.

Table 3. Inaccuracy measures between a corporation and clients

$I(X; C_1)$	$I(X; C_2)$	$I(X; C_3)$	$I(X; C_4)$	$I(X; C_5)$
0.9388	0.8460	0.9237	0.9151	1.0493

According to the inaccuracy measures presented in Table 3, client C_2 will be more suitable for this deal.

6. Conclusions

In this work we have proposed a new inaccuracy measure, called *fuzzy inaccuracy* in the setting of fuzzy set theory. This measure can be considered as a generalized version of fuzzy entropy proposed by De Luca and Termini [4]. An illustrative numerical example illustrated the applications of this inaccuracy measure in business. Parametric studies that introduce other flexibility criteria for the same membership functions, of this measure are also under study and will be reported separately.

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