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Supply Chain Management in Case of Batch Chemical Manufacturing, Optimal Distribution of Capacities, Concept and Software Realization

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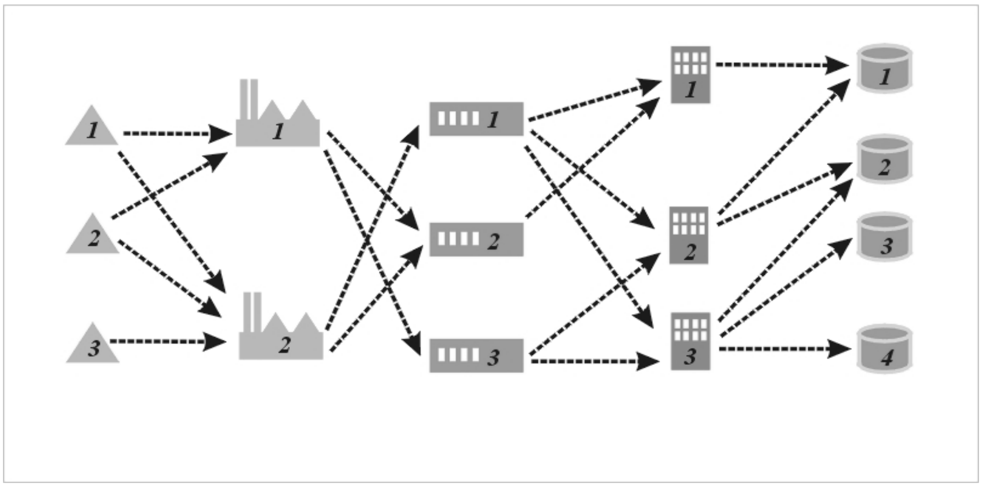
Abstract: *In recent years the problem of optimal management of multipurpose and multiproduct batch plants (MPBP) and their complexes in the chemical industry has an extreme increase. The work was focused on the merging of multipurpose plants into corporations (Supply chain) aimed at achievement of sustainable results of the unified system. In particular, the merging of chemical, especially pharmaceutical plants that exhibit specific features and the relevant problems of optimal performance has been targeted. The main objective solved is formulation of a planning strategy and schedules of multipurpose chemical plants, while accounting for the basic commercial requirements. A strategy, based on the decomposition approach, grounded on a two-stage optimal control task of the multipurpose system, has been proposed. Stage one considers definition of product portfolios corresponding to each individual plant of the complex.*

The feasibility of the suggested approach and its mathematical background in solving the first stage of the strategy are demonstrated by an example approximation of reality, based on software generated by MATLAB version 6.50.

Keywords: *Supply chain optimization, multipurpose and multiproduct batch plants.*

1. Introduction

The problem of optimal control of Multipurpose-multiProduct chemical Batch Plants (MPBP) has become recently important, this is also dictated by the current state of financial crisis worldwide. This requires the creation of flexible models responding to dynamical market in general. Formerly, only the optimal operation of single production units has been considered without paying attention to the complex interaction of the chain "supply-production-warehouse-distributor-market" (Fig. 1).



Suppliers-productions-warehouses-distributors-markets

Fig. 1. Supply chain structure

Nowadays, one could not achieve sustainable performance without optimal control along the whole chain. It is characteristic of the business nowadays that whole sets of multipurpose plants merge into corporations to enhance the corporate unit performance and to achieve stable economic parameters of the generated system. Such corporations are typical in the area of domestic chemistry, fine chemicals production and most often in the pharmaceutical industry. Only joint optimal coordination of the activities along the whole chain, combined with optimal operation of the separate units, could result in stable and predictable operation of the whole system. In order to overcome this problem, new methodologies should be proposed to solve the entire problem.

In recent years, the final solution of the overall problem, regarding MPBP, has been related to the ideas of ***Supply Chain*** (SC) optimization S h a h [10]. This concept is used by many adaptive target detection algorithms, which compare the signal intensity with an adaptive threshold with value depending on the noise level. The SC in itself presents both optimization of plant infrastructure and modeling, as well as analysis and development of production programs, accounting for the market demands and its probability nature. The reason for such activity is the enhanced competition in a global economy that makes companies respond to the market demands, financial crisis, while at the same time their production capacity is overloaded in order to achieve maximum effectiveness of commercial performance.

The problem of optimal management of multipurpose and multiproduct batch chemical plants, using the concept of SC for optimization of the plants infrastructure and development of production programs has been of extreme interest in the recent years. The subject has been treated frequently theoretically with relevant application in industry. The SC have been studied theoretically since the beginning of the 90-ies. The theoretical problems referred to in the world literature consider both optimal control tasks and SC design tasks, planning and optimal schedules formulae specific for the separate components in the chain. The concept of SC grew up at the beginning of 90-ies and has recently raised a lot of interest since the opportunity of an integrated management of the SC can reduce the unexpected events throughout the network and also affect decisively the profitability of all the members by which SC is designed. Typical characteristic of MPBP are namely the multivariant solutions, which allow obtaining different strategies in the process of SC optimal control. The SC-related problems belong to three main groups: The first one includes the chain analysis and policy formulation, the second one – planning and scheduling, and the third one is aimed at optimal infrastructure (network design). The analysis process is related to the design of the network of production units, warehouses, distributors and markets that belong to SC. The degrees of uncertainty are examined. Initially, the problem has been defined as a "classical" one by G e o f f r i o n and G r a v e s [1]. B r o w n et al. [2] have considered the design production aspect introducing the so called "open" and "closed" manufactures. K a l l r a t h [3] has described a model of simultaneous strategy and operative planning in a multiple production unit, comprising a network of plants. Tsiakis et al. [4] have shown the requirements regarding the uncertainty that could be introduced by the multiperiod model. They assumed that the future uncertainty could be "evaluated", and defined as a part of a previously set scenario "tree", where each scenario has a different sequence. The capacity planning problem is reduced to determination of the processes that will operate in the future, allowing inclusion of a new process. Such problems have been developed by C a m m et al. [5]. These authors have defined the trend of product cost domination among the processes. S a h i n i d i s et al. [6] have reported an approach of selection of processes operating in a network. L i u and S a h i n i d i s [7] underline the meaning of improvement of the solutions efficiency of these class problems. A h m e d and S a h i n i d i s [8] have focused upon model stability, involving the so called "punishment risk" defined as costs that exceed the expected costs relevant to the pre-expectations. A h m e d and S a h i n i d i s [9] include fast planning approximation that guarantees generation of a realizable solution. G u i l l é n et al. [11] have considered the problem of SC optimal design and multipurpose problem in the case of three-echelon SC: production-warehouses-market. They have matched various criteria to account for estimates such as maximum profit or the financial risk. These authors employ a multiperiod model and use a mathematical programming formulation. Arriving at a conclusion on the basis of the present literature survey the aim of the present study is to propose a method for optimal operative control of MPBP and complexes by accounting for the market demand and allowing flexible control along the elements of the chain in the case of five-echelon SC "suppliers-productions-warehouses-distributors-markets" [12, Fig. 1].

2. Types of problems in supply chain management

Different and varied types of problems arise in the management of supply chain, Fig. 2 illustrated the main areas of research.

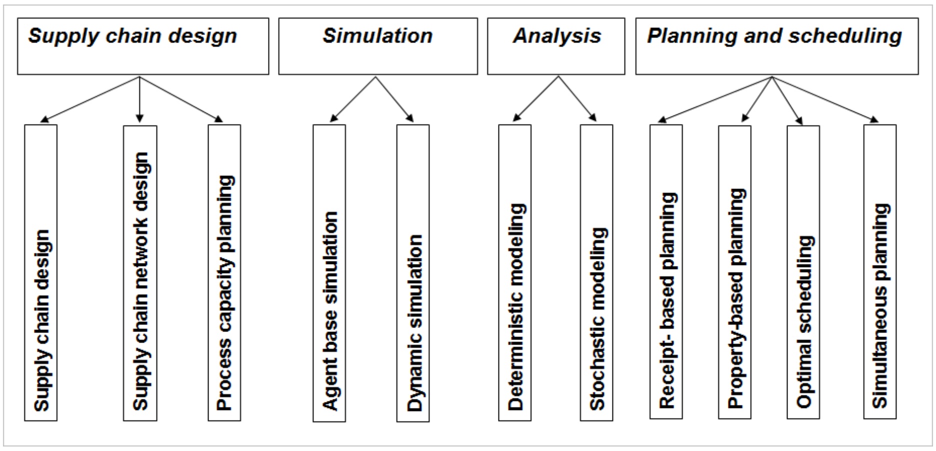


Fig. 2. Main research areas of Supply chain management

The processes of optimal management the supply chain network require three main objectives:

1) analysis and preparation;

of adequate and flexible models describing processes in the chain;

2) development of optimal production schedules;

3) optimal scheduling in real practice and obtain the objective targets in manufacturing.

It is important to remark that in recent years, especially from 2008 till now the problems which arise in this area are an extremely actual, this also appear from the financial and world crisis. Take into account this environment many of companies most frequently concentrate his efforts to reduce any production costs from the one side and also improve distribution processes so that to respond to market demand. All these problems cover the concept of supply chain management.

3. Problem statement

3.1. Characteristics of supply chain elements

The main advantages of supply chain are flexibility on the number of the participants in the chain and also the quantities and nomenclature of demanded products for each one of the participant. This gives an opportunity to the companies to simulate different scenarios and take simultaneously decision in critical situation in real practice.

3.2. The strategy

The main objective of SC is integration of all participants in such a way as to achieve optimal coordination between them S h a h [10]. The macro process is focused on achievement of optimal coordination between the chain components, illustrated in Fig. 1, aimed at optimal performance parameters, based on pre-determined criteria. To achieve this goal, the task can be fulfilled by consecutive solving of the sub tasks, as follows: The first stage includes an obligatory preliminary study of the market demands. During this stage, the requirements put forward by each potential customer in the planning horizon have to be determined.

The second stage includes determination of:

1) optimal product portfolio of each plant;
2) optimal raw materials supply to each plant;
3) optimal scheme of loading of warehouses with products;
4) optimal scheme of serving the distributors;
5) optimal scheme of serving the customers.

The third stage concerns the production schedules on plant scale corresponding to the product portfolio, obtained in the second stage. These steps are defined as part of the decomposition solution approach. It comes from the multiple-dimensional property of the task, as well as from the fact that it is practically insoluble by simultaneous procedure.

4. Mathmatical formulation of the problem

4.1. Model assumption

We assume that the supply chain network is fixed and consist of large data base of every one participant in the chain: of suppliers, plants, warehouses, distributors and customers. All available connections (binary or continuous) between the elements of the Supply Chain are set and can be canceled according to the market environment. We also assumed that the model, which is supposed in case of not piling up of materials in any one of the elements, which Supply Chain consists of, the following data are considered:

4.2. Data regarding the SC system

A set of products included in the ***SC production list*** – $NC = \{nc_n\}$ is a set of the names of all products that can be produced in the SC plant.

Planning horizon – ***H-Planning horizon*** of the SC system operation is considered.

Product storage time in the warehouses – ***T_storage*** is product maximum storage time in the warehouses is accepted.

4.3. Supply chain elements data

4.3.1. A set of suppliers of raw materials – $R = \{r_i\}$, $i = 1, \ldots, I$.

4.3.2. Capacity of the suppliers of raw materials for products $-\ \mathrm{NCI} = \{\mathrm{nci}_{ni}\}$

4.3.3. Allowable amount of raw materials provided by the relevant plant suppliers – $\mathrm{QCI}^U = \{\mathrm{qci}_{ni}^U\}, \mathrm{QCI}^L = \{\mathrm{qci}_{ni}^L\}$, where $\mathrm{qci}_{ni}^L, \mathrm{qci}_{ni}^U$ indicate the minimum and maximum amount of raw materials that can be delivered by the i-th supplier to n-th production unit during the planning horizon.

4.3.4. Possible relations between plants-suppliers – $\mathrm{XIJ} = \{\mathrm{xij}_{nij}\}$,

$$\mathrm{xij}_{nij} = \begin{cases} 1, \text{ if relation is established} \\ 0, \text{ otherwise.} \end{cases}$$

4.3.5. Prices of the raw material by product at the suppliers – $\mathrm{COST\_RM} = \{\mathrm{c\_rm}_{ni}\}$,

where $\mathrm{c\_rm}_{ni}$ is the price of unit n-th product raw material per unit n-th product at the i-th supplier.

4.3.6. Prices for raw materials transportation from the suppliers to the plants – $\mathrm{CJK} = \{\mathrm{cjk}_{njk}\}$, cjk_{njk} is the transportation price per unit of n-th product from j-th plant to k-th warehouse.

4.4. Data regarding the multipurpose plants in SC

4.4.1. A set of plants names where products are manufactured prior to sending them to the warehouses $-P = \{p_j\}, j = 1, ..., J$.

4.4.2. Production capacity related to products at the plants $-\ \mathrm{NCJ} = \{\mathrm{ncj}_{nj}\}$,

$$\mathrm{ncj}_{nj} = \begin{cases} 1, \text{ if product } n \text{ can be produced by the } j\text{-th plant} \\ 0, \text{ otherwise.} \end{cases}$$

4.4.3. Plant capacity for individual products

$\mathrm{QCJ}^U = \{\mathrm{qcj}_{nj}^U\}, \mathrm{QCJ}^L = \{\mathrm{qcj}_{nj}^L\}$ indicate the n-th product minimum and maximum amount that could be produced by the j-th plant in the planning horizon.

4.4.4. Matrix of plant's capacity – $\mathrm{QCJ\_Full} = \{\mathrm{qcj\_full}_j^U\}$, where $\mathrm{qcj\_full}_j^U$ indicates the mean value of the product manufactured in the planning horizon by the j-th plant included in SC by all possible unique companies.

4.4.5. Matrix, including the capacities of the individual lines relevant to the SC plants $\mathrm{Q\_Ind} = \{q_{nm}^U\}$ $\forall n, m,\ n = \mathrm{N\_Ind}_m$ where $\mathrm{N\_Ind}_m$ is the set of productions of a plant, related to individual lines. In cases where the individual lines involve batch processes, $\{q_{nm}^{\mathrm{U}}\}$ is determined by the equation:

(1) $$q_{nm}^{\mathrm{U}} = \left\lfloor \frac{H}{T_{nm}} \right\rfloor B_{nm} \quad \forall m, n = \{\mathrm{N\_Ind}_m\},$$

B_{nm}, T_{nm} indicate the batch size and the cycle time period of the specific line comprising relevant batch cycle.

4.4.5. Matrix, including the capacities-for-product of the multi-product lines of SC plants

The capacities of the plants' multiproduct lines for products are:
$\text{Q\_Ind} = \{q^{\text{U}}_{nm}\} \quad \forall n, m, p, n = \text{J\_multiproduct}_{mp}$, indicates the set of plant products manufactured at the relevant multi-product lines p. In case that these lines involve batch cycles, $\{q^{\text{U}}_{nm}\}$ is determined by the equation

(2) $$q^{\text{U}}_{nmp} = \left\lfloor \frac{H}{T_{nmp}} \right\rfloor B_{nmp} \qquad \forall m, p, n = \{\text{J\_multiproduct}_{mp}\},$$

B_{nmp}, T_{nmp} indicate the batch size and the cycle time period.

4.4.6. Possible relationship between the plants and the warehouses – YJK = $\{\text{yjk}_{njk}\}$,

$$\text{yjk}_{njk} = \begin{cases} 1, \text{for supply from } j\text{-th plant to the } k\text{-th warehouse for product } n\text{ - manufacturing,} \\ 0, \text{otherwise.} \end{cases}$$

4.4.7. Prices for product transportation from the plants to the warehouses $-\text{CJK} = \{\text{cjk}_{njk}\}$, cjk_{njk} is the transportation price per unit of n-th product from j-th plant to k-th warehouse.

4.4.8. Production cost of the products at the plants – $\text{PP\_PLANTS} = \{\text{pp}_{nj}\}$, pp_{nj} is the production cost for the unit of n-th product at j-th plant of the overall SC.

4.5. Data related to SC warehouses

4.5.1. A set of warehouses of SC $- S = \{s_k\}, k = 1,...,K.$

4.5.2. Prospects for product storage in the warehouses $-\text{NCK} = \{\text{nck}_{nk}\}$,

$$\text{nck}_{nk} = \begin{cases} 1, \text{for the case that the product can be stored in } k\text{-th warehouse } n\text{-manufacturing,} \\ 0, \text{otherwise.} \end{cases}$$

4.5.3. Product storage capacity of the warehouses $-\text{QCK}^U = \{\text{qck}^U_{nk}\}, \text{QCK}^L = \{\text{qck}^L_{nk}\}$, where $\text{qck}^L_{nk}, \text{qck}^U_{nk}$ are the minimum and maximum amount of n-th product to be stored in k-th warehouse.

4.5.4. Warehouse capacity for all products – $\text{VCK}^L = \{\text{vck}^L_k\}, \text{VCK}^U = \{\text{vck}^U_k\}$, where $\text{vck}^L_k, \text{vck}^U_k$ indicate the minimum and maximum operating capacity of k-th warehouse.

4.5.5. Possible relationship between the warehouses and the distribution centers $-\mathrm{ZKL}=\{\mathrm{zkl}_{nkl}\}$,

$$\mathrm{zkl}_{nkl}=\begin{cases}1, \text{for the case that } n\text{-th product can be delivered by } k\text{-th warehouse to} \\ \quad l\text{-th distribution centre,} \\ 0, \text{otherwise.}\end{cases}$$

4.5.6. Prices of product transportation from the warehouses to the distributors $-\ \mathrm{CKL}=\{\mathrm{cs}_{nkl}\}$, cs_{nkl} is the transportation price per unit of n-th product from the k-th warehouse to 1-th distribution centre.

4.5.7. Product storage prices $-\mathrm{PS}=\{\mathrm{ps}_{nk}\}$, ps_{nk} is the storage price per unit of n-th product in k-th warehouse per unit of time.

4.5.8. Storage capacity factors of products $-\ \mathrm{CR}=\{\mathrm{cr}_n\}$, where cr_n is the storage capacity required for unit of n-th product.

4.6. Data regarding the distributors in SC

4.6.1. A set of distributors in SC $-D\in\{d_l\}, l=1,...,L.$

4.6.2. Prospects for product handling at the distributor $-\ \mathrm{NCL}=\{\mathrm{ncl}_{nl}\}$,

$$ncl_{nkl}=\begin{cases}1, \text{for the case that } n\text{-th product can be handled by } l\text{-th distribution centre,} \\ 0, \text{otherwise.}\end{cases}$$

4.6.3. Handling capacity for products at the distributon $-\ \mathrm{QCL}^L=\{\mathrm{qcl}_{nl}^L\}$, $\mathrm{QCL}^U=\{\mathrm{qcl}_{nl}^U\}$, $\mathrm{qcl}_{nl}^L, \mathrm{qcl}_{nl}^U$ indicate the n-th product minimum and maximum amount that could be operated by i-th distributor within the planning horizon.

4.6.4. Relationships between the distributors and the markets $-\ \mathrm{GLM}=\{\mathrm{glm}_{nlm}\}$,

$$\mathrm{glm}_{nlm}=\begin{cases}1, \text{for the case that } n\text{-th product can be delivered to } l\text{-th distribution centre,} \\ 0, \text{otherwise.}\end{cases}$$

4.6.5. Prices of product transportation from the distributors to the markets $-\ \mathrm{CLM}=\{\mathrm{cg}_{nlm}\}$, cg_{nlm} is the transportation price per unit of n-th product from l-th distributor to m-th market.

4.6.6. Price for product handling at the distributor $-\ \mathrm{PD}=\{\mathrm{pd}_{nl}\}$, pd_{nl} is the price for handling and storage of unit of n-th product at l-th distributor.

4.7. Data regarding markets in the SC

4.7.1. A set of final markets in the SC $-\ C=\{c_m\}, m=1,...,M$, is a set of the names of the customers.

4.7.2. Markets requirements for product nomenclature $-\mathrm{NCM}=\{\mathrm{ncm}_{nm}\}$,

$$\mathrm{ncm}_{nm}=\begin{cases}1, \text{for the case that the } n\text{-th product can be supplied by } l\text{-th distributor,} \\ 0, \text{ otherwise.}\end{cases}$$

4.7.3. Demand satisfaction – $\mathrm{QCM}^U = \left\{\mathrm{qcm}_{nm}^U\right\}, \mathrm{QCM}^L = \left\{\mathrm{qcm}_{nm}^L\right\}$, qcm_{nm}^L, qcm_{nm}^U indicate the *n*-th product minimum and maximum amount required for the demand satisfaction of *m*-th market within the planning horizon.

4.7.4. Product market prices $\mathrm{COST\_CL} = \left\{\mathrm{cc}_{nm}\right\}$, where cc_{nm} is the final selling price of *n*-th product at *m*-th market.

4.8. Sets of independent variables

4.8.1. Amount of products produced in the company plants – $\mathrm{QNJ} = \left\{q_{nj}\right\}$, q_{nj} is the amount of *n*-th product, produced in *j*-th plant of the company during the planning horizon to be determined.

4.8.2. Flows of materials between the nodes of the network $-\mathrm{XIJ} = \left\{x_{nij}\right\}$ – raw material supply ratios from the supplier to the plant, where $0 \le x_{nij} \le 1$.

4.8.3. Warehouse capacity loading ratios at the plants – $\mathrm{YJK} = \left\{y_{njk}\right\}$, where $0 \le y_{njk} \le 1$.

4.8.4. Warehouse loading ratio at the distributors – $\mathrm{ZKL} = \left\{z_{nkl}\right\}$, $0 \le z_{nkl} \le 1$.

4.8.5. Market loading ratios at the warehouse – $\mathrm{GLM} = \left\{g_{nlm}\right\}$, $0 \le g_{nlm} \le 1$.

4.9. Basic relationship

4.9.1. Raw material balance equation is given by

(3) $$\mathrm{QR}_{ni} = \sum_{j=1}^{J} x_{nji} \cdot q_{nj} \quad \forall i, n .$$

4.9.2. Product mass balance equation related to the items in the warehouses during the warehouses storage time intervals is given by

(4) $$\mathrm{QS}_{nk} = \sum_{j=1}^{J} y_{njk} \cdot q_{nj} \quad \forall k,\ n,$$

$$\mathrm{QS\_Storage}_{nk} = \frac{QS_{nk}\ cr_n}{\mathrm{T\_storage}} \quad \forall k,\ n.$$

4.9.3. Product mass balance equation related to items at the distributors

(5) $$\mathrm{QD}_{nl} = \sum_{k=1}^{K} \left(z_{nkl}\ \mathrm{QS}_{nk}\right) \ \forall l,\ n,$$

$$\mathrm{QD}_{nl} = \sum_{k=1}^{K} \left(z_{nkl} \sum_{j=1}^{J} y_{njk}\ q_{nj} \right) \ \forall l,\ n..$$

4.9.4. Demand balance equation

(6) $$\mathrm{QM}_{nm} = \sum_{l=1}^{L} \left(g_{nlm}\ \mathrm{QD}_{nl} \right) \quad \forall m,\ n,$$

$$\mathrm{QM}_{nm} = \sum_{l=1}^{L} \left(g_{nlm} \sum_{k=1}^{K} \left(z_{nkl} \sum_{j=1}^{J} y_{njk}\ q_{nj} \right) \right) \quad \forall m,\ n.$$

4.10. Capacity constraints

4.10.1. Capacity constraints, product suppliers are

(7) $$\mathrm{nci}_{ni}^{L} \le \mathrm{QR}_{ni} \le \mathrm{nci}_{ni}^{U} \quad \forall n,\ i.$$

4.10.2. Capacity constraints, product warehouses are

(8) $$\mathrm{qck}_{nk}^{L} \le \mathrm{QS\_Storage}_{nk} \le \mathrm{qck}_{nk}^{U} \quad \forall n,\ k.$$

4.10.3. Capacity constraints, product warehouses, warehouses time are

(9) $$\mathrm{vck}_{k}^{L} \le \sum_{n=1}^{N} \mathrm{QS\_Storage}_{nk} \le vck_{k}^{U} \quad \forall k.$$

4.10.4. Capacity constraints, product distributors are

(10) $$\mathrm{qck}_{nk}^{L} \le \mathrm{QD}_{nl} \le \mathrm{qcl}_{nl}^{U} \quad \forall n, l.$$

4.10.5. Capacity constraints, product markets are

(11) $$\mathrm{qcm}_{nm}^{L} \le \mathrm{QM}_{nm} \le \mathrm{qcm}_{nm}^{U} \quad \forall n,\ m.$$

4.10.6. Product portfolio feasibility constraints

In the general case one expects that a specific multi-purpose plant of the SC comprises a set of individual batch-operating lines, a set of multi-product lines and finally a multipurpose line intended for a group of products not-included in the previous groups. Referring to the latter group, it is characteristic that it may follow different scenarios of various production rates. Each group of products imposes different constraints, which ensure that the optimal production programme would be fulfilled.

4.10.7. Product portfolio feasibility constraints for individual batch lines

The amounts of products q_{nm} that are being manufactured by using individual lines by the separate plants of the SC have to answer the following set of inequalities:

(12) $$q_{nm} \le q_{nm}^{U} \quad \forall m,\ n \in \mathrm{J\_ind}_{m},$$

$\mathrm{J\_ind}_{m}$ denotes the set of products that are manufactured by individual lines in the plants, while q_{nm}^{U} is pre-determined by using (1).

4.10.8. The product portfolio feasibility constraints for multiproduct batch systems are

(13) $$\sum_{n \in \mathrm{J\_multiproduct}_{mp}} \frac{q_{nm}}{Q_{nm}} \le H \quad \forall m,\, p,\, n \in \mathrm{J\_multiproduct}_{mp},$$

Q_{nm} (in kg/h) indicates the throughput of m-th line following n-th product when it is being manufactured at p-th multi-product line and it is operating continuously.

4.10.9. The set of constraints relevant to the multiproduct lines with batch cycles are

(14) $$\sum_{n \in \mathrm{J\_mproduct}_{mp}} \left\lfloor \frac{q_{nm}}{B_{nm}} \right\rfloor T_{nm} \le H \quad \forall m,\, p,\, n \in \mathrm{J\_mproduct}_{mp},$$

B_{nm}, T_{nm} are the dimensional factor and the limiting time interval of the relevant product manufactured in the relevant plant.

4.10.10. The product portfolio feasibility constraints for multipurpose batch systems are

(15) $$\mathrm{qcj\_full}_j^L \le \sum_{n \in \mathrm{J\_mproduct}_m} q_{nm} \le \mathrm{qcj\_full}_j^U \quad \forall m, n \in \mathrm{J\_mproduct}_m,$$

$\mathrm{J\_mproduct}_m$ indicates the sets of products, manufactured by the multi-purpose lines of the relevant plants.

4.11. Functional constraints

4.11.1. The relationship imposing condition of delivery to a specific plant by a set of suppliers of a predetermined amount of the specific product is

(16) $$\sum_{i=1}^{I} x_{nji} = 1 \quad \forall j, n.$$

4.11.2. The constraint imposing the condition of a specific product manufactured by any plant, to be distributed at a predetermined ratio among the warehouses is

(17) $$\sum_{k=1}^{K} y_{njk} = 1 \quad \forall n, j.$$

4.11.3. The constraint imposing the condition of a specific product of any warehouse to be distributed at predetermined ratios among the distributors

(18) $$\sum_{l=1}^{L} z_{nkl} = 1 \quad \forall n, k.$$

4.11.4. The constraint imposing the condition of a specific product from any distributor to be distributed at predetermined ratios among the markets is

(19) $$\sum_{m=1}^{M} g_{nlm} = 1 \quad \forall n,\ l.$$

4.12. The bounds on the variables are

(20) $$0 \le y_{njk} \le 1 \quad \forall n, k, j,$$

(21) $$0 \le z_{nkl} \le 1 \quad \forall n, k, l,$$

(22) $$0 \le g_{nlm} \le 1 \quad \forall n, m, l,$$

(23) $$\mathrm{qcj}_{nj}^{\mathrm{L}} \le q_{nj} \le \mathrm{qcj}_{nj}^{\mathrm{U}} \quad \forall n, j.$$

4.13. Objective function

4.13.1. Price estimates

The relationship of the product net values at the plants is

(24) $$\mathrm{COST}_{nj} = \mathrm{pp}_{nj} + \sum_{i=1}^{I} \left(x_{nij} \left(\mathrm{c\_rm}_{ni} + \mathrm{cij}_{nij} \right) \right) \quad \forall n, j.$$

4.13.2. The relationship of the product mean values at the warehouses is

(25) $$\mathrm{CS}_{nkj} = \mathrm{COST}_{nj} + \mathrm{cjk}_{njk} + \mathrm{ps}_{nk} \quad \forall n, j, k.$$

This is the product price, including the cost of product transport from the plant to the warehouse and the handling and storage costs. The total value of *n*-th product, stored in the warehouse can be determined by the equation

(26) $$\mathrm{PKN}_{nk} = \sum_{j=1}^{J} \left(\mathrm{CS}_{nkj} y_{njk} q_{nj} \right) \quad \forall n, k.$$

The mean price of unit of product at the outlet of the warehouse will be

(27) $$\mathrm{Price\_K}_{nk} = \frac{\mathrm{PKN}_{nk}}{\mathrm{QS}_{nk}} \quad \forall n, k.$$

4.13.3. The relationship of the mean price of supply to the markets is

(28) $$\mathrm{CS}_{nkl} = \mathrm{Price\_K}_{nk} + \mathrm{cs}_{nkl} + \mathrm{pd}_{nl} \quad \forall n, k, l.$$

This is the product price, including the transport cost from the warehouse to the distributor and the handling and storage price at the distributor. The total price of products, stored and handled at the distributor, can be determined by the equation

(29) $$\mathrm{PKL}_{nl} = \sum_{k=1}^{K} \left(\mathrm{CS}_{nkl}\ z_{nkl}\ \mathrm{QS}_{nk} \right) \quad \forall n, l.$$

4.13.4. Relationship of profit evaluation at the market

(30) $$\mathrm{Price\_M}_{nml} = \mathrm{Price\_D}_{nl} + \mathrm{cg}_{nlm} \quad \forall n,\ m, l.$$

The profit from the purchase of a specific product, delivered by a specific distributor to a specific customer will be determined as follows:

(31) $$\text{Profit}_{nml} = \left(\text{cc}_{nm} - \text{Price\_M}_{nml}\right) g_{nlm}\, QD_{nl} \quad \forall n, m, l\,.$$

Then the total profit from selling a specific product to a specific customer will be

(32) $$\text{Profit\_NM}_{nm} = \sum_{l=1}^{L} \text{Profit}_{nml} \quad \forall n, m.$$

The profit at a specific market for selling off all the products of its list will be

(33) $$\text{Profit\_M}_{m} = \sum_{n=1}^{N} \text{Profit\_NM}_{nm} \quad \forall m\,.$$

The profit, realized by the performance of the whole company will be

(34) $$\text{Profit\_C} = \sum_{m=1}^{M} \text{Profit\_M}_{m},$$

or in a developed form

(35) $$\text{Profit\_C} = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{l=1}^{L} \left(\text{cc}_{nm} - \text{Price\_M}_{nml}\right) g_{nlm}\, \text{QD}_{nl}\,.$$

4.13.5. The objective function aimed at company profit NPV maximization would be

(36) $$\text{MAX(Profit\_C)}.$$

5. Problem statement for profit maximization

The maximum of the objective function (36) upon system restrictions (7)-(23) involving the sets of independent (and continuous) variables x_{nji}, y_{njk}, z_{nkl}, g_{nlm} is to be sought.

6. Case study

SC comprising of two plants, three eventual suppliers at different distances from the plant, three warehouses, three distributors and four markets, is illustrated in Fig. 1. The plants can produce nine different products. Three of them are manufactured by using individual lines, while the rest are manufactured by using multiproduct lines. Tables 1-5 contain the main data, needed to solve the problem by using the mathematical formulation proposed.

Using the methodology described, an optimal SC performance, aimed at profit maximization, can be determined. With regard to the configuration of SC elements and allowable product relationships (connections) illustrated in Fig. 1, the problem is trans-formed into a task for continuous nonlinear programming with the following specific dimensions: 189 independent continuous variables, 72 linear constraints of the equality type, 4 non-linear constraints of the equality type , 234 non-linear constraints of the non-equality type.

Table 1. Lower and upper capacity bounds of SC elements

Elements of SC/Products	MIN/MAX								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Supplier1	0/200	0/120	0/180	0/550	0/840	0/320	0/500	0/300	0/85
Supplier2	0/100	0/220	0/120	0/400	0/340	0/120	0/350	0/130	0/180
Supplier3	0/300	0/80	0/100	0/340	0/440	0/220	0/90	0/80	0/255
Plant1	0/100	0/180	0/200	0/140	0/130	0/90	0/190	0/180	0/300
Plant2	0/200	0/250	0/300	0/240	0/240	0/220	0/250	0/280	0/400
Warehouse1	0/200	0/250	0/300	0/240	0/240	0/220	0/250	0/280	0/400
Warehouse2	0/300	0/80	0/100	0/340	0/440	0/220	0/90	0/80	0/255
Warehouse3	0/100	0/220	0/120	0/400	0/340	0/120	0/350	0/130	0/180
Distributor1	0/300	0/80	0/100	0/340	0/440	0/220	0/90	0/80	0/255
Distributor2	0/100	0/180	0/200	0/140	0/130	0/90	0/190	0/180	0/300
Distributor3	0/200	0/250	0/300	0/240	0/240	0/220	0/250	0/280	0/400
Market1	0/300	0/180	0/100	0/140	0/140	0/110	0/90	0/80	0/25
Market2	0/300	0/280	0/200	0/130	0/350	0/90	0/190	0/40	0/155
Market3	0/300	0/480	0/300	0/240	0/230	0/360	0/130	0/50	0/234
Market4	0/300	0/580	0/100	0/340	0/560	0/160	0/310	0/30	0/70

Table 2. Production costs of products at SC plants

Elements of SC/Products	Production costs of products at SC plants								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Supplier1	2	2.2	2.1	2.4	2.5	1.6	1.9	2	2.3
Supplier2	2.1	2	2.2	2.4	2.5	1.6	1.9	2	2.3
Supplier3	2.1	2	2.2	2.4	2.5	1.6	1.9	2	2.3
Plant1	18	12	11	40	24	25	72.34	30	38
Plant2	22	14	15	46	28	23	62.34	32	35
Warehouse1	2.1	2.0	2.2	2.4	2.5	1.6	1.9	2	2.3
Warehouse2	2.1	2.0	2.2	2.4	2.5	1.6	1.9	2	2.3
Warehouse3	2.1	2.0	2.2	2.4	2.5	1.6	1.9	2	2.3
Distributor1	2.1	2.4	2.2	2.4	2.5	1.6	1.9	2	2.3
Distributor2	2.2	2.1	2.2	2.4	2.5	1.6	1.9	2	2.3
Distributor3	2.3	2.3	2.2	2.4	2.5	1.6	1.9	2	2.3
Market1	2.4	2.5	2.7	2.4	2.8	1.6	1.9	2.3	2.5
Market2	2.1	2.0	2.2	2.4	2.5	1.6	1.9	2	2.3
Market3	2.1	2.0	2.2	2.4	2.5	1.6	1.9	2	2.3
Market4	2.1	2.0	2.2	2.4	2.5	1.6	1.9	2	2.3

Table 3. Raw materials prices at suppliers

Elements of SC/Products	Raw materials prices per unit of product at various suppliers								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Supplier1	10	15	23	24	22	26	30	17	23
Supplier2	12	13	24	18	23	24	34	12	21
Supplier3	11	12	25	20	24	25	38	18	23

Table 4. Product prices on the market

Elements of SC/Products	End price of various products on the market								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Market1	56	68	66	87	74	94	130	81	87
Market2	52	70	68	86	70	91	136	88	92
Market3	51	69	69	89	72	98	140	78	67
Market4	54	65	64	87	76	99	148	86	60

Table 5. Transportation costs to Demand Sources for products

Supplier-Plant	**Plant1**								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Supplier1	1.3	1.3	1.3	1.5	1.5	1.5	1.4	1.4	1.4
Supplier2	1.7	1.7	1.7	1.4	1.4	1.4	1.8	1.8	1.8
Supplier3	3.2	3.2	3.2	3.2	3.2	3.2	3.7	3.7	3.7
Supplier-Plant	Plant2								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Supplier1	2.3	2.3	2.3	2.5	2.5	2.5	2.4	2.4	2.4
Supplier2	2.7	2.7	2.7	2.1	2.1	2.1	2.8	2.8	2.8
Supplier3	1.2	1.2	1.2	1.2	1.2	1.2	1.7	1.7	1.7
Plant -Warehouse	Warehouse1								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Plant 1	7	7	7	7	7	7	7	7	7
Plant 2	5	5	5	5	5	5	5	5	5
Plant -Warehouse	Warehouse2								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Plant 1	6	6	6	6	6	6	6	6	
Plant 2	7	7	7	7	7	7	7	7	7
Plant -Warehouse	Warehouse3								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Plant 1	7	7	7	7.3	7.3	7.3	7	7	7
Plant 2	4	4	4.2	4.3	4.3	4.7	4.7	4.7	4.7
Warehouse-Distributor	Distributor 1								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Warehouse 1	0.62	0.62	0.62	0.92	0.92	0.92	1.3	1.3	1.3
Warehouse 2	1.38	1.38	1.38	1.82	1.82	1.82	1.45	1.45	1.45
Warehouse 3	4.9	4.9	4.9	5.3	5.3	5.3	4.4	4.4	4.4
Warehouse-Distributor	Distributor 2								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Warehouse 1	0.62	0.62	0.62	0.92	0.92	0.92	2.3	2.3	2.3
Warehouse 2	1.38	1.38	1.38	0.82	0.82	0.82	1.45	1.45	1.45
Warehouse 3	4.9	4.9	4.9	5.3	5.3	5.3	1.4	1.4	1.4

Table 5 (c o n t i n u e d)

Warehouse-Distributor	Distributor 3								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Warehouse 1	0.62	0.62	0.62	0.92	0.92	0.92	2.3	2.3	2.3
Warehouse 2	1.38	1.38	1.38	0.82	0.82	0.82	1.45	1.45	1.45
Warehouse 3	4.9	4.9	4.9	5.3	5.3	5.3	1.4	1.4	1.4
Distributor-Market	Market 1								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Distributor 1	0.62	0.62	0.62	0.68	0.68	0.68	0.63	0.63	0.63
Distributor 2	3.2	3.2	3.2	2.8	2.8	2.8	2.8	2.8	2.8
Distributor 3	1.3	1.3	1.3	1.6	1.6	1.6	1.8	1.8	1.8
Distributor-Market	Market 2								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Distributor 1	3.2	3.2	3.2	2.8	2.8	2.8	2.8	2.8	2.8
Distributor 2	0.62	0.62	0.62	0.68	0.68	0.68	0.63	0.63	0.63
Distributor 3	3	3	3	2.6	2.6	2.6	3.8	3.8	3.8
Distributor-Market	Market 3								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Distributor 1	3	3	3	2.6	2.6	2.6	3.8	3.8	3.8
Distributor 2	3.2	3.2	3.2	2.8	2.8	2.8	2.8	2.8	2.8
Distributor 3	0.62	0.62	0.62	0.68	0.68	0.68	0.63	0.63	0.63
Distributor-Market	Market 4								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Distributor 1	3.2	3.2	3.2	2.8	2.8	2.8	2.8	2.8	2.8
Distributor 2	0.62	0.62	0.62	0.68	0.68	0.68	0.63	0.63	0.63
Distributor 3	3	3	3	2.6	2.6	2.6	3.8	3.8	3.8

Table 6. Optimal product portfolio and optimal distribution by SC elements

Elements of SC/Products	**Optimal product portfolio and SC distribution elements**								
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Supplier1	(53.125)	(63.635)	(38.67)	(79.11)	(69.94)	(85.10)	(142.00)	(63.90)	(2.00)
Plant1	41.829	39.882	19.66	38.05	62.168	70.72	115.28	45.547	1.67
Plant2	11.295	23.75	19.01	41.056	7.766	14.378	26.725	18.358	0.326
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Supplier2	(99.26)	(21.06)	(67.70)	(53.21)	(38.75)	(115.06)	(107.78)	(37.83)	(19.5)
Plant1	20.11	11.23	34.03	18.218	3.713	18.666	12.921	5.291	6.60
Plant2	79.152	9.827	33.67	34.989	35.04	96.40	94.861	32.54	12.931
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Supplier3	(33.43)	(37.52)	(32.29)	(26.42)	(78.82)	(107.72)	(47.09)	(4.51)	(9.66)
Plant1	0	0	0	0	0	0	0	0	0
Plant2	33.43	37.52	32.29	26.42	78.82	107.72	47.09	4.51	9.66
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Plant1	(61.94)	(51.12)	(53.69)	(56.27)	(65.88)	(89.38)	(128.70)	(50.84)	(8.27)
Warehouse 1	0	0	0	0	0	0	0	0	0
Warehouse 2	54.96	18.58	26.81	53.08	51.85	49.12	17.71	24.93	3.468
Warehouse 3	6.97	32.53	26.87	3.18	14.03	40.26	110.48	25.90	4.807
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Plant2	(123.88)	(71.10)	(84.97)	(96.46)	(121.63)	(218.5)	(168.68)	(55.41)	(22.92)
Warehouse 1	43.89	19.75	47.277	73.167	56.01	204.099	144.10	41.75	10.445
Warehouse 2	79.58	51.355	37.70	23.296	65.618	14.401	24.58	13.65	12.474
Warehouse 3	0	0	0	0	0	0	0	0	0

Table 6 (continued)

	P1	P2	P3	P4	P5	P6	P7	P8	P9
Warehouse1	(43.89)	(19.75)	(47.27)	(73.16)	(56.01)	(204.09)	(144.11)	(41.75)	(10.44)
Distributor1	3.368	9.821	24.353	48.995	9.796	19.476	9.171	15.989	4.595
Distributor2	8.946	3.425	13.852	22.567	21.989	66.750	6.132	20.613	5.274
Distributor3	31.579	6.504	9.071	1.604	24.229	117.87	128.80	5.153	0.575
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Warehouse2	(134.95)	(69.94)	(64.51)	(76.38)	(117.47)	(63.52)	(42.29)	(38.58)	(15.94)
Distributor1	134.95	69.94)	64.51	76.38	117.47	63.52	42.29	38.58	15.94
Distributor2	0	0	0	0	0	0	0	0	0
Distributor3	0	0	0	0	0	0	0	0	0
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Warehouse3	(6.97)	(32.53)	(26.87)	(3.18)	(14.03)	(40.26)	(110.48)	(25.90)	(4.80)
Distributor1	0	0	0	0	0	0	0	0	0
Distributor2	0.127	14.253	9.18	0.454	8.444	16.861	60.34	11.83	2.63
Distributor3	6.84	18.278	17.699	2.729	5.585	23.601	50.144	14.07	2.172
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Distributor1	(138.32)	(79.76)	(88.86)	(125.38)	(127.27)	(83.00)	(51.47)	(54.57)	(20.53)
Market1	138.32	79.76	88.86	125.38	127.27	83.00	51.47	54.57	20.53
Market 2	0	0	0	0	0	0	0	0	0
Market 3	0	0	0	0	0	0	0	0	0
Market4	0	0	0	0	0	0	0	0	0
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Distributor2	(5.07)	(17.68)	(23.03)	(23.02)	(30.42)	(83.41)	(66.47)	(32.45)	(7.90)
Market1	1.22	7.06	10.86	13.73	11.76	21.32	37.94	25.35	4.13
Market 2	4.85	10.61	12.16	9.28	18.66	62.09	28.52	7.09	3.776
Market 3	0	0	0	0	0	0	0	0	0
Market4	0	0	0	0	0	0	0	0	0
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Distributor3	(38.42)	(24.78)	(26.77)	(4.33)	(29.81)	(141.47)	(178.94)	(19.22)	(2.74)
Market1	0	0	0	0	0	0	0	0	0
Market 2	9.949	17.91	6.41	0.354	6.98	21.48	38.38	17.07	0.988
Market 3	22.73	3.239	13.37	1.08	17.97	66.33	54.21	1.01	1.27
Market4	5.739	3.633	6.98	2.89	4.85	53.65	86.33	1.137	0.48

7. Software

Thus formulated, the mathematical programming task is a high-dimensional one and its resolution requires the use of a dual-step optimization method. It consists of a two-stage procedure, the first one performing a modified method of random search. The best solution obtained is applied as a starting point further. At the second stage, the method of combined search, related to the FMINCON module of code MATLAB v. 6.50 is activated. The overall problem is solved by a "Supply Chain" code, developed by the author.

8. Conclusions

The following conclusions could be drawn:

1. The study reveals the characteristics of MPBPs and their complexes as optimal management control.
2. A five-echelon SC accounting fully for the real situation has been revealed.

3. Based on the decomposition approach, a strategy of decision making in the area of MPBP optimal control is proposed.

4. The model of optimal batch determination of the SC components, taking into account for the potentials of the individual elements to fulfill the settings by composing optimal production schedules was developed.

5. Based on the method and its mathematical description, new software "Supply Chain" based on MATLAB version 6.50 is developed.

6. Supported by package"Supply Chain", an illustrative example has been solved in order to prove SCM efficiency.

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