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Alternative Approaches for Target Velocity Estimation Using the Hough Transform in MIMO Radar Systems

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Abstract: *Recently it has been shown that Multiple-Input Multiple-Output (MIMO) antenna systems have the potential to significantly improve the performance of communication systems compared to single antenna systems. An effective method of multiple-input multiple-output radar target detection and parameter estimation is proposed in this paper. Such contemporary approach for moving target velocity estimation is the application of a mathematical transform of the received input signals. The Hough transforms is very suitable for this case. Its application allows target parameter estimation maintaining low levels of the signal to noise ratio. In the present paper an original algorithm for two stage velocity estimation of a moving on a straight line target is proposed. To estimate the efficiency of the proposed algorithm a comparative analysis of the achieved results using two techniques – Doppler and Hough transform is carried out. A Monte-Carlo analysis of the two velocity estimation techniques is done based on the same parameters of surveillance radar. The numerical and graphical results show that both estimators make possible to estimate velocity with the same accuracy. An alternative three stage combined approach for velocity estimation is also considered. This algorithm combines the advantages of both Doppler and Hough detectors, and decreases vie computational burden. The effectiveness of the algorithms proposed is formulated in terms of quality parameters – the accuracy estimation and the probability of detection. The quality parameters are estimated using the Monte-Carlo simulation approach. Compared are the results of detection achieved by both analytical and simulational approaches.*

Keywords: *Multiple-Input Multiple-Output (MIMO), Radar detector, Doppler velocity estimator, Hough transform, Randomly arriving impulse interference, Probability of detection, Probability of false alarm.*

1. Introduction

In the conventional radar systems, a known waveform is transmitted by an omni directional antenna and a target reflects some of the transmitted energy toward an array of sensors that is used to estimate some unknown parameters e.g. range or speed considered by Fishler et al. [8]. In the Multiple-Input Multiple-Output (MIMO) radar systems, the receiver enjoys the fact that the average (over all information streams) Signal To Noise Ratio (SNR) is more or less constant, whereas in conventional systems, which transmit all their energy over a single path, the received SNR varies considerably. The model for MIMO system according to Fishler et al. [8] focuses on the effect of the target spatial properties ignoring range and Doppler effects. The advantages of the MIMO radar systems have been analyzed in other Fishler's papers [15, 16]. The concept of MIMO radar can be used to increase the spatial diversity of the system.

Godrich, Haimovich and Blum [9] presented a comparative study of coherent and non-coherent target localization techniques for MIMO radar systems with widely distributed elements. A structure of MIMO radar real-time imaging algorithm is proposed by Wang et al. [13]. In paper [10] is considered the joint optimization of waveforms and receiving filters in the MIMO radar for the case of extended target in clutter.

The mutual information between the received waveforms and the target impulse response has been optimized by properly designing the transmitting waveforms. This idea has been extended to the MIMO radar case in paper of Yang and Blum [12]. The corresponding robust design has also been proposed in another paper of these authors. However, in [12] Yang and Blum study the case when the effect of the clutter is ignored.

De Maio et al. [11] propose an optimal radar code which considers detection probability, Doppler frequency estimation accuracy, peak-to-average-power ratio and the range resolution.

Unlike the traditional SIMO (Single-Input Multiple-Output) radar, which can only transmit scaled versions of a signal waveform, the MIMO radar is capable of transmitting arbitrary waveforms considered by Bliss and Forsythe [14].

Several advantages have been demonstrated by different authors, including excellent interference rejection capability. One of these studies is contributed by Mecca, Ramakrishnan and Krolik [17].

Serbetli and Yener in [18] apply optimization technique in different fields such as multi-user transceiver design. Similar technique has been used to solve an MSE minimization problem in multi-user transceivers.

The modern mathematical methods for target detection and trajectory parameters estimation, which use mathematical transformation of received signals, allow designing new highly effective algorithms for MIMO radar signal processing. As a result, extremely precise estimates of moving target parameters can be obtained in conditions of very dynamic radar environment. An approach for linear trajectory target detection by means of Hough transformed coordinates, obtained for few sequential scans of the observation area is considered in [1]. According to this approach, the method for target detection uses a limited set of preliminary chosen patterns of a linear target trajectory. The set of target distance measurements is transformed to the pattern space (parameter space) by means of the Hough transform. The association of measurements to a special pattern is done by estimation of the data extracted from the target signals connected to this pattern. Thus the trajectory parameters of the targets, moving in the observation area, are determined through parameters of the corresponding pattern.

Behar, Kabakchiev and Doukovska [2] propose a structure of the Hough detector, which may by used in a mono-impulse radar for target detection or initial target linear trajectory formation in conditions of stationary interference with known or unknown intensities.

In Kabakchiev, Doukovska and Garvanov [6] and in Doukovska and Kabakchiev [7], a comparative analysis of the performance of different types of signal processors used in the algorithm of Hough detector is carried out. The probability characteristics of Hough detector with fixed threshold synthesis for homogeneous interference with known power are compared to five other Hough detector types with one and two-dimensional CFAR processors. These five structures maintain constant false alarm rate in conditions of homogeneous interference with unknown power and randomly arriving impulse interference with known parameters (non-homogeneous background).

In conventional radar systems, the radial velocity of targets in the radar directional beam is measured using the Doppler Effect. In each range resolution cell, velocity measurements are made by transmitting a pulse train towards a target over a very short period of time, and measuring relative target movement between each pulse. The number of pulses used is usually known as packet size and the frequency in which they are transmitted as pulse repetition frequency. The Doppler radar processes a train of received pulses determining the average phase-shift between successive pulses within a pulse packet. This is typically done by means of a 1D Fast Fourier Transform (FFT), which is performed independently for each range resolution cell, using all pulses within a packet. The radial velocity of the target is evaluated based on knowledge of the radar frequency, speed of light, pulse repetition frequency and average Doppler phase-shift. Because Doppler radar is sampling systems, a maximum radial velocity that can be measured without ambiguity is limited by the pulse repetition frequency and the radar wavelength. The velocity resolution depends on the maximal unambiguous velocity and the packet size, i.e. $V\max$ and N.

An alternative approach for velocity estimation of targets moving towards or down radar can be realized using the Hough detector proposed by Carlson, Evans and Wilson [1]. Using this approach, an original two-stage algorithm for simultaneous target detection and its radial velocity estimation is proposed and tested by Behar, Doukovska and Kabakchiev [3]. At the first stage, the target is detected using the Hough detector. At the second stage, the target radial velocity is found using the estimate of the Hough space parameter, which is found at the former stage. The effectiveness of the algorithm for the combined "detection-estimation" algorithm is formulated in terms of both quality parameters – the probability of detection and the accuracy of velocity estimation. The quality parameters of the detection algorithm are evaluated by means of Monte-Carlo simulations.

In paper [4], the aim of the study is to compare the accuracy of velocity estimation provided by the same radar with two different approaches. The first of them, which is the traditional approach, provides the instant velocity estimate using the Doppler effect in each current scan. The other approach provides the average velocity estimate by performing the data obtained for several scans using the Hough transform. The maximal absolute velocity error provided by the Doppler estimator varies inversely to the number of channels used in the FFT transform. By analogy, the maximal absolute velocity error provided by the Hough estimator varies inversely to the number of channels used in the Hough transform. At the first stage of investigation the maximal velocity errors are calculated analytically for the two estimators. At the second stage the velocity errors are estimated using Monte-Carlo simulations. The comparative analysis of the estimation accuracy provided by the two estimators, Doppler and Hough, is done for the same parameters of surveillance radar and the same probabilities of detection and false alarm. The estimation accuracy is evaluated in terms of bias and percent errors.

In paper [5] D o u k o v s k a proposes a combined Doppler-Hough algorithm for velocity estimation in order to decrease the computational burden and to meet the accuracy requirements. This original three-stage algorithm combines the advantages of both detectors, Doppler and Hough, while decreasing the computational burden. At the first stage the Doppler processing is performed in order to compute the crude estimate of a target velocity for several consequential scans. For this purpose, the FFT with small number of points can be used. At the second stage the fine velocity estimate is determined using the crude estimates obtained at the first stage. This requires determining the boundary values of the Hough parameter space using the crude velocity estimates obtained by the Doppler processing. Then, the Hough parameter space is incrementally sampled in order to meet the needed accuracy requirements. The final velocity estimate is calculated by the Hough detector that assures the needed accuracy of estimation.

The algorithms proposed are studied using Monte-Carlo simulations. The effectiveness of the estimation algorithm is evaluated in terms of the following parameters – the accuracy of estimation and the probability of detection.

The research work is performed in MATLAB computational environment.

2. Target detection and velocity estimation

In conventional radar systems the radial target velocity is measured using the Doppler Effect. The principal structure of a Doppler estimator is shown in Fig. 1. The incoming radio frequency signal is demodulated down to a center frequency of zero prior to pulse compression and Doppler filtering. This is down to reduce computational burden, since the demodulated signal can be down sampled to reduce the amount of data needed for storage. The demodulated signal is usually referred as complex envelope or IQ-data, where I-data is a real part and Q-data is an imaginary part of a complex envelope.

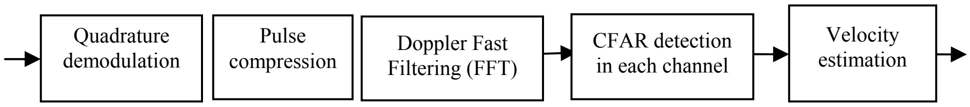

Fig. 1. The structure of Doppler detector/estimator

After pulse compression, the complex amplitude of all pulses received from a target is processed into Doppler velocity filters which are used to determine velocity. The envelope at the output of a filter with maximal envelope is compared with an adaptive Constant False Alarm Rate (CFAR) detection threshold. If this theshold is exceeded, the radial velocity of the target is evaluated by

(1) $$V_{\text{target}} = \frac{\lambda F_{\text{PRF}}}{2N_{\text{FFT}}} n \text{, where } F_{\text{PRF}} < \frac{c}{2R_{\max}}.$$

The parameter F_{PRF} in (1) is the Pulse Repetition Frequency, λ is the wavelength, c is the velocity of light, $R_{\max}$ is the maximal unambiguous range, N_{FFT} is the number of points in the FFT transform and n is the channel number, in which the envelope exceeds the preliminary determined detection threshold. A main problem associated with this technique is velocity ambiguity because Doppler phase-shifts exceeding π are aliased. For that reason, the maximal and minimal unambiguous target velocities are determined by

(2) $$V_{\text{targ,min}} = \delta V_{\text{NFFT}} = \frac{\lambda F_{\text{PRF}}}{2N_{\text{FFT}}} \text{, and } V_{\text{targ,max}} = \frac{\lambda F_{\text{PRF}}}{4}.$$

According to (1), the minimal target velocity, or velocity resolution, is determined by the number of point in the FFT transform.

In modern radar system an alternative approach for target velocity estimation can be realized using the Hough transform. Consider radar that provides range, azimuth and elevation as a function of time. Time is sampled by the scan period, but resolution cells sample range, azimuth and elevation. The trajectory of a target that moves in the same "azimuth-elevation" resolution cell, is a straight line specified by several points (r, t) in the range-time r–t data space, as it is shown in Fig. 2. Besides, in the r–t space, the target trajectory can be specified through the other parameters – the angle θ of its perpendicular from the data space origin and the distance ρ from the origin to the line along the perpendicular.

The Hough transform maps all points (r, t) of the r–t space into curves in the $\rho - \theta$ parameter space (Hough parameter space) as follows:

(3) $$\rho = r\cos\theta + t\sin\theta$$

where r and t are point coordinates in the r–t space, ρ and θ are parameters that specify a straight line in the Hough parameter space.

The mapping of a line into the Hough parameter space can be considered as stepping through θ from 0° to 180° and calculating the corresponding ρ. The parameter space showing several sinusoids corresponding to different points in the r–t space is shown in Fig. 3. The trigonometric manipulations of (3) lead to the other form of the Hough transform

(4) $$\rho = \sqrt{r^2 + t^2}\sin\left(\theta + \text{arctg}\frac{r}{t}\right).$$

The mapping by (2) results in a sinusoid with an amplitude and phase dependent on coordinates in the *r–t* space of a point that is mapped. The maximum value for ρ is equal to the length of the diagonal across the *r–t* data space. Equation (3) is the simpler version that is actually used for mapping.

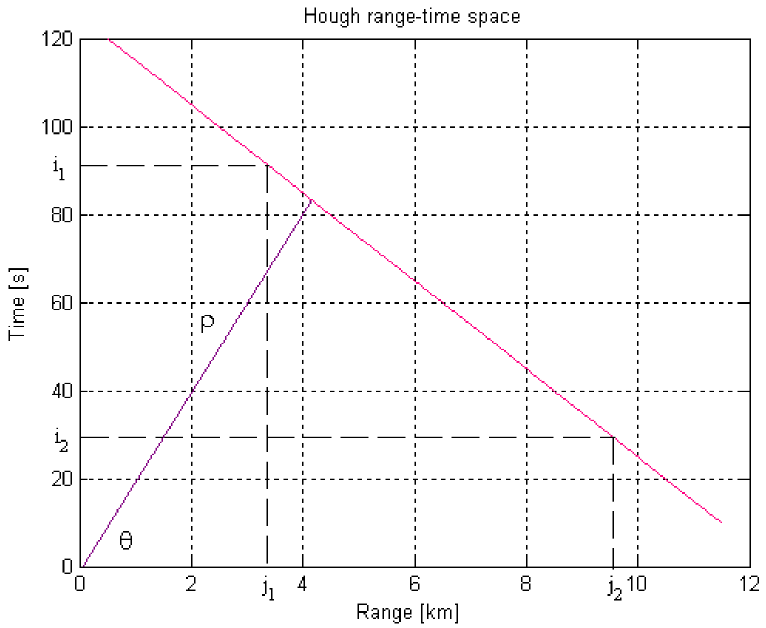

Fig. 2. Range-time space

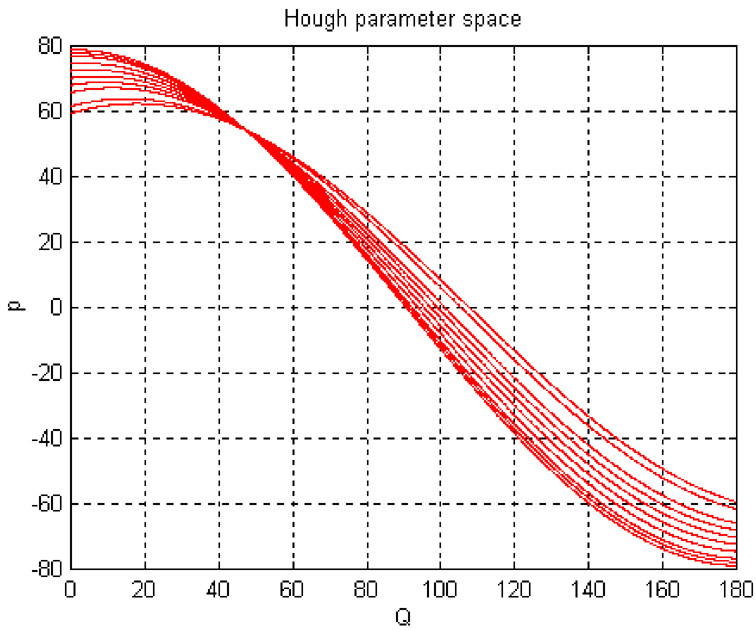

Fig. 3. Hough parameter space

Each (ρ, θ) point in the Hough parameter space corresponds to a single straight line in the r–t data space. Any one of the sinusoidal curves in the Hough parameter space corresponds to the set of all possible lines in the data space through the corresponding data point. If a straight line exists in the r–t space, this line is represented in the Hough parameter space as a point of intersection of all the mapped sinusoids. The slope of the target trajectory presented in Fig. 2 is determined by the radial velocity of the target

(5) $$V = \frac{(j_2 - j_1)\delta R}{(i_2 - i_1)t_{SC}} = \mathrm{tg}\theta \frac{\delta R}{t_{SC}}$$

where $(i_1\delta R, j_1 t_{SC})$ and $(i_2\delta R, j_2 t_{SC})$ are coordinates of two points in the r–t space that belong to the target trajectory, δR is the range resolution cell and t_{SC} is the scan period. According to equations (1), (2), (3), the principal structure of the Hough detector/estimator is shown in Fig. 4.

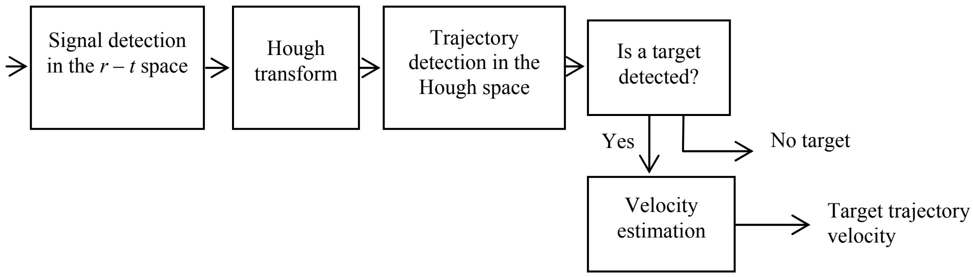

Fig. 4. The structure of Hough detector/estimator

In the r–t space, a low primary threshold is set, and any range-time cell with a value exceeding this threshold is mapped into the $\rho - \theta$ parameter space using (3). The parameter space is sampled in ρ and θ dimensions. When a primary threshold crossing in any (r, t) cell is mapped into the parameter space, its signal power is added into (ρ, θ) cells that intersect the corresponding sinusoidal curve in the parameter space. In this way, in the Hough parameter space the accumulator point at the intersection of several sinusoids will reach a high value. A secondary threshold applied to each point in the $\rho - \theta$ parameter space can be use to declare detection of a target trajectory. The point $(\hat{\rho},\ \hat{\theta})$ where the secondary threshold is exceeded specifies the detected trajectory of a target.

According to (5), the estimate of radial velocity of a target can be evaluated as:

(6) $$\hat{V} = \frac{\delta R}{t_{SC}} \mathrm{tg}\hat{\theta}$$

where δR is the range resolution cell, t_{SC} is the scan period.

3. Simulation algorithm

The effectiveness of target detection and velocity estimation provided by the algorithm proposed can be expressed in terms of two quality parameters – the detection probability characteristics and the accuracy of velocity estimation. In order to evaluate statistically these quality parameters, a simulation algorithm for testing of the new detector/estimator is developed:

Step 1. The *r–t* space is quantized. To do this, the following data is needed – the range resolution cell (δR), scan time (t_{SC}), and the number of scans (N_{SC}). The quantized *r–t* space is of size $N \times M$, where $N = N_{SC}$ and $M = \dfrac{R_k - R_n}{\delta R}$.

Step 2. The hypothesis matrix (IndTr) is formed as follows:

$$\begin{cases} \mathrm{IndTr}(i,j) = 1, \; j = Vt_{SC} i / \delta R, \\ \mathrm{IndTr}(i,j) = 0, \; j \neq Vt_{SC} i / \delta R. \end{cases} \tag{7}$$

The number of nonzero elements K_{target} in the hypothesis matrix IndTr equals the number of all the target positions in the *r–t* space:

$$K_{\mathrm{target}} = \sum_{i=1}^{N} \sum_{j=1}^{N} \mathrm{IndTr}(i,j) / \mathrm{IndTr}(i,j) \neq 0. \tag{8}$$

Step 3. The process of target detection in each cell of the *r–t* space is simulated. As a result, the following matrix whose each element indicates whether the target is detected or not in the corresponding cell of the *r–t* space, is formed:

$$\mathrm{Det}^q(i,j) = \begin{cases} 1, \text{ target is detected,} \\ 0, \text{ target is not detected,} \end{cases} \tag{9}$$

where p is the simulation cycle number.

Step 4. The ρ–θ parameter space is quantized. It is a matrix of size $K \times L$. The parameters K and L are determined by the number of discrete values of the θ parameter, which is sampled in the interval (θ_1, θ_2) with sampling step $\Delta\theta$, and the size of the *r–t* space,

$$K = 2\sqrt{N^2 + M^2}, \quad L = \frac{\theta_2 - \theta_1}{\Delta\theta}. \tag{10}$$

Step 5. All the nonzero elements of the matrix Det^q are performed using the Hough transform. In such a way, the *r–t* space is mapped into the $\rho - \theta$ parameter space. The resulting matrix is $\{\mathrm{Ht}\}_{K,L}^q$.

Step 6. A target trajectory is detected. This is done by comparing the value of each element of the parameter space, i.e. of matrix $\{\mathrm{Ht}\}_{K,L}^q$, with the fixed threshold T_M. It means that the decision rule "T_M out of N_{SC}" is applied to each element in the parameter space. According to this criterion, the linear target trajectory specified as a point $(\hat{\rho}, \hat{\theta})$ in the Hough parameter space is detected if and only if the value $\mathrm{Ht}^q(\hat{\rho}, \hat{\theta})$ exceeds the threshold T_M,

$$\mathrm{DetHo}^q(i,j) = \begin{cases} 1, \; \mathrm{Ht}^q(i,j) > T_M, \\ 0, \quad \text{otherwise.} \end{cases} \tag{11}$$

Step 7. That is performed in case when a target trajectory is detected at the former step. At this step the target radial velocity is estimated as follows:

(12) $$\hat{V} = \frac{\delta R}{t_{\mathrm{SC}}} \mathrm{tg}\left(\hat{\theta}\right)$$

where $\hat{\theta}$ is the Hough parameter, where the target trajectory is detected.

Step 8. In order to estimate both the probability characteristics and the accuracy of velocity estimation, Steps 1-7 are repeated N_p times.

The false alarm probability in the *r–t* space is estimated as

(13) $$\hat{P}_{\mathrm{fa}} = \frac{1}{\left(NM - K_{\mathrm{target}}\right)N_q} \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{l=1}^{N_q} \left\{\mathrm{Det}^q(i,j) / \mathrm{IndTr}(i,j) \neq 1\right\}.$$

The target detection probability in the *r–t* space is estimated as

(14) $$\hat{P}_{\mathrm{d}} = \frac{1}{K_{\mathrm{target}} + N_q} \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{l=1}^{N_q} \left\{\mathrm{Det}^q(i,j) / \mathrm{IndTr}(i,j) = 1\right\}.$$

The false alarm probability in the ρ–θ space is estimated as

(15) $$\hat{P}_{\mathrm{FA}} = \frac{1}{KLN_q} \sum_{i=1}^{K}\sum_{j=1}^{L} \left\{\mathrm{DetHo}^q(i,j) / i \neq I,\, j \neq J\right\}.$$

The probability of trajectory detection in the ρ–θ space is estimated as

(16) $$\hat{P}_{\mathrm{D}} = \max_{I,J} \sum_{l=1}^{N_q} \mathrm{DetHo}^q(i,j).$$

4. Simulation results

In this section we apply the simulation algorithm described above to typical surveillance radar. The goal is to analyze statistically the algorithm for target detection and velocity estimation. In order to obtain the statistical estimates of the basic quality parameters (probability characteristics and accuracy of velocity estimation), the following data was used in simulations: scan period – t_{SC} =6 s; number of scans – N_{SC}=20; range resolution cell – δR=150 m and 1500 m; size of the range-time space – 128×20 elements; interval for variation of the θ parameter – θ_1=0°, θ_2=180°; probability of signal detection in the *r–t* space – P_{D}=0.9; probability of false alarm per cell in the range-time space – P_{fa}=0.0001; decision rule for trajectory detection in the Hough parameter space – $T_{\mathrm{M}}/N_{\mathrm{SC}}$=7/9 and $T_{\mathrm{M}}/N_{\mathrm{SC}}$=7/20; target velocity – V_{target}=333 m/s; the average signal-to-noise ratio – SNR= 37 dB; number of simulation cycles – 1000.

According to (12), the theoretical accuracy of velocity estimation (ΔV) can be expressed as

(17) $$\Delta V_i = V_{i+1} - V_i = \frac{\delta R}{t_{\mathrm{sc}}}\left(\mathrm{tg}\theta_{i+1} - \mathrm{tg}\theta_i\right)$$

where $\theta_{i+1} = i\Delta\theta$, $\theta_i = (i-1)\Delta\theta,\ i = 1, 2, ..., L$.

Therefore, the accuracy of estimation is mainly determined by the sampling rate of the parameter θ and also depends on the sampling interval of the *r–t* space. It means that for given δR and t_{SC}, the sampling interval $\Delta\theta$ should be chosen in such a way in order to meet the requirements for the accuracy of velocity estimation.

The theoretical accuracy that can be reached depending on the sampling interval of the parameter θ is presented in Figs. 5 and 6 – for the range resolution cell of 1500 m and 150 m, respectively. The theoretical accuracy of velocity estimation as a function of the target velocity to be estimated is presented for six different variants of $\Delta\theta$.

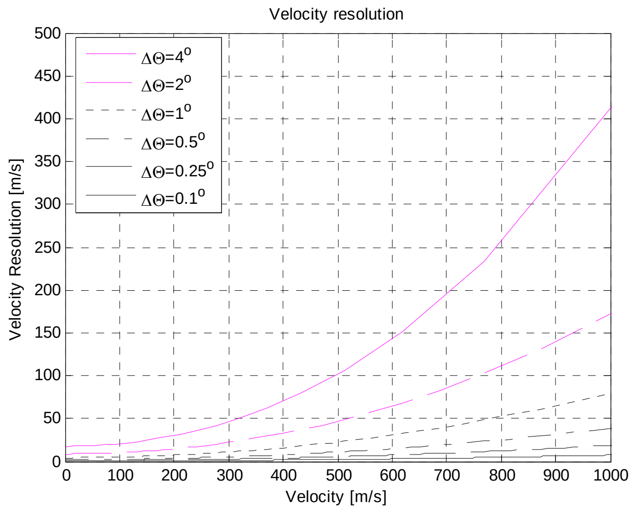

Fig. 5. Velocity resolution (δR=1500 m)

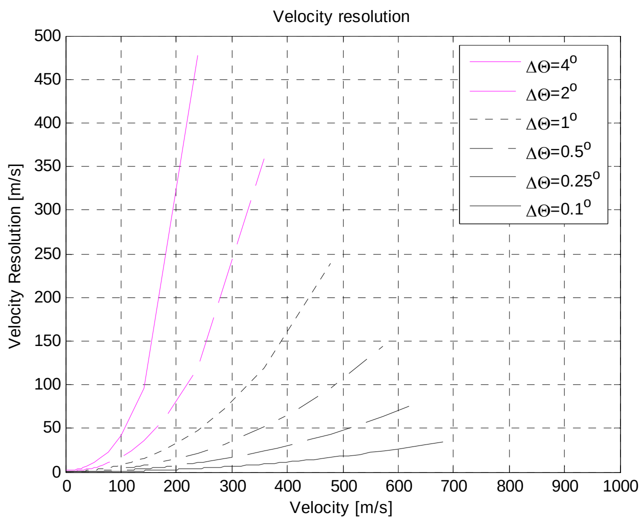

Fig. 6. Velocity resolution (δR=150 m)

The averaged velocity estimates, obtained in simulations for a target velocity of 333 m/s, are shown in Fig. 7. The velocity estimates are plotted as a function of the sampling interval $\Delta\theta$. For comparison, the averaged velocity estimate is plotted for two values of the range resolution cell – 150 m and 1500 m. The absolute errors of velocity estimation, calculated for two values of the range resolution cell are shown in Fig. 8. They are also plotted as a function of the sampling interval $\Delta\theta$.

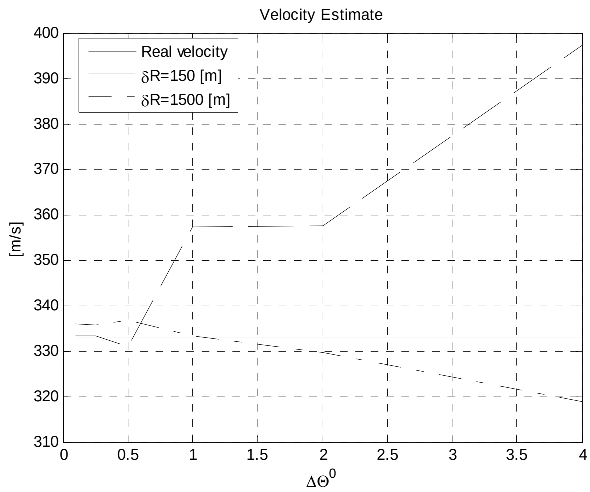

Fig. 7. Average velocity estimate

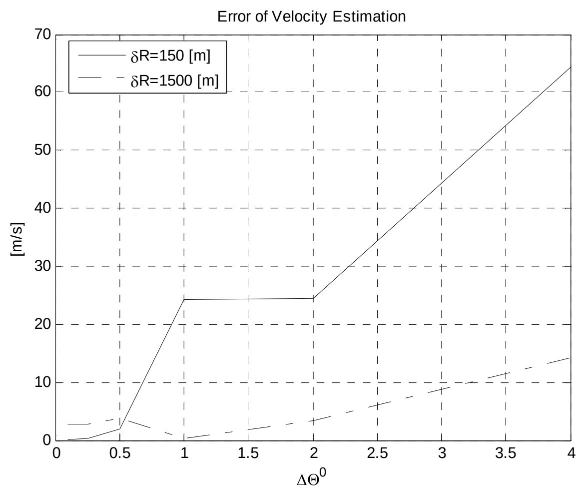

Fig. 8. Absolute error of velocity estimation

The numerical results that correspond to the graphical results are summarized in Table 1 and Table 2. The velocity estimates calculated for a target moving in straight line with velocity of 333 m/s are presented for six sampling interval of the parameter θ. In addition, Table 1 and Table 2 contain the estimates of both probability characteristics, the probability of signal detection in the r–t space (P_d) and the probability of trajectory detection in the Hough parameter space (P_D).

For this simulated example the $r–t$ space contains 128 range resolution cells. It means that the general number of points specifying the target trajectory in the $r–t$ space can be calculated as

(18)
$$N_{\text{point}} = \begin{cases} \dfrac{128\delta R}{V_{\text{target}} t_{\text{SC}}}, & \text{if } \dfrac{128\delta R}{V_{\text{target}} t_{\text{SC}}} < 20 \\ 20, & \text{otherwise.} \end{cases},$$

If t_{SC}=6 s and V_{target}=333 m/s, then N_{point} = 9 – for δR=150 m and N_{point} = 20 – for δR =1500 m. Therefore, the decision rule applied to trajectory detection in the Hough parameter space is "7 out of 9" – in case of the range resolution cell of 150 m, and "7 out of 20" – in case of the range resolution cell of 1500 m. For that reason the probability of trajectory detection presented in Table 2 is greater than that presented in Table 1.

Table 1. Velocity and probability estimates for value δR=150 m

$\Delta\theta°$	Real velocity – (V_{real}) Real parameter – (θ_{real})	V_{ave} (m/s)	ΔV (m/s)	P_d	P_D
0.1	V_{real} =333 m/s θ_{real} = 85.7066°	333.1697	0.1697	0.932	0.956
0.25		333.2808	0.2808	0.909	0.940
0.5		330.9933	2.0067	0.912	0.935
1.0		357.2151	24.2151	0.916	0.474
2.0		357.5167	24.5167	0.914	0.485
4.0		397.3636	64.3636	0.912	0.006

Table 2. Velocity and probability estimates for value δR=1500 m

$\Delta\theta°$	Real velocity – (V_{real}) Real parameter – (θ_{real})	V_{ave} (m/s)	ΔV (m/s)	P_d	P_D
0.1	V_{real} =333 m/s θ_{real} = 53.1026°	335.844	2.844	0.912	1
0.25		335.7617	2.7617	0.915	1
0.5		336.7357	3.7357	0.918	1
1.0		333.3181	0.3181	0.917	1
2.0		329.5216	3.4784	0.919	1
4.0		318.832	14.168	0.920	1

The values of velocity resolution and both errors of velocity estimation of the Hough estimator, calculated for velocities within the range (33-298 m/s), are plotted in Figs. 9-11. Calculations are made for six different variants of $\Delta\theta$ – 1°; 0.8°; 0.6°; 0.4°; 0.2° and 0.1°.

Comparative analysis of velocity resolution shown in Fig. 9 shows the advantages of the Hough detector to maintain the needed accuracy of estimation by appropriate sampling rate of the parameter θ in the Hough parameter space. When the velocity under estimation lies in the velocity region of the Doppler estimator (4-117 m/s), the Hough estimator maintains higher accuracy measurement if the sampling step of the parameter θ is chosen to be less than 1°.

Despite of that the absolute and relative errors provided by the Hough estimator, grow with increasing the velocity under estimation, the sampling steps of the parameter θ less than 0.4° guarantee the relative estimation errors less than 1%, for velocities within the velocity range of the Hough detector.

In order to decrease computational burden of the Hough estimator, the velocity estimation in the Hough parameter space can be done in two stages. At the first stage the parameter θ is sampled with a large sampling step, i.e. $\Delta\theta$>> 1°. In such a widely sampled Hough parameter space, the crude velocity estimate $\widehat{V}$ is found, i.e. $\widehat{V} \in [V_1, V_2]$. At the second stage the parameter $\theta \in [\theta_1, \theta_2]$ is sampled with a very fine sampling step, i.e. $\Delta\theta$<<1°. The variation range of the parameter θ corresponds to the velocity range $[V_1, V_2]$ found at the former stage:

(19) $$\theta_1 = atg(\frac{V_1 \cdot t_{SC}}{\delta R}),\ \theta_2 = atg(\frac{V_2 \cdot t_{SC}}{\delta R}).$$

At the second stage of velocity estimation, the fine velocity estimate is found, and the sampling step $\Delta\theta$ must be chosen in order to meet the accuracy requirement.

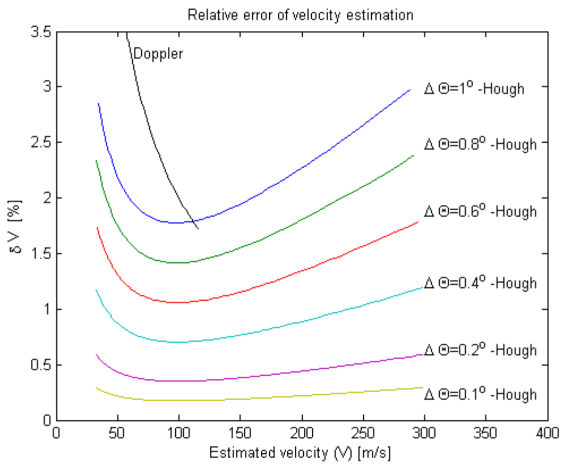

Fig. 9. Relative error of velocity estimation

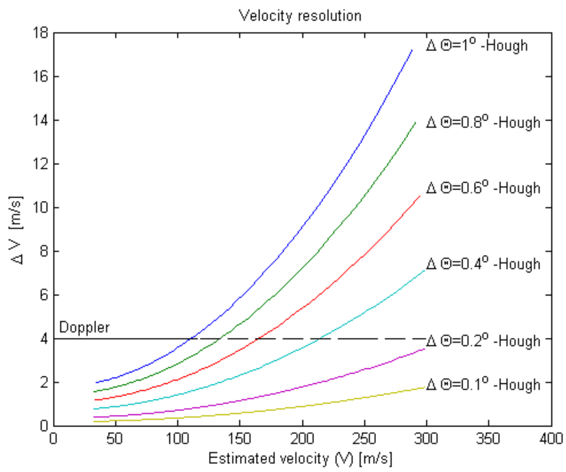

Fig. 10. Radial velocity resolution

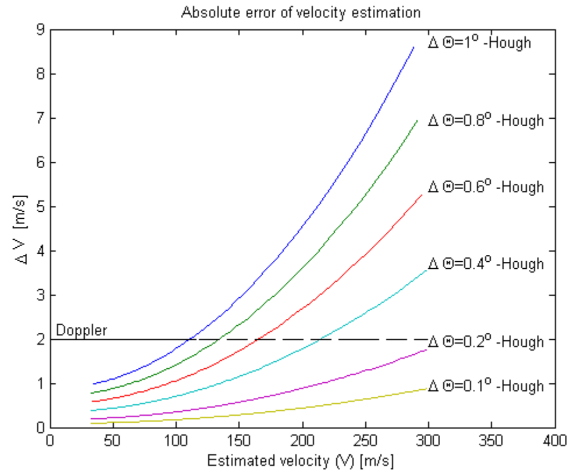

Fig. 11. Absolute error of velocity estimation (m/s)

The comparative analysis of the two velocity estimation techniques, Doppler and Hough, is carried out by Monte-Carlo simulations. The aim of simulations is to evaluate the accuracy measures provided by the two estimators for the same parameters of surveillance radar. The velocity estimation errors (bias error $\Delta\hat{V}$ and percent error $\delta\hat{v}$) are calculated as

(20) $$\Delta\hat{V} = V - \sum_{k=1}^{Kcycle} \hat{V}_k / K_{\text{cycle}}, \text{ and } \delta\hat{V} = \left|\Delta\hat{V}\right| / V \cdot 100\%,$$

where $\hat{V}_k$ is the velocity estimate calculated in the k-th cycle of simulation, and K_{cycle}=10000 is the total number of simulations. In simulations, the following radar parameters are used: wavelength – 1.875 m; pulse repetition frequency – 250 Hz; scan period – 10 s; packet size – 56; probability of detection in a range cell P_D>0.9; false alarm probability – $P_{FA}=10^{-6}$. According to (2), for λ=1.875 m and F_{PRF}= 250 Hz, the maximal unambiguous radial velocity capable of measurement by the Doppler estimator is $V_{\text{targ,max}}$=117 m/s. In our study, the two errors of velocity estimation, bias and percent, are calculated for the velocity of 100 m/s. The probability of detection and velocity estimation is calculated as

(21) $$P_D = \text{probability}\{P_{\text{sig}} > H_D \;\&\; \left|V - \hat{V}\right| \le \delta V_{\text{NFFT}}\}.$$

Since the velocity resolution of the Doppler estimator depends on the parameter N_{FFT} both errors determined by (20), are calculated as functions of the parameter N_{FFT}.

The errors of velocity estimation are calculated according to (20), for the velocity of 100 m/s, as in case of the Doppler estimator. The probability of target detection in the r–t space is maintained to be 0.9, as it is required in the Doppler estimator. According to (12), the maximal bias error provided by the Hough estimator depends on the sampling rate of the parameter θ and a scale factor ($\delta R / t_{SC}$):

(22) $$\Delta V_{\text{Hough}i} = (V_{i+1} - V_i)/2 = \frac{\delta R}{2t_{\text{SC}}}\left(\text{tg}\,\theta_{i+1} - \text{tg}\,\theta_i\right), \quad \text{under } V_i < V < V_{i+1},$$

where $\theta_{i+1} = i\Delta\theta$, $\theta_i = (i-1)\Delta\theta$, $i = 1, 2, ..., L$, $L = (\theta_2 - \theta_1)/\Delta\theta$.

The maximal errors (bias and percent) obtained analytically by (22) are plotted in Figs. 12 and 13 for seven different values of the parameter $\Delta\theta$. The numerical results obtained by simulations, i.e. the estimation errors and the probability of detection are presented in Table 3 and plotted on Fig. 15.

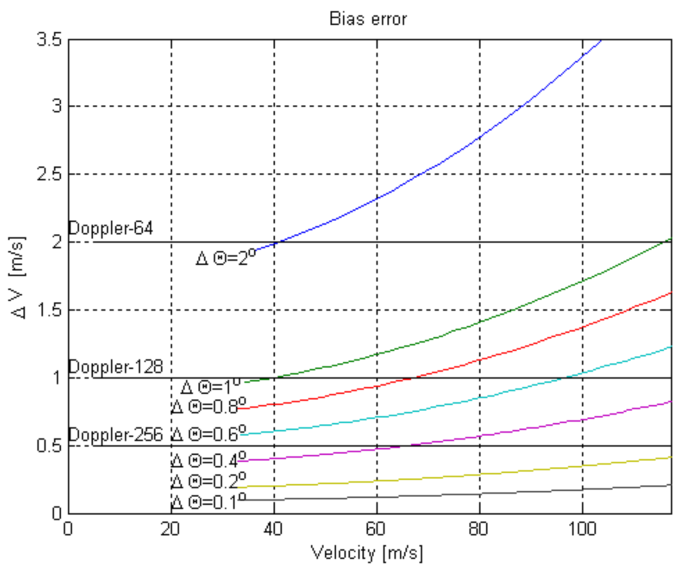

Fig. 12. Maximal bias errors obtained analytically

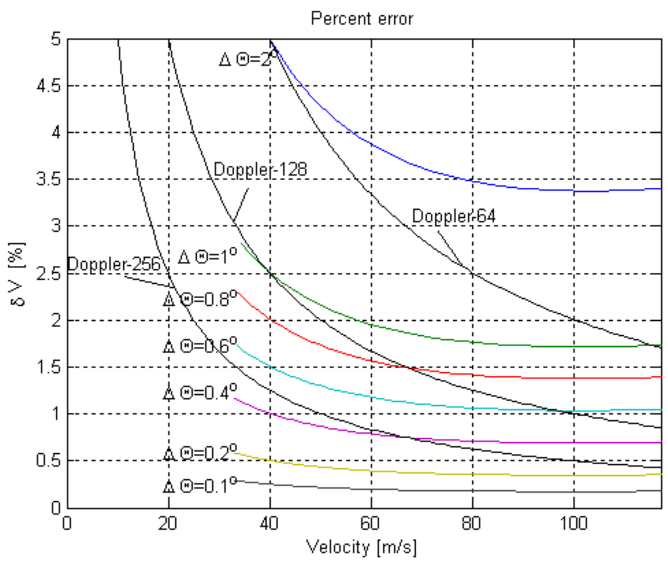

Fig. 13. Maximal percent errors obtained analytically

Table 3. The accuracy and probability measures obtained in simulations

Doppler estimator				Hough estimator			
N_{FFT}	$\Delta\hat{V}$ (m/s)	$\delta\hat{V}$ (%)	$\hat{P}_D$	$\Delta\theta$	$\Delta\hat{V}$ (m/s)	$\delta\hat{V}$ (%)	$\hat{P}_D$
64	–1.123	1.123	0.9999	4°	–3.1135	3.1135	0.904
128	0.7080	0.7080	0.8802	3°	–1.2674	1.2674	0.906
256	–0.2075	0.2075	0.9176	2°	0.6988	0.6988	0.905
512	–0.2075	0.2075	0.5383	1°	0.5795	0.5795	0.906
1024	0.0214	0.0214	0.4859	0.8°	0.0647	0.0647	0.906
2048	0.0214	0.0214	0.2513	0.6°	0.3818	0.3818	0.905
4096	0.0214	0.0214	0.1254	0.4°	0.4142	0.4142	0.905
				0.2°	0.3321	0.3321	0.905
				0.1°	0.3847	0.3847	0.906

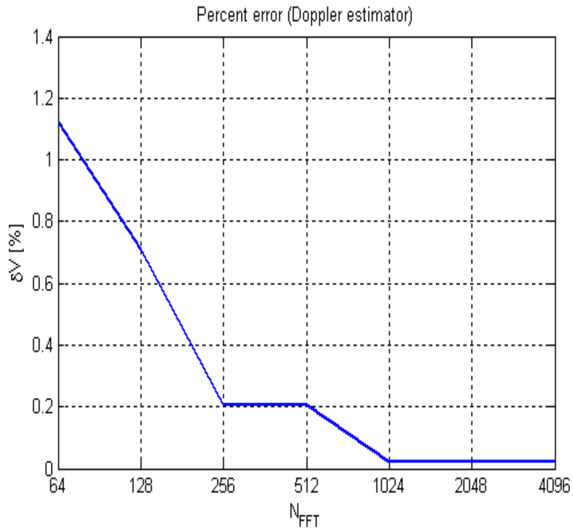

Fig. 14. Percent errors (Doppler estimator)

In order to illustrate that the estimation errors increase with the growth of the velocity under estimation, the estimation errors are also calculated for velocity of 200 m/s as it is shown in Fig. 15. Comparative analysis of errors presented in Table 1 and Figs. 14 and 15 shows that both estimators make possible to estimate radial velocity with the same accuracy. The Doppler estimator with 128 velocity channels provides the same percent errors of velocity estimation ($\delta\hat{V} = 0.7\%$) that are provided by the Hough estimator whose parameter space is sampled by 2°.

The numerical results obtained also show that the further enlargement of the number velocity channels in the Doppler estimator (N_{FFT}>256) does not permit the improvement of the accuracy of velocity estimation because in that case the probability of detection abruptly falls. It is also shown that the further decrease of the sampling step $\Delta\theta$ in the parameter space of the Hough detector ($\Delta\theta$<1°) does not improve the accuracy of velocity estimation. Therefore the optimal sampling step $\Delta\theta$ should be chosen in such a way in order to meet the requirements for accuracy of velocity estimation.

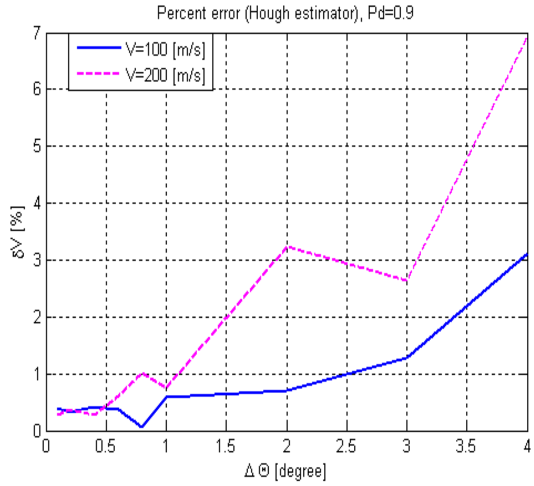

Fig. 15. Percent errors (Hough estimator)

The results achieved when analyzing the considered methods for estimation of a moving target velocity allow a combined two stages approach to be proposed. At the first stage a Doppler technique for "rough" velocity estimation is used. The second stage uses a Hough evaluation to achieve more accurate values. Another advantage of the proposed method is the possibility for simultaneous of target and trajectory detection.

5. Conclusions

An original algorithm for moving target detection and velocity estimation is presented and evaluated in the paper. In order to test and study the new algorithm, the simulation algorithm based on the Monte-Carlo approach is developed. The graphical and numerical result show that the quality parameters strongly depend on discretization not only of the *r–t* space but of the Hough parameter space as well. It is also shown that the discretization of both spaces (*r–t* and ρ–θ) should be optimized in order to meet the requirements for both quality parameters – the probability of target trajectory detection and the accuracy of velocity estimation.

A comparative Monte-Carlo analysis of the two velocity estimation techniques is done on the base of the same parameters of surveillance radar. The numerical results obtained by simulations prove the rough estimates calculated analytically at the first stage of investigations. They show that both estimators make possible to estimate velocity with the same accuracy. The optimal parameter N_{FFT}, for the Doppler estimator, or $\Delta\theta$, for the Hough estimator, must be chosen carefully in order to meet the accuracy requirements for velocity estimation. For given parameters of surveillance radar, the Hough detector has some advantage over the Doppler estimator because it allows estimation of velocities out of the velocity range of the Doppler estimator (i.e. 200 m/s).

Also a combined algorithm for moving target velocity estimation is presented in this paper. In order to test and study, the simulation algorithm based on the Monte-Carlo approach is developed. The quality parameters of the algorithm proposed are expressed in terms of the probability of target and trajectory detection and the accuracy of velocity estimation. The graphical and numerical results show that the quality parameters strongly depend on discretization not only of the r–t space, but of the Hough parameter space as well. It is also shown that the discretization of both spaces (r–t and Hough) should be optimized in order to meet the requirements for both quality parameters – the probability of target trajectory detection and the accuracy of velocity estimation.

The obtained results can be successfully applied for radar target detection and in the existing communication network receivers that use pulse signals.

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