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On One Class of Intuitionistic Fuzzy Implications

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Abstract: *In this paper a new class of intuitionistic fuzzy implications is introduced. Fulfillment of some axioms and properties together with Modus Ponens and Modus Tollens inference rules are investigated. A negation induced by an implication is presented.*

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1. Introduction

In 1983 Krassimir Atanassov has presented[1] a concept of a kind of vague sets, called *Intuitionistic Fuzzy Sets* (IFS). The concept directly alluded to the concept of *Fuzzy Sets* (FS) introduced in 1965 by L.A. Zadeh. IFS, however, differ from FS, because independence of the membership degree and non-membership degree of the element x to the set A was introduced. While in FS a non-membership degree of elements x to the FS A is (typically) $1-\mu_A(x)$, where $\mu_A(x)$ is a membership degree, Atanassov introduced the separate values $\mu_A(x)$ and $\nu_A(x)$ of memberships and non-memberships of x to the IFS A.

In Intuitionistic Fuzzy Logic (IFL) the truth-value of variable x is given by an ordered pair $\langle a, b\rangle$, where $a, b, a+b \in [0, 1]$. The numbers a and b are interpreted as the degrees of validity and non-validity of x. We denote the truth-value of x by $V(x)$.

[1] A t a n a s s o v, K. Intuitionistic Fuzzy Sets. VII ITKR's Sci. Session, Sofia (June 1983), deposed in Central Science-Technical Library of Bulgarian Academy of Sciences, Hπ 1697/84 (in Bulgarian). IFS became widely known after publication [1].

The variable with a truth-value *true* in classical logic is denoted by $\underline{1}$ and the variable *false* – by $\underline{0}$. For these variables holds also $V(\underline{1}) = \langle 1, 0\rangle$ and $V(\underline{0}) = \langle 0, 1\rangle$.

We call the variable x an Intuitionistic Fuzzy Tautology (IFT), if and only if for $V(x) = \langle a, b\rangle$ holds: $a \geq b$ and, similarly, an Intuitionistic Fuzzy co-Tautology (IFcT), if holds: $a \leq b$.

For every x we can define the value of negation of x in the typical form $V(\neg x) = \langle b, a\rangle$.

It is clear that a IFcT could be defined by IFT and $\neg$.

An important operator of IFL is intuitionistic fuzzy implication. A t a n a s s o v [2, 4, 5] noted more than a hundred different intuitionistic fuzzy implications. A t a n a s s o v a [6] presented an additional one.

Definition 1.

The fuzzy implication (see:[8, 9]) is a mapping I: $[0, 1]^2 \to [0, 1]$ where for $p_1, p_2, p, q_1, q_2, q \in [0, 1]$ holds:

(i1FL) if $p_1 \leq p_2$ then $I(p_1, q) \geq I(p_2, q)$,

(i2FL) if $q_1 \leq q_2$ then $I(p, q_1) \leq I(p, q_2)$,

(i3FL) $I(0, q) = 1$,

(i4FL) $I(p, 1) = 1$,

(i5FL) $I(1, 0) = 0$.

Applying this definition to the IFL first we will introduce some ordering relation for the intuitionistic truth-value.

For $V(x) = \langle a, b\rangle$ and $V(y) = \langle c, d\rangle$ where $a, b, c, d, a+b, c+d \in [0, 1]$, we denote $V(x) \preceq V(y)$ if and only if $a \leq c$ and $b \geq d$.

In the case of IFL the conditions (i1FL)-(i5FL) for implication $\Rightarrow$ are given in the form:

(i1) if $V(x_1) \preceq V(x_2)$ then $V(x_1 \Rightarrow y) \succeq V(x_2 \Rightarrow y)$,

(i2) if $V(y_1) \preceq V(y_2)$ then $V(x \Rightarrow y_1) \preceq V(x \Rightarrow y_2)$,

(i3) $\underline{0} \Rightarrow y$ is an IFT,

(i4) $x \Rightarrow \underline{1}$ is an IFT,

(i5) $\underline{1} \Rightarrow \underline{0}$ is an IFcT.

2. Main results

Now we introduce a parametric class of fuzzy intuitionistic implications.

Theorem 1. An intuitionistic logical connective with a truth-value:

$$V(x \to_\gamma y) = \langle \frac{b+c+\gamma}{2\gamma+1}, \frac{a+d+\gamma-1}{2\gamma+1} \rangle,$$

where $\gamma \in \Re$, $\gamma \geq 1$, is an intuitionistic fuzzy implication fulfilling Definition 1 with (i1)-(i5). Implication $\to_\gamma$ is not presented in the previous bibliography (known to the author).

P r o o f.

Preliminary note: $\langle \frac{b+c+\gamma}{2\gamma+1}, \frac{a+d+\gamma-1}{2\gamma+1} \rangle$ holds IFS conditions because

1′) $$0 \le \frac{1}{3} \le \frac{\gamma}{2\gamma+1} \le \frac{b+c+\gamma}{2\gamma+1} \le \frac{2+\gamma}{2\gamma+1} \le 1,$$

2′) $$0 \le \frac{\gamma-1}{2\gamma+1} \le \frac{a+d+\gamma-1}{2\gamma+1} \le \frac{\gamma+1}{2\gamma+1} \le \frac{2}{3} \le 1,$$

3′) $$0 \le \frac{1}{3} \le \frac{2\gamma-1}{2\gamma+1} \le \frac{b+c+\gamma}{2\gamma+1} + \frac{a+d+\gamma-1}{2\gamma+1} \le \frac{2\gamma+1}{2\gamma+1} = 1.$$

Conditions:

(i1) If $\langle a_1, b_1\rangle = V(x_1) \preceq V(x_2) = \langle a_2, b_2\rangle$ therefore $a_1 \le a_2$ and $b_1 \ge b_2$, so

$$\frac{b_1+c+\gamma}{2\gamma+1} \ge \frac{b_2+c+\gamma}{2\gamma+1} \text{ and } \frac{a_1+d+\gamma-1}{2\gamma+1} \le \frac{a_2+d+\gamma-1}{2\gamma+1}$$

and consequently $V(x_1 \to_\gamma y) \succeq V(x_2 \to_\gamma y)$.

(i2) If $\langle c_1, d_1\rangle = V(y_1) \preceq V(y_2) = \langle c_2, d_2\rangle$ therefore $c_1 \le c_2$ and $d_1 \ge d_2$, so

$$\frac{b+c_1+\gamma}{2\gamma+1} \le \frac{b+c_2+\gamma}{2\gamma+1} \text{ and } \frac{a+d_1+\gamma-1}{2\gamma+1} \ge \frac{a+d_2+\gamma-1}{2\gamma+1}$$

and consequently $V(x_1 \to_\gamma y) \preceq V(x_2 \to_\gamma y)$.

(i3) It is, by definition, $V(\underline{0} \to_\gamma y) = \langle \frac{1+c+\gamma}{2\gamma+1}, \frac{d+\gamma-1}{2\gamma+1} \rangle$.

Because $\frac{1+c+\gamma}{2\gamma+1} \ge \frac{d+\gamma-1}{2\gamma+1}$ is equivalent for an inequality $c–d \ge –2$, and this holds therefore $\underline{0} \Rightarrow y$ is an IFT.

(i4) It is $V(x \to_\gamma \underline{1}) = \langle \frac{b+1+\gamma}{2\gamma+1}, \frac{a+\gamma-1}{2\gamma+1} \rangle$.

Because $\frac{b+1+\gamma}{2\gamma+1} \ge \frac{a+\gamma-1}{2\gamma+1}$ is equivalent for an inequality $b–a \ge –2$, and this holds therefore $x \Rightarrow \underline{1}$ is an IFT.

(i5) It is $V(\underline{1} \to_\gamma \underline{0}) = \langle \frac{\gamma}{2\gamma+1}, \frac{\gamma+1}{2\gamma+1} \rangle$. Because $\frac{\gamma}{2\gamma+1} \le \frac{\gamma+1}{2\gamma+1}$ therefore $\underline{1} \Rightarrow \underline{0}$ is an IFcT. ■

In recent literature (various authors give these axioms following [10], p. 308, 310; see also [2, 4, 5, 6]), besides (i1)-(i5), the following axioms are also postulated:

(i6) $V(\underline{1} \Rightarrow y) = V(y)$,

(i7) $V(x \Rightarrow x) = V(\underline{1})$,

(i8) $V(x \Rightarrow (y \Rightarrow z)) = V(y \Rightarrow (x \Rightarrow z))$,

(i9) $V(x \Rightarrow y) = V(\underline{1}) \Leftrightarrow V(x) \preceq V(y)$,

(i10) $V(x \Rightarrow y) = V(N(y) \Rightarrow N(x))$, while N is a some negation, (i11) $\Rightarrow$ is a continuous function,

where x, y, z are variables with a truth-value $V(x) = \langle a, b\rangle$, $V(y) = \langle c, d\rangle$, $V(z) = \langle e, f\rangle$ and $a, b, c, d, e, f, a+b, c+d, e+f \in [0, 1]$.

Theorem 2. Implication $\to_\gamma$

a) does not satisfy (i6), (i7), (i8),

b) does not satisfy (i9), but if $V(x \to_\gamma y) = V(\underline{1})$ then $V(x) \preceq V(y)$,

c) satisfies (i11) and (i10) with $N = \neg$.

Proof:

(i6)
$$V(\underline{1} \to_\gamma y) = \langle \frac{c+\gamma}{2\gamma+1}, \frac{d+\gamma}{2\gamma+1} \rangle \neq \langle c, d\rangle.$$

(i7)
$$V(x \to_\gamma x) = \langle \frac{a+b+\gamma}{2\gamma+1}, \frac{a+b+\gamma-1}{2\gamma+1} \rangle \neq \langle 1, 0\rangle.$$

(i8)
$$V(x \to_\gamma (y \to_\gamma z)) =$$
$$= \langle \frac{(2\gamma+1)(b+\gamma)+d+e+\gamma}{(2\gamma+1)^2}, \frac{(2\gamma+1)(a+\gamma-1)+c+f+\gamma-1}{(2\gamma+1)^2} \rangle \neq$$
$$\neq \langle \frac{(2\gamma+1)(d+\gamma)+b+e+\gamma}{(2\gamma+1)^2}, \frac{(2\gamma+1)(c+\gamma-1)+a+f+\gamma-1}{(2\gamma+1)^2} \rangle =$$
$$= V(y \to_\gamma (x \to_\gamma z)).$$

It is easy to show that the equality (i8) does not hold.

For a counterexample let us assume $a = b = 0.1$, $c = d = 0.2$, $e = f = 0.3$.

(i9) If $V(x \to_\gamma y) = V(\underline{1})$, i.e. $\frac{b+c+\gamma}{2\gamma+1} = 1$ and $\frac{a+d+\gamma-1}{2\gamma+1} = 0$,

therefore $b+c = 1+\gamma$ and $a+d = 1-\gamma$ what holds only for $\gamma = 1$, $a = d = 0$, $b = c = 1$, therefore
$$V(x) = \langle 0, 1\rangle \preceq \langle 1, 0\rangle = V(y).$$

In the other direction, if $V(x) \preceq V(y)$ i.e., $a \le c$ and $b \ge d$, then not necessarily $V(x \to_\gamma y) = V(\underline{1})$. The counterexample: $a = d = 0.1$, $b = c = 0.2$, $\gamma = 1$.

(i10) Because $V(N(x)) = \langle b, a\rangle$, $V(N(y)) = \langle d, c\rangle$ then
$$V(N(y) \to_\gamma N(x)) = \langle \frac{b+c+\gamma}{2\gamma+1}, \frac{a+d+\gamma-1}{2\gamma+1} \rangle = V(x \to_\gamma y).$$

(i11) Arithmetic operations are continuous due to both arguments. ■

It is also easy to check that the implication $\to_\gamma$ does not satisfy the classical (two-valued) logic axioms.

Namely $V(\underline{0} \to_\gamma \underline{0}) = V(\underline{1} \to_\gamma \underline{1}) = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1} \rangle \neq V(\underline{1})$, $V(\underline{1} \to_\gamma \underline{0}) = \langle \frac{\gamma}{2\gamma+1}, \frac{\gamma+1}{2\gamma+1} \rangle \neq V(\underline{0})$ and $V(\underline{0} \to_\gamma \underline{1}) = \langle \frac{2+\gamma}{2\gamma+1}, \frac{\gamma-1}{2\gamma+1} \rangle \neq V(\underline{1})$ (except $\gamma = 1$). But we notice that $\underline{0} \to_\gamma \underline{0}$, $\underline{1} \to_\gamma \underline{1}$ and $\underline{0} \to_\gamma \underline{1}$ are IFTs, however $\underline{1} \to_\gamma \underline{0}$ is an IFcT.

As we can see the implication $\to_\gamma$ is not a generalization of the classical implication.

Let us introduce now some IFL-case of axioms (i6)-(i10) in the form:

(i6IFL) 1^0) $\underline{1} \Rightarrow y$ is an IFT iff y is an IFT,

2^0) $\underline{1} \Rightarrow y$ is an IFcT iff y is an IFcT,

(i7IFL) $x \Rightarrow x$ is an IFT,

(i8IFL) 1^0) $x \Rightarrow (y \Rightarrow z)$ is an IFT iff $y \Rightarrow (x \Rightarrow z)$ is an IFT,

2^0) $x \Rightarrow (y \Rightarrow z)$ is an IFcT iff $y \Rightarrow (x \Rightarrow z)$ is an IFcT,

(i9IFL) $x \Rightarrow y$ is an IFT iff $V(x) \preceq V(y)$,

(i10IFL) 1^0) $x \Rightarrow y$ is an IFT iff $N(y) \Rightarrow N(x)$ is an IFT,

2^0) $x \Rightarrow y$ is an IFcT iff $N(y) \Rightarrow N(x)$ is an IFcT.

Theorem 3. Implication $\to_\gamma$

a) satisfies (i6IFL), (i7IFL) and (i10IFL).

b) does not satisfy (i8IFL) and (i9IFL), but if $V(x) \preceq V(y)$ then $x \to_\gamma y$ is an IFT.

Proof:

(i6IFL) 1^0) $\underline{1} \to_\gamma y$ is an IFT so $\frac{c+\gamma}{2\gamma+1} \geq \frac{d+\gamma}{2\gamma+1}$ therefore $c \geq d$ hence y is an IFT,

2^0) $\underline{1} \to_\gamma y$ is an IFcT so $\frac{c+\gamma}{2\gamma+1} \leq \frac{d+\gamma}{2\gamma+1}$ therefore $c \leq d$ hence y is an IFcT.

(i7IFL) $V(x \to_\gamma x) = \langle \frac{a+b+\gamma}{2\gamma+1}, \frac{a+b+\gamma-1}{2\gamma+1} \rangle$, and $\frac{a+b+\gamma}{2\gamma+1} \geq \frac{a+b+\gamma-1}{2\gamma+1}$ holds,

therefore $x \to_\gamma x$ is an IFT.

(i8IFL) 1^0) Let $a = 0.5$, $b = 0.4$, $c = 1$, $d = 0$, $e = 0$, $f = 1$.

It is $\frac{(2\gamma+1)(b+\gamma)+d+e+\gamma}{(2\gamma+1)^2} \geq \frac{(2\gamma+1)(a+\gamma-1)+c+f+\gamma-1}{(2\gamma+1)^2}$ but not

$$\frac{(2\gamma+1)(d+\gamma)+b+e+\gamma}{(2\gamma+1)^2} \geq \frac{(2\gamma+1)(c+\gamma-1)+a+f+\gamma-1}{(2\gamma+1)^2}$$

therefore $x \to_\gamma (y \to_\gamma z)$ is an IFT while $y \to_\gamma (x \to_\gamma z)$ is not an IFT.

So neither equivalence nor implication holds.

2^0) A counterexample similar to 1^0).

(i9IFL) Condition: $x \to_\gamma y$ is an IFT does not entails $V(x) \preceq V(y)$.

For example: if a = 0.4, b = 0.3, c = 0.4 and d = 0.5 then $\frac{b+c+\gamma}{2\gamma+1} \geq \frac{a+d+\gamma-1}{2\gamma+1}$ but not $V(x) \preceq V(y)$ because $a \leq c$ and not $b \geq d$.

In turn, if $V(x) \preceq V(y)$ then $c-a+b-d \geq 0$ so $c-a+b-d \geq -1$ therefore $\frac{b+c+\gamma}{2\gamma+1} \geq \frac{a+d+\gamma-1}{2\gamma+1}$ which means that $x \to_\gamma y$ is an IFT .

(i10IFL) It is a simple consequence of (i10). ■

There exist two basic rules of inference. They are Modus Ponens and Modus Tollens rules. These are the tautologies, given in the two-valued logic in the form: $(p \wedge (p \Rightarrow q)) \Rightarrow q$ and $((p \Rightarrow q) \wedge N(q)) \Rightarrow N(p)$ respectively.

The Modus Ponens in the IFL-case is as follows: if x is an IFT and $(x \Rightarrow y)$ is an IFT then y is an IFT. Similar, the Modus Tolens in the IFL-case is: if $(x \Rightarrow y)$ is an IFT and y is an IFcT then x is an IFcT.

Theorem 4. Implication $\to_\gamma$

a) does not satisfy Modus Ponens in the IFL-case

b) does not satisfy Modus Tolens in the IFL-case.

Proof – by a counterexample:

a) Let a = 0.5, b = 0.5, c = 0, d = 1. Then x is an IFT and $x \to_\gamma y$ is an IFT because $\frac{b+c+\gamma}{2\gamma+1} = \frac{0.5+\gamma}{2\gamma+1} \geq \geq \frac{1.5+\gamma-1}{2\gamma+1} = \frac{a+d+\gamma-1}{2\gamma+1}$ while y is not an IFT.

b) Let a = 1, b = 0, c = 0.5, d = 0.5. Then $x \to_\gamma y$ is an IFT because

$$\frac{b+c+\gamma}{2\gamma+1} = \frac{0.5+\gamma}{2\gamma+1} \geq \frac{1.5+\gamma-1}{2\gamma+1} = \frac{a+d+\gamma-1}{2\gamma+1}$$

and y is an IFcT while x is not an IFcT. ■

Remark. For $V(x) = \langle 1, 0\rangle$ if $x \to_\gamma y$ would be an IFT, i.e., $\frac{b+c+\gamma}{2\gamma+1} \geq \frac{a+d+\gamma-1}{2\gamma+1}$ we would have $c \geq d$ which means that Modus Ponens rule in the IFL case holds.

Similarly, for $V(y) = \langle 0, 1\rangle$ we have $\frac{b+\gamma}{2\gamma+1} \geq \frac{a+\gamma}{2\gamma+1}$ therefore $b \geq a$ which means that Modus Tolens rule in the IFL case holds.

One of the fundamental tautology of classical logic is the relationship between an implication and negation. This relationship says that the truth-value of a negation of the variable x is equal to the value of the logical implications of the antecedent x and the consequent *false*. Symbolically, this tautology is written in the form of $N(x) \Leftrightarrow (x \Rightarrow 0)$. Using this relationship we can, for every intuitionistic fuzzy implication, designate a corresponding negation, called a generated (induced) negation.

Theorem 5. Negation N_γ generated by $\to_\gamma$ is expressed by the formula

$$V(N_\gamma(x)) = \langle \frac{b+\gamma}{2\gamma+1}, \frac{a+\gamma}{2\gamma+1} \rangle.$$

Proof – by the definition of $\to_\gamma$.

Remarks:

R1. If x is an IFT then $N_\gamma(x)$ is an IFcT and if x is an IFcT then $N_\gamma(x)$ is an IFT.

R2. Negation N_γ is not involutive. Moreover, $V(N_\gamma(N_\gamma(x))) = V(x)$ only for a=b=0.5.

R3. Negation N_γ does not satisfy the classical axioms

$$V(N_\gamma(\underline{0})) = V(\underline{1}) \text{ and } V(N_\gamma(\underline{1})) = V(\underline{0}) .$$

R4. Negation N_γ satisfies properties $V(N_\gamma(\neg x)) = V(\neg(N_\gamma(x)))$, in particular

$$V(\neg(N_\gamma(\underline{1}))) = V(N_\gamma(\underline{0})) \quad \text{and} \quad V(\neg(N_\gamma(\underline{0}))) = V(N_\gamma(\underline{1})).$$

R5. Negation N_γ satisfies property $V(N_\gamma(\underline{0}))\ @\ V(N_\gamma(\underline{1})) = \langle 0.5, 0.5\rangle$ where @ is an operator given by Atanassov in [3] by the formula $x@y = \langle \frac{a+c}{2}, \frac{b+d}{2} \rangle$.

We denote $N_\gamma^{1}(x) = N_\gamma(x)$ and $N_\gamma^{m+1}(x) = N_\gamma(N_\gamma^{m}(x))$ for any $m \in \boldsymbol{N}_+$.

Theorem 6. Negation N_γ holds for a natural number $k \geq 1$ the relationship

a) $$V(N_\gamma^{2k}(x)) = \langle \frac{2a+(2\gamma+1)^{2k}-1}{2(2\gamma+1)^{2k}}, \frac{2b+(2\gamma+1)^{2k}-1}{2(2\gamma+1)^{2k}} \rangle$$

b) $$V(N_\gamma^{2k-1}(x)) = \langle \frac{2b+(2\gamma+1)^{2k-1}-1}{2(2\gamma+1)^{2k-1}}, \frac{2a+(2\gamma+1)^{2k-1}-1}{2(2\gamma+1)^{2k-1}} \rangle.$$

P r o o f:

For k = 1 it holds

$$V(N_\gamma^{2k-1}(x)) = \langle \frac{2b+2\gamma}{2(2\gamma+1)}, \frac{2a+2\gamma}{2(2\gamma+1)} \rangle = V(N_\gamma(x)) = \langle \frac{b+\gamma}{2\gamma+1}, \frac{a+\gamma}{2\gamma+1} \rangle$$

and

$$V(N_\gamma^{2k}(x)) = \langle \frac{2a+(2\gamma+1)^{2}-1}{2(2\gamma+1)^{2}}, \frac{2b+(2\gamma+1)^{2}-1}{2(2\gamma+1)^{2}} \rangle = V(N_\gamma(N_\gamma(x))).$$

We assume that for k>1 is

$$V(N_\gamma^{2k-1}(x)) = \langle \frac{2b+(2\gamma+1)^{2k-1}-1}{2(2\gamma+1)^{2k-1}}, \frac{2a+(2\gamma+1)^{2k-1}-1}{2(2\gamma+1)^{2k-1}} \rangle.$$

Then

$$V(N_\gamma^{2k}(x)) = V(N_\gamma(N_\gamma^{2k-1}(x))) =$$

$$= \langle \frac{1}{2\gamma+1}\left(\frac{2a+(2\gamma+1)^{2k-1}-1}{2(2\gamma+1)^{2k-1}}+\gamma\right), \frac{1}{2\gamma+1}\left(\frac{2b+(2\gamma+1)^{2k-1}-1}{2(2\gamma+1)^{2k-1}}+\gamma\right) \rangle =$$

$$= \langle \frac{2a+(2\gamma+1)^{2k}-1}{2(2\gamma+1)^{2k}}, \frac{2b+(2\gamma+1)^{2k}-1}{2(2\gamma+1)^{2k}} \rangle .$$

The proof for $V(N_\gamma^{2k-1}(x))$ is analogous.

Based on principle of mathematical induction Theorem 6 is valid for every $k \in \boldsymbol{N}_+$. ■

Corollary 1. $\lim_{m\to\infty} V(N_\gamma^{m}(x)) = \langle 0.5, 0.5\rangle$.

Corollary 2. Axiom (i10) is satisfied for negation N_γ only for $V(x) = V(y) = \langle a, 1-a\rangle$.

Generally, (i10) does not hold because

$$V(x \to_\gamma y) = \langle \frac{b+c+\gamma}{2\gamma+1}, \frac{a+d+\gamma-1}{2\gamma+1} \rangle \neq \langle \frac{b+c+2\gamma^2+3\gamma}{(2\gamma+1)^2}, \frac{a+d+2\gamma^2+\gamma-1}{(2\gamma+1)^2} \rangle =$$
$$= V(N_\gamma(y) \to_\gamma N_\gamma(x)).$$

Axiom (i10IFL) does not hold also but properties 1) and 2) in the form

1) if $x \to_\gamma y$ is an IFT then $N_\gamma(y) \to_\gamma N_\gamma(x)$ is an IFT,

2) if $N_\gamma(y) \to_\gamma N_\gamma(x)$ is an IFcT then $x \to_\gamma y$ is an IfcT,

are valid.

This is because the fact $x \to_\gamma y$ is an IFT means that

$$\frac{b+c+\gamma}{2\gamma+1} \geq \frac{a+d+\gamma-1}{2\gamma+1}$$

what is equivalent to

$$\frac{b+c+2\gamma^2+3\gamma}{(2\gamma+1)^2} \geq \frac{a+d+2\gamma^2+\gamma-1+\gamma}{(2\gamma+1)^2}$$

which entails $\frac{b+c+2\gamma^2+3\gamma}{(2\gamma+1)^2} \geq \frac{a+d+2\gamma^2+\gamma-1}{(2\gamma+1)^2}$ which is a prerequisite to $N(y) \to_\gamma N(x)$ being an IFT.

In turn, if $N_\gamma(y) \to_\gamma N_\gamma(x)$ is an IFcT, i.e.,

$$\frac{b+c+2\gamma^2+3\gamma}{(2\gamma+1)^2} \leq \frac{a+d+2\gamma^2+\gamma-1}{(2\gamma+1)^2}$$

what is equivalent to

$$\frac{b+c+\gamma+2\gamma}{2\gamma+1} \leq \frac{a+d+\gamma-1}{2\gamma+1},$$

it is also

$$\frac{b+c+\gamma}{2\gamma+1} \leq \frac{a+d+\gamma-1}{2\gamma+1}$$

which is a prerequisite to $x \to_\gamma y$ being an IFcT.

3. Conclusion

In the paper a new class of fuzzy intuitionistic implications with their basic properties is presented. The fulfillment of some axioms and properties together with Modus Ponens and Modus Tollens inference rules are investigated. The negation induced by implication is presented. These implications may be the subject of further research, both in terms of their properties or comparisons with other intuitionistic fuzzy implications, and possible applications.

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