BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **10**, No 1

Sofia • 2010

Hough Detector Threshold Analysis in the Presence of a Randomly Arriving Impulse Interference[1]

Lyubka Doukovska, Ivan Garvanov

Institute of Information Technologies, 1113 Sofia
E-mails: doukovska@iit.bas.bg   igarvanov@yahoo.com

Abstract: *In this paper a technique of Hough detector threshold procedure for moving target detection under conditions of randomly arriving impulse interference with a Poisson distributed flow and Raleigh amplitude distribution is proposed. The expressions of detection and false alarm probability are derived for a highly fluctuating Swerling II target. A comparative analysis of the performance of a Hough detector with fixed threshold and other five Hough detection structures keeping constant false alarm rates is done. These are a CA CFAR (Cell Averaging Constant False Alarm Rate), an EXC CFAR (Excision Constant False Alarm Rate), a CFAR BI (Constant False Alarm Rate with Binary Integration), an EXC CFAR BI (Excision Constant False Alarm Rate with Binary Integration) and an API CFAR (Adaptive censoring Post detection Integration Constant False Alarm Rate). A method for losses estimation, which allows choosing of optimal detector parameters, is developed. The estimates are obtained of the effectiveness of Hough detector in the presence of randomly arriving impulse interference and they are compared to patterns researched by other authors. The achieved results can be successfully applied for radar target detection and in the existing communication network receivers that use pulse signals.*

Keywords: *Radar detector, Hough detector, Randomly arriving impulse interference, Probability of detection, Probability of false alarm, Detectability profits (losses).*

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[1] This work is supported by projects BG051PO001/07/3.3-02-7/08, DTK02/28/2009 and IIT-010097.

1. Introduction

The modern radar techniques for moving target detection include comparison between the moving target signal and a preliminary determined detection threshold under conditions of Randomly Arriving Impulse Interference (RAII) with unknown intensity. Signal detection in noise or clutter is a very important part of target detection procedure. In theory the noise and clutter background will be described by a statistical model with e.g. Rayleigh or exponentially distributed random variables of known average noise power. But in practical applications this average noise or clutter power is absolutely unknown and some statistical parameter can additionally vary over range, time and azimuth. In automatic radar detection, the signal received is sampled in range and frequency. Each sample is placed in an array of range and Doppler resolution cells. The clutter background in the cell under test is estimated by averaging the outputs of the nearby resolution cells (range and/or Doppler). The target detection is declared if the signal value exceeds a preliminary determined threshold. The detection threshold is obtained by scaling the noise level estimate with a constant T_α to achieve a desired probability of false alarm P_{FA}.

The detection process is based on a statistical analysis, which guarantees target detection with Constant False Alarm Rate (CFAR). For the first time a structure with CFAR algorithm is proposed in 1968 year by Finn and Johnson [2] (Cell Averaging CFAR). This CFAR processor is very effective in case of stationary and homogeneous interference. The presence of strong impulse interference can cause drastic degradation in the performance of the CA CFAR processor. Such type of interference is non-stationary and non-homogenous and it is often caused by adjacent radar or other radio-electronic devices. In a non-homogenous environment, the detection performance and the false alarm regulation properties of CA CFAR detector may be seriously degraded.

This concept is used by many adaptive target detection algorithms, which compare the signal intensity with an adaptive threshold with value depending on the noise level. During the last few years a lot of different structured CFAR algorithms appeared [4, 14].

The detection performance of CFAR processors is proposed by Hou in [3] for the case of homogeneous environment and chi-square family of fluctuating target models (Swerling I, II, III, IV). In our paper we study the situation for a highly fluctuating target − Swerling II type target model detection under conditions of intensive RAII.

In recent years the algorithms that extract information for target's behavior through mathematical transformation of the signals reflected from a target became very popular. Modern methods for target detection and trajectory parameters estimation by using mathematical transformation of received signals allow designing of new highly effective algorithms for radar signal processing. As a result, extremely precise estimates of moving targets parameters can be obtained in conditions of very dynamic radar environment. An approach for target detection by means of Hough transform of the target coordinates obtained for few sequential scans of the observation area is considered in [1]. For target detection, the method discussed uses a limited set of preliminary chosen patterns of a linear target trajectory. The set of target distance measurements is transformed to the pattern space (parameter space) by means of Hough transform. The association of measurements to a special pattern is done by parameter estimation of the data extracted from the signals for the target distances related to this pattern. Thus the trajectory parameters of the targets moving in the observation area are determined through parameters of the corresponding pattern.

Hough detector performance depends very much on the randomly arriving impulse interference caused by different sources. The occurrence of these disturbances, even with low probability, worsens detector performance. In such cases the Hough detector with constant threshold doesn't support constant false alarm rate. This causes an increase in the value of the detection threshold. Thus the detection probability is diminished. The usage of CFAR processors together with a Hough detector would improve the probability characteristics ensuring the constant false alarm rate [5-14].

In this paper a comparative analysis of the performance of different types of CFAR detectors used in the algorithm of Hough detector is carried out. This structure gives a possibility of keeping constant false alarm rate in presence of random arriving impulse interference. In our study we consider the limit case when increasing the probability of appearance changes the distribution law from Poisson to binominal. The binominal model is more general than Poisson distribution model [4]. The change of distribution law and parameters of RAII makes impossible keeping the constant false alarm rate of the Hough detector with fixed threshold and leads to worsen detection process.

In the presented paper some threshold determination procedures for several types of Hough detector structures with CFAR processors are investigated, in order to choose the most efficient one under conditions of intensive RAII.

The research work is performed in MATLAB computational environment.

2. Signal model

Using Carlson's approach [1], we obtain new results for detection performance in Hough space, for a target model of Swerling II type in RAII. The signal in the reference window is assumed to be with Poisson distribution and has the following Probability Density Function (PDF) [4]:

$$f_{sP}(x)=\frac{(1-e_0)}{\lambda_0(1+s)}\exp\left(\frac{-x}{\lambda_0(1+s)}\right)+\frac{e_0}{\lambda_0(1+s+r_j)}\exp\left(\frac{-x}{\lambda_0(1+s+r_j)}\right) \tag{1}$$

where s is the per pulse average Signal-to-Noise Ratio (SNR), λ_0 is the average power of the receiver noise, r_j is the average Interference-to-Noise Ratio (INR), e_0 is the probability of appearance of RAII.

Under conditions of binomial distribution of pulse interference, the probability of interference-plus-noise occurrence in the background environment is $2e_0(1-e_0)$. The probability of appearance of two interferences in a single cell is ${e_0}^2$ and having only noise probability, it is $(1-e_0)^2$, where $e_0 = 1-\sqrt{1-t_c F}$, F is the average repetition frequency of pulse interference and t_c is the length of pulse transmission [4].

The distribution is binomial when the probability of pulse interference is above 0.1 [4]. In these situations the outputs of the reference window are observations from statistically independent exponential random variables. Consequently, the Probability Density Function (PDF) of the reference window outputs may be defined by:

$$f_{sB}(x) = \frac{(1-e_0)^2}{\lambda_0(1+s)}\exp\left(\frac{-x}{\lambda_0(1+s)}\right) + \frac{2e_0(1-e_0)}{\lambda_0(1+s+r_j)}\exp\left(\frac{-x}{\lambda_0(1+s+r_j)}\right) + \\ + \frac{{e_0}^2}{\lambda_0(1+s+2r_j)}\exp\left(\frac{-x}{\lambda_0(1+s+2r_j)}\right) \tag{2}$$

where λ_0 is the average power of the receiver noise and r_j/λ_0 is the per pulse average Interference-to-Noise Ratio.

On next two figures simulation examples for Poisson and for binomial distributions of pulse interference are shown, with equally values s=70 dB, $\lambda_0 = 1$, r_j=30 dB, e_0=0.1.

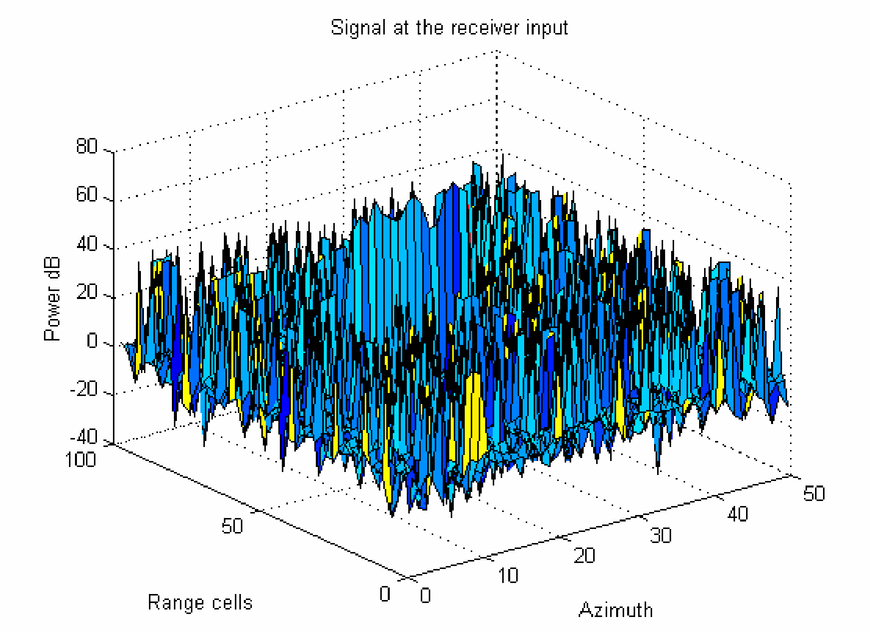

Fig. 1. Example for Poisson distribution of pulse interference

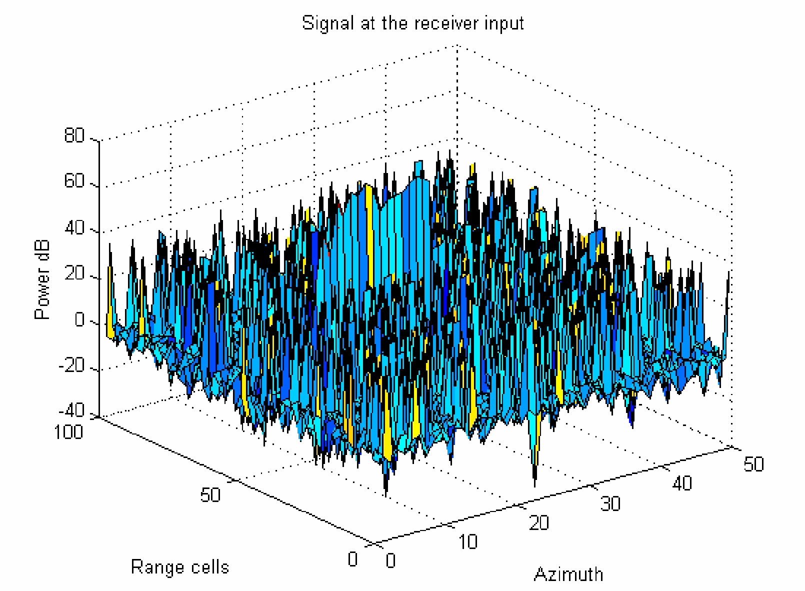

Fig. 2. Example for binominal distribution of pulse interference

3. CFAR processor statistical analisys

In a contemporary radar, the target detection is made by the signal processor after preliminary detection and sampling of the input signals. During the processing consequently the following procedures are carried out: filtration, adaptive moving target detection, non-coherent integration and adaptive detection summary signal by comparing the value of the integrated signal with a preliminary determined adaptive threshold.

The target detection is declared if the signal value exceeds this threshold. The threshold is formed by current estimating the noise level in the reference window. As an estimate of the noise level the estimate proposed by Finn and Johnson in [2] is often used. This estimate is formed by averaging the outputs of the reference cells surrounding the test cell. Thus a CFAR is maintained in the process of detection. On Fig. 3 the general structure of an adaptive CFAR processor is shown.

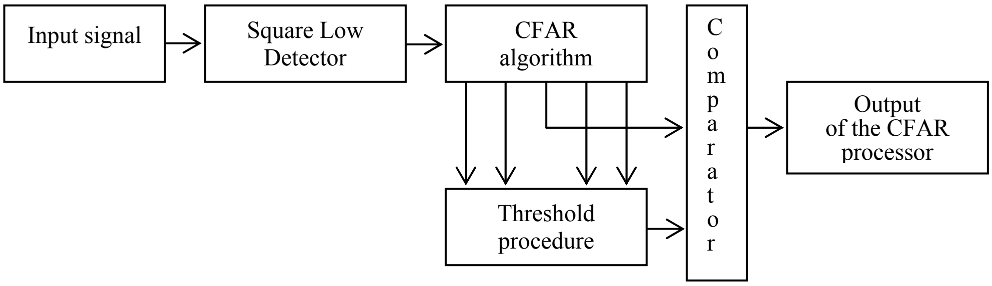

Fig. 3. General structure of an adaptive CFAR processor

The presence of strong impulse interference can cause drastic degradation in the performance of the CFAR processor. Such type of interference is non-stationary and non-homogenous and it is often caused by an adjacent radar or any other radio-electronic devices. There are a lot of publications on increasing of the efficiency of CFAR processors under these conditions, which makes this problem very actual [3, 4, 14].

In modern radars systems, keeping constant false alarm rates, the target is detected according to the following algorithm [2]:

$$\begin{cases} H_1 : \Phi(q_0) = 1, & q_0 > T_\alpha V, \\ H_0 : \Phi(q_0) = 0, & q_0 < T_\alpha V, \end{cases} \tag{3}$$

where H_1 is the hypothesis that the test resolution cell contains the echoes from the target and H_0 is the hypothesis that the test resolution cell contains the randomly arriving impulse interference only, V is the noise level estimation. The constant T_α is a scale coefficient, which is determined in order to maintain a given constant false alarm rate. Fig. 4 presents one example of the adaptive threshold procedure for one-dimensional CFAR processor under conditions of randomly arriving impulse interferences.

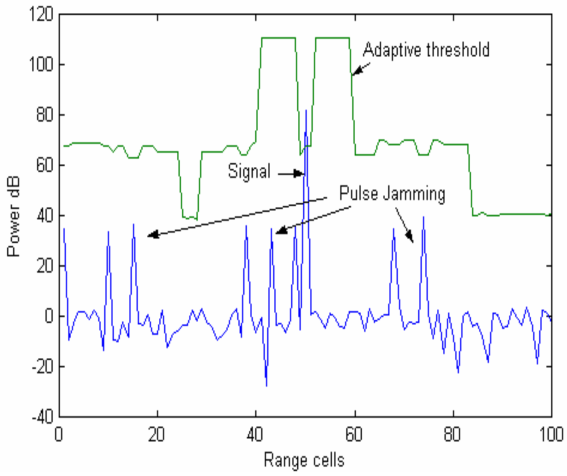

Fig. 4. Adaptive threshold procedure for one-dimensional CFAR processor

The different CFAR structures make use of different algorithms for noise level estimation – V, [5-14]. In this paper several types of signal processors – a CA (Cell Averaging), an EXC (EXCision), a BI (Binary Integration), an EXC BI (Excision with Binary Integration) and an API (Adaptive censoring Post detection Integration) are analyzed.

4. Hough detector analysis

The basic concept of using the Hough transform to improve radar target detection in white Gaussian noise is firstly introduced by Carlson, Evans and Wilson in [1]. In this paper it is proved that different CFAR processors used for signal detection on the homogeneous background of unknown intensity and under the presence of randomly arriving impulse interference with known parameters improve the detection performance. In such CFAR processors it is usually assumed that the noise amplitude is a Rayleigh distributed variable and the power, therefore, is an exponentially distributed variable. As shown in papers [5-13], such CFAR processors combined with a conventional Hough detector can improve the detection probability characteristics in conditions of randomly arriving impulse interference. This is a very severe situation from a radar point of view. The analysis of the performance of the Hough detector with different structures of one-dimensional and two-dimensional CFAR processors – CA CFAR (Cell Average), EXC CFAR (EXCision), CFAR BI (Binary Integration), EXC CFAR BI (EXCision and binary integration) and API CFAR (Adaptive Post Integration) is done in [5-13].

The general structure of an adaptive Hough detector with binary integration of data in the Hough parameter space is shown on Fig. 5.

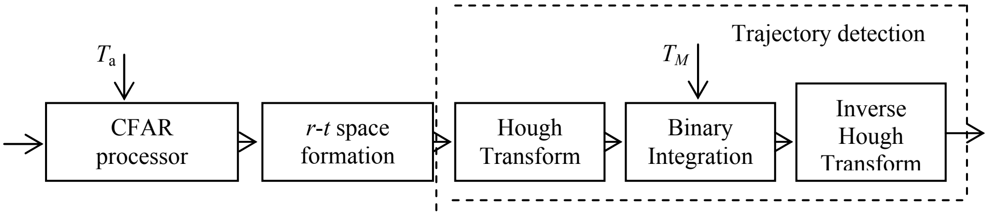

Fig. 5. Structure of an adaptive Hough detector with CFAR processor

In the paper presented, we consider the effectiveness of target detection procedure provided by the proposed CFAR algorithm. This effectiveness can be expressed by the quality parameters – the detection probability characteristics [1]. In papers [5-13] a detailed statistical analysis of the probability characteristics of different Hough structures is considered.

Due to the essence of the used mathematical transform and the existing constraints, the presentation of the cumulative probability of target detection in Hough parameter space P_{D} can not be expressed as a simple Bernoulli sum [1].

The probability of detection in the Hough parameter space – P_{D} can be calculated by Brunner's method [1]. For N_s scans we have:

$$P_{\text{D}} = \sum_{i=T_M}^{N_S} P_{\text{d}}^{*}(i, N_S), \tag{4}$$

where T_M is the detection threshold in Hough parameter space and P_{d}^{*} is the detection probability of one of the studied CFAR processors.

Let us assume that L pulses hit the target, which is modeled according to Swerling II case. The received signal is sampled in range by using N+1 resolution cells resulting in a matrix with N+1 rows and L columns. Each column of the data matrix consists of the values of the signal obtained for L pulse intervals in one range resolution cell. Let us also assume that the first N/2 and the last N/2 rows of the data matrix are used as a reference window in order to estimate the "noise-plusinterference” level in the radar test resolution cell. In this case the samples of the reference cells result in a matrix X of size $N\times L$. The test cell or the radar target image includes the elements of the $N/2+1$ row of the data matrix and is a vector Z of length L.

The probability of detection for a CA CFAR processor for target of case Swerling II, according to [9] is:

$$P^{*}{}_{\mathrm{d_{CA}}}=\sum_{i=1}^{N}C_N^i e_0^i\left(1-e_0\right)^{N-i}\left\{\frac{e_0}{\left(1+\frac{\left(1+r_j\right)T_{\mathrm{CA}}}{1+r_j+s}\right)^{i}\left(1+\frac{T_{\mathrm{CA}}}{1+r_j+s}\right)^{M-i}}+\right. \tag{5}$$

$$\left.+\frac{1-e_0}{\left(1+\frac{\left(1+r_j\right)T_{\mathrm{CA}}}{1+s}\right)^{i}\left(1+\frac{T_{\mathrm{CA}}}{1+s}\right)^{M-i}}\right\}$$

where T_{CA} is the threshold constant for CA CFAR processor.

The probability of detection for an EXC CFAR processor for target model Swerling II, according to [8] is

$$P^{*}{}_{\mathrm{d_{EXC}}}=\sum_{k=1}^{N}C_N^k P_E^k\left(1-P_E\right)^{N-k}\left\{\left(1-e_0\right)M_V\left(\frac{T_{\mathrm{EXC}}}{\lambda_0\left(1+s\right)},\,k\right)+e_0M_V\left(\frac{T_{\mathrm{EXC}}}{\lambda_0\left(1+r_j+s\right)},\,k\right)\right\}, \tag{6}$$

where $M_V(.)$ is the moment-generating function and T_{EXC} is a predetermined scale factor for EXC CFAR processor.

The moment-generating function (mgf) of the noise level estimate V, may be obtained as, [4]:

$$M_V\left(U\right)=\sum_{k=1}^{N}\binom{N}{k}P_E{}^{k}\left(1-P_E\right)^{N-k}M_V\left(U,k\right), \tag{7}$$

where the probability that a sample x_i survives at the excisor output is calculated as

$$P_E=1-\left(1-e_0\right)\exp\left(\tfrac{-B_E}{\lambda_0}\right)-e_0\exp\left(\tfrac{-B_E}{\lambda_0\left(1+r_j\right)}\right). \tag{8}$$

The function $M_V\left(U\,,k\right)$ is the conditional mgf of the estimate V where k is the number of samples survived at the excisor output.

The probability of pulse train detection for CFAR BI and EXC CFAR BI processors is evaluated in such noise situation as in [7, 10] by

$$P^{*}{}_{\mathrm{d_{BI}}}=\sum_{l=M}^{L}C_L^l P^{*}{}_{\mathrm{d_{CA/EXC}}}{}^{l}\left(1-P^{*}{}_{\mathrm{d_{CA/EXC}}}\right)^{L-l} \tag{9}$$

where M is binary decision rule, $P^{*}{}_{\mathrm{d_{CA/EXC}}}$ is the probability of pulse detection, which may be found using the expressions (5) or (6) – for CA CFAR processor and for EXC CFAR processor with Poisson impulse noise. The probability of the false alarm is evaluated by (9), setting $s = 0$.

The probability of detection for API CFAR processor according to [6] is:

$$\begin{aligned}
P^{*}_{\mathrm{d_{API}}} = {} & \sum_{k=1}^{N}\binom{N}{k}(1-e_0)^k e_0^{N-k}\sum_{l=1}^{L}\binom{L}{l}(1-e_0)^l e_0^{L-l}\sum_{i=0}^{l-1}\binom{k+i-1}{i}\frac{T_{\mathrm{API}}^{i}(1+s)^k}{(T_{\mathrm{API}}+1+s)^{k+i}} + \\
& + \sum_{k=1}^{N}\binom{N}{k}(1-e_0)^k e_0^{N-k} e_0^{L}\sum_{i=0}^{L-1}\binom{k+i-1}{i}T_{\mathrm{API}}^{i}(1+r_j+s)^k (T_{\mathrm{API}}+1+r_j+s)^{-(k+i)} + \\
& + \sum_{l=1}^{L}\binom{L}{l}(1-e_0)^l e_0^{L-l} e_0^{N}\sum_{i=0}^{l-1}\binom{N+i-1}{i}T_{\mathrm{API}}^{i}\left(\frac{1+s}{1+r_j}\right)^{N}\left(T_{\mathrm{API}}+\frac{1+s}{1+r_j}\right)^{-(N+i)} + \\
& + e_0^{N} e_0^{L}\sum_{i=0}^{L-1}\binom{N+i-1}{i}T_{\mathrm{API}}^{i}\left(\frac{1+r_j+s}{1+r_j}\right)^{N}\left(T_{\mathrm{API}}+\frac{1+r_j+s}{1+r_j}\right)^{-(N+i)}
\end{aligned} \tag{10}$$

where T_{API} is a predetermined scale factor for API CFAR processor that provides a constant false alarm rate (P_{FA}).

5. Numerical results

The presented results are obtained after detailed simulational analysis of Hough detector performance in RAII conditions. The experimental environment is similar to the one considered in other papers of the team [5-14]. In order to analyze the quality of the Hough detector we consider a radar with parameters, similar to those in [1]. The Carlson's approach, using the Brunner's method for calculating the probability of detection in Hough parameter space was developed further in order to maintain constant false alarm probability at the output of the Hough detector.

Fig. 6 shows the probability of false alarm for a Hough detector with a constant detection threshold achieved for non homogeneous interference with parameters – average power of the receiver noise λ_0=1, average Interference-to-Noise Ratio INR r_j=30 dB, probability of appearance of impulse interference with average length in the range cells $e_0 = 0 \div 0.1$, number of reference cells $N = 256$ and for value of binary rule in Hough parameter space – T_M=2/20.

Having no constant false alarm rate causes, an adaptive threshold determination procedure to be applied. The suitable scalar factor was chosen iteratively, according to the given noise environment.

Application of an adequate threshold processing allows the obtaining of a very high detection performance of the Hough detector in noise intensive environment. Choosing the appropriate threshold constants assures good detection results even for low values of the SNR.

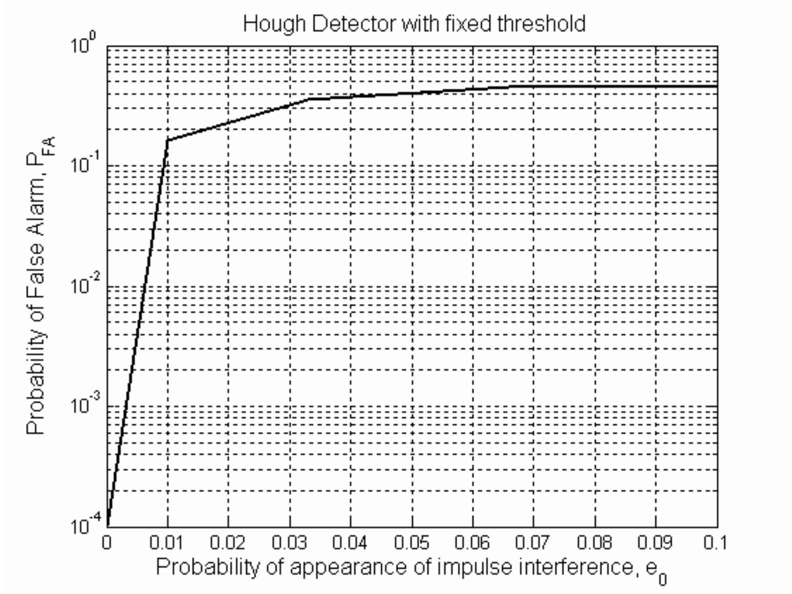

Fig. 6. Probability of false alarm for Hough detector with fixed threshold

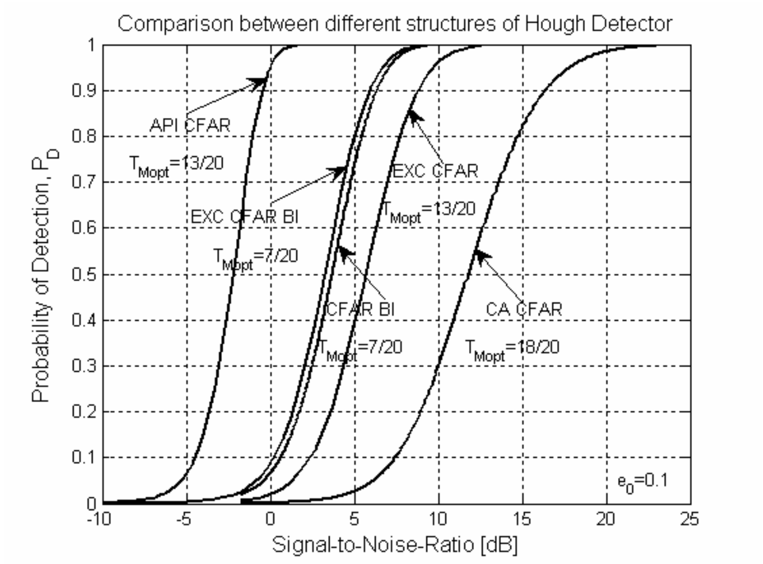

Fig. 7. Probability of detection for different structures of Hough detector

Table 1 presents the obtained threshold constants in equal experimental conditions for the different detection structures and different values of the binary rule in the Hough parameter space. The Hough detector structures are CA Hough CFAR, EXC Hough CFAR, Hough CFAR BI, EXC Hough CFAR BI and API Hough CFAR. Threshold constants are calculated for different Hough detectors and for fixed values – $P_{\mathrm{FA}}= 10^{-4}$, $e_0=0.1$, $r_j= 30$ dB, $N= 16$, $L = 16$.

Table 1

Hough detectors	T_M=2/20	T_M=$T_{M\text{opt}}$/20
CA Hough CFAR	672	1.186
EXC Hough CFAR	21880	3.225
Hough CFAR BI	0.000494	0.0000858
EXC Hough CFAR BI	1.1285	0.3161
API Hough CFAR	7.5	1.535

For comparison on Fig. 7 the probability of detection of different structures of Hough detector, calculated for optimal values of binary rule in Hough parameter space is shown – $T_M = T_{M\text{opt}}/20$ and also for the following environment parameter values – average power of the receiver noise λ_0=1, average INR r_j=30 dB, probability for the appearance of impulse interference with average length e_0=0.1, N=16, L=16 and for probability of false alarm $P_{\text{FA}} = 10^{-4}$.

6. Conclusions

The paper presented considers the results obtained by the proposed adaptive threshold determination procedure and analysis of different Hough detector structures in intensive RAII environment. The need of an adequate threshold analysis procedure allowing better detection results for low values of the SNR, is considered.

The optimal threshold values for different input conditions are estimated. The value of the test resolution cell and the probability of false alarm over the average detection threshold are studied. The application of censoring techniques in the detection algorithm improves the Hough detectors effectiveness.

The results obtained may have significant practical application for Hough detectors working in conditions of RAII. The obtained results show that Hough detector with API CFAR processor is the most effective under these conditions.

As a final conclusion the results achieved in the presented paper confirm once again the necessity of synthesis of new algorithms for moving targets detection, assuring robustness and higher efficiency of the radar systems. The results obtained in this paper could practically be used in radar and communication networks.

References

1. Carlson, B., E. Evans, S. Wilson. Search Radar Detection and Track with the Hough Transform. Parts I, II, III. – IEEE Trans., Vol. **AES-30**, 1994, 102-124.
2. Finn, H., R. Johnson. Adaptive Detection Mode with Threshold Control as a Function of Spatially Sampled Clutter Estimation. – RCA Review, Vol. **29**, 1968, No 3, 412-464.
3. Hou, X., N. Morinaga, T. Namekawa. Direct Evaluation of Radar Detection Probabilities.– IEEE Trans., Vol. **AES-23**, 1987, No 4, 418-423.
4. Akimov, P., F. Evstratov, S. Zaharov. Radio Signal Detection. – Radio and Communication, Moscow, 1989, 195-203 (in Russian).
5. Kabakchiev, C., L. Doukovska, I. Garvanov. Hough Radar Detectors in Conditions of Intensive Pulse Jamming. – Sensors & Transducers Magazine, Special Issue “Multisensor Data and Information Processing”, 2005, 381-389.

6. Kabakchiev, C., I. Garvanov, L. Doukovska. Adaptive Censoring CFAR PI Detector with Hough Transform in Randomly Arriving Impulse Interference. – Cybernetics and Information Technologies, Vol. **5**, 2005, No 1, 115-125.
7. Kabakchiev, C., I. Garvanov, L. Doukovska. Excision CFAR BI Detector with Hough Transform in Presence of Randomly Arriving Impulse Interference. – In: Proc. of the International Radar Symposium – IRS'05, Berlin, Germany, 2005, 259-264.
8. Doukovska, L., C. Kabakchiev. Performance of Hough Detectors in Presence of Randomly Arriving Impulse Interference. – In: Proc. of the International Radar Symposium – IRS'06, Krakow, Poland, 2006, 473-476.
9. Kabakchiev, C., L. Doukovska, I. Garvanov. Cell Averaging Constant False Alarm Rate Detector with Hough Transform in Randomly Arriving Impulse Interference. – Cybernetics and Information Technologies, Vol. **6**, 2006, No 1, 83-89.
10. Doukovska, L. Hough Detector with Binary Integration Signal Processor. – Compt. Rend. Acad. Bulg. Sci., Vol. **60**, 2007, No 5, 525-533.
11. Doukovska, L., V. Behar, C. Kabakchiev. Hough Detector Analysis by Means of Monte Carlo Simulation Approach. – In: Proc. of the International Radar Symposium – IRS'08, Wroclaw, Poland, 2008, 103-106.
12. Doukovska, L. Moving Target Hough Detector in Pulse Jamming. – Cybernetics and Information Technologies, Vol. **7**, 2007, No 1, 67-76.
13. Doukovska, L. Hough Detector with One-dimensional CFAR Processors in Randomly Arriving Impulse Interference. – In: Proc. of Distributed Computer and Communication Networks, International Workshop, Sofia, Bulgaria, 2006, 241-254.
14. Behar, V., C. Kabakchiev, L. Doukovska. Adaptive CA CFAR Processor for Radar Target Detection in Pulse Jamming. – Journal of VLSI Signal Processing, Vol. **26**, 2000, 383-396.
15. Rohling, H. Radar CFAR Thresholding in Clutter and Multiple Target Situation. – IEEE Trans., Vol. **AES-19**, 1983, No 4, 608-621.