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Fuzzy Algorithms for Selection of Bidding Strategies

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***Abstract**: The subject of the work is investigating algorithms for multicriteria decision making with fuzzy logic. The aim of the work is proving that algorithms with fuzzy criteria can successfully be used for solving the task of multicriteria ordering of bidding strategies in an auction. Two algorithms have been compared: one with aggregation operators (ATOKRI) and one with attitude alternative-criterion (FTNA). The major similarity between the two algorithms is that they both use fuzzy relations as a tool for choosing the optimal alternative. The key difference between them is in the subjectivism rate and the influence of the decision maker's attitude towards the compared alternatives and criteria. Despite the differences in the obtained strategy orders, both algorithms give results that are close to those of the "classical" algorithms for multicriteria analysis. Therefore, the two algorithms offer efficient ways for finding a solution of the task of multicriteria selection of bidding strategy in an auction under incomplete and uncertain information and changing conditions.*

***Keywords**: Decision making, fuzzy logic, multi criteria analysis.*

1. Introduction

The behaviour of the agents in an auction is based on their bidding strategies. Strategy is a methodology which the agent implements to achieve its goals while following the auction rules. Strategies are private and are chosen by auction participants (agents' owners). Various protocols for bidding are used in practice, so there is no universal strategy for successful negotiation. A given strategy can be efficient under one protocol and inefficient under another. The creation of an optimal strategy for Continuous Double Auction (CDA) is a complex task that still challenges electronic commerce researchers. The aim creating strategies that would pick out the "right" deal sides so that CDA effectiveness would be maximized and there would be quick deal price convergence toward the equilibrium price. The goal of this work is to investigate two alternative algorithms for multicriteria analysis with fuzzy logic for bidding strategy selection in CDA. The described methods for decision making can be used in electronic auctions not only for preliminary selection of the most suitable strategy from a given set of agent strategies, but also for changing the strategy used during auction. In Section 1 the current investigations of the problem are reviewed. Section 2 represents a comparative analysis between the two algorithms on the basis of a conducted experimental investigation. In the last section some conclusions are made about the applicability of the methods suggested and the future work perspectives are outlined.

2. Related work

For detailed investigation of intelligent agents' behaviour as participants in electronic commerce, various auction models have been simulated. The suggested methods for bidding strategies evaluation are based on two different approaches. The first one of them is essentially comparison between strategies' efficiency in a static agent population (the strategies are selected in advance and cannot be changed once an auction has begun). The second approach investigates agent populations, in which a change in the strategy used is allowed after the beginning of an auction, using replicator dynamics. Anthony and Jennings, who follow the first approach, generate biddings that depend on the following parameters: time left till the end of the auction; number of open auctions; agent's intention towards a deal and agent's attitude towards risk. The combination of these four indices through relative weighting coefficients, defined by the user, gives a bidding strategy. Later, the two authors suggest a genetic algorithm for search of an efficient strategy from the solution set defined by the specific market conditions [2]. As followers of the second approach, Walsh et al. use an evolutionary variant of game theory and investigate experimentally an agent's preference towards three bidding strategies. The major shortcoming of their work is that the time complexity of the suggested algorithms for strategy comparison depends exponentially on the number of the investigated strategies [12]. Muchnick and Solomon create the NatLab platform using the principle of Markov's nets. In order to make a smooth transition from computer simulation to experimental economics, they generate eight different bidding strategies using avatars. To make the emulation more realistic, the system accomplishes adaptive actualization of the avatars [4]. Posada and Lopez suggest a portfolio comprised of three alternative bidding strategies. For strategy selection they propose two heuristics – imitation and take-the-best. Imitation heuristic uses social learning, taking into account the past collective experience. The other heuristic, take-the-best, uses individual rational learning taking into account previous experiences of the agent [11]. Goyal et al. use the term "attitude" analogically to the typical to agent technology terms "intention" and "commitment". Each agent has a definite attitude towards the bidding process. This helps him adapt to the market dynamics more quickly. In order to choose a proper bidding, a set of individual bids is generated in advance. The agents' attitudes towards the set of criteria and bids take part in the multicriteria procedure for bidding selection [3]. The subject of indetermination and uncertainty in decision making is interpreted in literature sources from different points of view. Penev suggests and develops the idea of adaptivity through an heuristic approach in the search for an optimal solution. Through several heuristics, he speeds up the process of finding a real function optimum [5]. Angelova systematizes and generalizes various methods for decision making with fuzzy logic [1]. Peneva and Popchev suggest a series of algorithms for decision making with fuzzy criteria and aggregating operators [6-8]. Popchev and Radeva develop a specific procedure for multicriteria order of the criteria for evaluation of economic objects. In this way the most suitable indicators for multi criteria order according to the Decision Maker (DM) are selected [9, 10]. The current work investigates the applicability of two algorithms with fuzzy criteria with the common purpose of solving the task of multicriteria ordering of bidding strategies in an auction. A prototype of a software program for decision making with fuzzy logic is used as a tool for conducting the investigation.

3. Algorithms for multicriteria ranking with fuzzy relations

The two algorithms that are being compared can be formally described as follows.

3.1. Aggregation operators' algorithm ATOKRI

We will investigate three varieties of ATOKRI, formed on the basis of usage of weighting coefficients – weighting coefficients-real numbers, without weighting coefficients and weighting coefficients-real functions.

3.1.1. Exact criteria with weighting coefficients-real numbers (ATOKRI1)

Step 1. As the criteria evaluation can be expressed in different measurement units , for their unification procedures are used, which transform the values of each criterion in fuzzy preference relations [6-8]. For this purpose, the following transforming function is used:

(1)
$$\mu_k(a_i, a_j) = \begin{cases} 1 & \text{if } i = j, \\ 0.5 + \dfrac{x_{ik} - x_{jk}}{2(\max\limits_i \{x_{ik}\} - \min\limits_i \{x_{ik}\})} & \text{if i} \neq j, \end{cases}$$

in which x_{ik}, x_{jk}, i, j = 1, 2, …, n, k = 1, 2, …, m, are the evaluations of the alternatives a_i and a_j, i, j = 1, 2, …, n, according to the criterion c_k, k = 1, 2, …, m. The obtained fuzzy relations are R_k, k = 1, 2, …, m. If a certain criterion c_k, is minimizing, in order for the alternatives to be sorted in a descending order, the complement to the relation $R'_k = 1 - R_k$ is calculated, in other words, for this relation a new membership function is calculated using the formula $\mu_k'(a_i, a_j) = 1 - \mu_k(a_i, a_j)$.

Step 2. All relations $R_1, R_2, ..., R_m$ are combined so that an aggregated relation ***R*** with the following matrix can be obtained with the membership function:

(2) $$\mu(a_i, a_j) = \mathrm{Agg}\{\mu_1(a_i, a_j), \mu_2(a_i, a_j), ..., \mu_m(a_i, a_j)\}.$$

Each element from matrix ***R*** is calculated by the aggregation operators' formula with weighting coefficients. The following operators are used: WMean, WGeom, WMaxMin and WMinMax with weighting coefficients-real numbers as the values for $\mu_k(a_i, a_j)$ are taken from (1). If w is the set of criteria weights and

(3) $$w=\{w_1, w_2,..., w_m\}, \text{ then } w_k\in[0, 1] \text{ for } k = 1, 2,\dots, m \text{ and } \sum_{k=1}^{m} w_i = 1.$$

The calculations with the operators WMean, WGeom, WMaxMin and WMinMax for degree of membership to each of the aggregated relations R of the pair (a_i, a_j) are as follows:

(4) $$\mu(a_i, a_j) = \mathrm{WMean}\ \{\mu_1(a_i, a_j), \mu_2(a_i, a_j),..., \mu_m(a_i, a_j)\} = \sum_{k=1}^{m} w_k \mu_k(a_i, a_j);$$

(5) $$\mu(a_i, a_j) = \mathrm{WGeom}\{\mu_1(a_i, a_j), \mu_2(a_i, a_j), ..., \mu_m(a_i, a_j)\} = \prod_{k=1}^{m} [\mu_k(a_i, a_j)]^{w_k};$$

(6) $$\mu(a_i, a_j) = \mathrm{WMaxMin}\ \{\mu_1(a_i, a_j), \mu_2(a_i, a_j), ..., \mu_m(a_i, a_j)\} = \max_k\{\min(\mu_k(a_i, a_j), w_k)\};$$

(7) $$\mu(a_i, a_j) = \mathrm{WMinMax}\ \{\mu_1(a_i, a_j), \mu_2(a_i, a_j), ..., \mu_m(a_i, a_j)\} = \min_k\{\max(\mu_k(a_i, a_j), w_k)\}.$$

For the last two operators, WMaxMin (6) and WMinMax (7) the weights of the criteria are recalculated so that they belong to the interval [0, 1] and the largest of them to be equal to 1 by the formula:

(8) $$w_k' = \frac{w_k}{\max\limits_{k=1}^{m}\{w_k\}}.$$

Four aggregated relations are obtained, e.g. four matrices of type R. Each of these matrices is recalculated so that matrices R' are obtained in the following way:

(9) if $\mu(a_i, a_j) \geq \mu(a_j, a_i)$, then $\mu'(a_i, a_j) = \mu(a_i, a_j)$ and $\mu'(a_j, a_i) = 0$.

Step 3. The obtained preference relations $R_1, R_2, \dots, R_m$ are fuzzy complete orders. Taking into consideration the relations between the properties of R_k, $k = 1, 2, \dots, m$, and ***R***, the aggregated relation ***R***, obtained by the WMean operator is a fuzzy complete order, and these obtained by the next three operators are fuzzy rearrangements. Every asymmetric fuzzy rearrangement *R′* of ***R***, e.g., if condition (9) is fulfilled, is a fuzzy partial order. *R′* can be rearranged into a triangular matrix. After the triangular matrix *R′* is rearranged, a relation is obtained which represents a fuzzy linear arrangement. A non-fuzzy order of the alternatives is the same as their order in the title raw of the obtained table and is the solution to the problem of multicriteria arrangement.

3.1.2. Exact criteria without weighting coefficients (ATOKRI2)

In contrast with *ATOKRI1*, here in **Step 2** membership degrees of these relations, transformed by weight criteria, are calculated. In the experiment conducted the calculations are done with a *t*-norm $T_p(x, y) = xy$ and *t*-co-norm $S_p(x, y) = x + y - xy$.

Step 3. For combining of the obtained relations the aggregating operators MaxMin, MinAvg and Gamma are used:

$$\begin{aligned}\mu(a_i, a_j) &= \mathrm{MaxMin}\{\mu_1(a_i, a_j), \mu_2(a_i, a_j), ..., \mu_m(a_i, a_j)\} = \\ &= \alpha \max_k\{\mu_k^w(a_i, a_j)\} + (1-\alpha)\min_k\{\mu_k^w(a_i, a_j)\};\end{aligned} \tag{10}$$

$$\begin{aligned}\mu(a_i, a_j) &= \mathrm{MinAvg}\ \{\mu_1(a_i, a_j), \mu_2(a_i, a_j), ..., \mu_m(a_i, a_j)\} = \\ &= \sum_{k=1}^{m} \mu_k(a_i, a_j) + (1-\lambda)\min_k\{\mu_k(a_i, a_j)\};\end{aligned} \tag{11}$$

$$\begin{aligned}\mu(a_i, a_j) &= \mathrm{Gamma}\{\mu_1(a_i, a_j), \mu_2(a_i, a_j), ..., \mu_m(a_i, a_j)\} = \\ &= \left[\prod_{k=1}^{m} \mu_k^w(a_i, a_j)\right]^{1-\gamma}\left[1-\prod_{k=1}^{m}(1-\mu_k^w(a_i, a_j))\right]^{\gamma}.\end{aligned} \tag{12}$$

In **Step 4** the non-fuzzy orders are generated.

3.1.3. Exact criteria with weighting coefficients-real functions (ATOKRIF)

Unlike the previous two variants, the weighting coefficients in ATOKRIF are not real numbers, but they are described by real functions. To determine the membership degree of the pair (a_i, a_j) in **Step 2** the following formula is used:

$$\mu_k^w(a_i, a_j) = \begin{cases} 1 & \text{if } a_i = a_j \\ \dfrac{f_k(x_{ij}^k)x_{ij}^k}{S(a_i, a_j)} & \text{if } a_i \neq a_j \end{cases} \qquad i, j = 1, 2, ..., n, \quad k = 1, 2, ..., m, \tag{13}$$

and the weighting functions *f* are of three types:

– linear

$$f_k(x_{ij}^k) = 1 + \beta_k x_{ij}^k, \quad k = 1, 2, ..., m;$$

– parametric linear

$$f_k(x_{ij}^k) = \alpha_k \frac{1+\beta_k x_{ij}^k}{1+\beta_k},\ k=1, 2, ..., m;$$

and

– quadratic

(14) $$f_k(x_{ij}^k) = 1 + (\beta_k - \gamma_k)x_{ij}^k + \gamma_k (x_{ij}^k)^2, \quad k=1, 2, ..., m.$$

To calculate the aggregated relations that are corresponding to the three weighting coefficients, a generalized mixture operator is used in **Step 3**:

(15) $$\mu^w(a_i, a_j) = \mathrm{Agg}(\mu_1^w(a_i, a_j),\quad \mu_2^w(a_i, a_j), ..., \mu_m^w(a_i, a_j)) = \begin{cases} 1 & a_i = a_j \\ \sum_{k=1}^m \dfrac{f_k(x_{ij}^k)x_{ij}^k}{S(a_i, a_j)} & a_i \neq a_j \end{cases},\quad i,\ j=1, 2, ..., n.$$

3.2. Algorithm with fuzzy techniques and negotiable attitudes (FTNA)

Step 1. The DM determines the relative weighting coefficients of the criteria for each strategy using the method of analytical hierarchic process. The values of the weights depend on the degree of importance of the given criterion. The fuzzy relations (matrices for comparison) of the criteria are completed according to the degree of importance of the paired criteria. Evaluations vary in the range from 1 to 9: 1 – insignificantly important; 3 – more important; 5 – equally important; 7 – substantially more important; 9 – absolutely more important, and ranks 2, 4, 6, 8 represent values that are between the given ones. The weight of each criterion is given by the formula of geometric mean of the corresponding row of the comparison matrix. If we let w be the set of weights and $w=\{w_1, w_2, ..., w_m\}$, then here we will also have $w_i \in [0, 1]$ for $i=1, 2, ..., m$ and $\sum_{i=1}^{m} w_i = 1$.

Step 2. The agent's attitude towards the bidding strategies and the criteria for their evaluation are determined. Here "attitude" represents the preference of agent k to choose a strategy i with criterion j. The evaluations of the attitudes $a_{ij}{}^k$, $i = 1, 2, \ldots, n$, $j = 1, 2, ..., m$, $k = 1, 2, ..., l$, are presented through linguistic terms, such as "very low", "low", "average", "high", and "very high". The fuzzy agent relations towards strategies and criteria are completed in the attitude matrices $A_k = (a_{ij})^k_{n\times m}$.

Step 3. Each attitude matrix is aggregated into attitude vector A_i, $i = 1, 2,..., n$, as follows:

$$A_i^k = w_1^k a_{i1}^k + w_2^k a_{i2}^k + ... + w_n^k a_{in}^k.$$

Step 4. We assume that all the agents are equally important and calculate the normalized vector of the fuzzy solution r. The normalized weight of the agents D_k, $k = 1, 2, ..., l$, is denoted by (ν_1, ν_2 , ..., ν_l):

$$(r_1, r_2, \dots, r_n) = (v_1, v_2, \dots, v_l)\begin{pmatrix} a_1^1 & a_2^1 & \dots & a_n^1 \\ a_1^2 & a_2^2 & \dots & a_n^2 \\ \dots & \dots & \dots & \dots \\ a_1^l & a_2^l & \dots & a_n^l \end{pmatrix}.$$

Step 5. The elements of the normalized vector of the fuzzy solution r_i are positive triangular fuzzy numbers and belong to the interval [0, 1]. Then we calculate the distance between the fuzzy solutions r_i and the perfect ones, a positive and a negative solution. Let r^+ be the fuzzy positive perfect solution, r^- – the fuzzy negative perfect solution and r^+=(1, 1, 1) and r^-=(0, 0, 0). The distances d_i^+ between r_i and r^+ and d_i^- between r_i and r^- are calculated: $d_i^+ = d(r_i, r^+)$ and $d_i^- = d(r_i, r^-)$, where d is the distance between two fuzzy numbers. To calculate d the vertex method is used.

Step 6. To determine the rank of each strategy, the coefficient of closeness is calculated with the formula:

$$\mathrm{CC}_i = \frac{1}{2}\,(d_i^+ + (1 - d_i^-)) \quad \text{for } i = 1, 2, ..., n.$$

The strategy with the largest coefficient of closeness is the most appropriate one for bidding at that moment.

4. Numerical example and analysis of the results

Given:

– Ten strategies for agents' bidding in CDA:

A_1 – snipping strategy (Snipping),
A_2 – strategy with fixed markup (L),
A_3, A_4, A_5 – three strategies with different historical prices treatment H_3, H_4, H_5,
A_6 – zero intelligence unconstrained (ZIU),
A_7 – zero intelligence with budget constraints (ZIC),
A_8 – zero intelligence plus (ZIP),
A_9 – risk based strategy (RB) and
A_{10} – strategy with genetic algorithm (GA);

– Three criteria for strategies' evaluation:

C_1 – time complexity,
C_2 – price prediction and
C_3 – risk attitude.

For ATOKRI1 and ATOKRI2 algorithms:

– strategies' valuation for each criterion x_{ij}, i=1, 2, ..., 10, j=1, 2, 3, where x_{ij} are real numbers;

– weighting coefficients w_j, j=1, 2, 3 and w_j are real numbers;

– α, λ and χ – real coefficients for ATOKRI2 algorithm.

For ATOKRIF algorithm:

– weighting functions' coefficients α_j, β_j и χ_j, j = 1, 2, 3.

The input data for the ATOKRI algorithm are visualized in Fig. 1.

For FTNA algorithm:

– five bidding agents and their individual attitude matrices strategy-criterion;

– ten comparison matrices for each pair of criteria (one matrix for each strategy).

Fig. 2 shows a fragment of the input data for the FTNA algorithm.

0.42	0.39	0.19	- weighting coefficients from (3)
1.00	0.93	0.45	- transformed weighting coefficients from (6)
1	1	1	- all criteria are maximizing

Criterion / Strategy	C_1	C_2	C_3	
Snipping	6.3	0.5	0.8	- the evaluation matrix $\{x_{ij}\}$, i=1,2,..,10, j=1,2,3
L	2.1	0.5	0.8	
H_1	4.7	6.7	0.8	
H_2	5.8	7.0	1.4	
H_3	7.0	8.0	1.9	
ZIU	8.6	7.0	2.2	
ZIP	10.1	1.6	1.6	
ZIC	8.6	4.4	1.6	
RB	8.1	11.4	14.2	
GA	7.9	14.5	14.5	

functions' coefficients from (14):

0.2	0.7	0.1	$-\alpha_1, \beta_1, \chi_1$
0.6	0.3	0.1	$-\alpha_2, \beta_2, \chi_2$
0.3	0.5	0.2	$-\alpha_3, \beta_3, \chi_3$

Fig. 1. Input data for ATOKRI algorithm

Comparison matrix for strategy A_1

Snipping	C_1	C_2	C_3
C_1	5	8	9
C_2	2	5	5
C_3	1	5	5
Sum:	8	18	19

Fuzzy relation to the strategy A_1

0.63	0.44	0.47
0.25	0.28	0.26
0.13	0.28	0.26
1.00	1.00	1.00

Strategy's weighting coefficients:	Strategy's normalized weighting coefficients	
0.51	0.52	$w_{A_1C_1}$
0.26	0.27	$w_{A_1C_2}$
0.21	0.21	$w_{A_1C_3}$
0.98	1.00	

Comparison matrix for strategy A_2

L	C_1	C_2	C_3
C_1	5	9	9
C_2	1	5	5
C_3	1	5	5
Sum:	7	19	19

Fuzzy relation to the strategy A_2

0.71	0.47	0.47
0.14	0.26	0.26
0.14	0.26	0.26
1.00	1.00	1.00

0.54	0.56	$w_{A_2C_1}$
0.21	0.22	$w_{A_2C_2}$
0.21	0.22	$w_{A_2C_3}$
0.97	1.00	

Attitude matrix of $Agent^1$ to select strategy i under criteria j

$Agent^1$	C_1	C_2	C_3
Snipping	VH	VL	VL
L	VL	VL	VL
H_1	ML	M	VL
H_2	M	M	VL
H_3	MH	M	VL
ZIU	H	M	L
ZIP	VH	VL	VL
ZIC	H	H	VL
RB	L	VH	VH
GA	VL	VH	VH

Fig. 2. Fragment of the input data for FTNA algorithm

Find: the strategy order according to the criteria with the ATOKRI and FTNA algorithms.

Results:

In Table 1 the multi criteria orders obtained with ATOKRI (ten variants depending on the used aggregating operator) and FTNA algorithms are shown. The results obtained with one of the "classical" methods are also printed – the method of Linear Combination of Private Criteria (LCPC). The grey cells show the rank of the strategy in the same column (column of matrix R').

Table 1. Results from the multi criteria analysis with the ATOKRI, FTNA and LCPC algorithms

Exact criteria with weighting coefficients-real numbers (ATOKRI1)										
WMean (4)										
Order	A_{10}	A_9	A_6	A_5	A_7	A_8	A_4	A_3	A_1	A_2
Value		0.540	0.631	0.530	0.510	0.500	0.539	0.537	0.544	0.610
Rank	0	1	2	3	4	5	6	7	8	9
WGeom (5)										
Order	A_{10}	A_9	A_6	A_5	A_7	A_8	A_4	A_3	A_1	A_2
Value		0.537	0.610	0.527	0.473	0.493	0.525	0.537	0.525	0.597
Rank	0	1	2	3	3	3	5	7	8	9
WMaxMin (6)										
Order	A_6	A_9	A_{10}	A_5	A_4	A_7	A_8	A_1	A_3	A_2
Value		0.531	0.513	0.732	0.575	0.693	0.594	0.644	0.600	0.721
Rank	4	4	4	5	7	7	7	8	8	9
WMinMax (7)										
Order	A_{10}	A_9	A_6	A_5	A_4	A_7	A_8	A_1	A_2	A_3
Value		0.511	0.929	0.511	0.518	0.493	0.500	0.529	0.500	0.500
Rank	0	1	2	3	4	4	4	7	7	7

Table 1 (c o n t i n u e d)

Exact criteria without weighting coefficients (ATOKRI2)										
MaxMin (α = 0.5) (10)										
Order	A_{10}	A_9	A_6	A_7	A_8	A_5	A_4	A_3	A_1	A_2
Value		0.527	0.539	0.525	0.518	0.508	0.572	0.557	0.535	0.649
Rank	0	1	2	3	4	5	6	7	8	9
MinAgv (λ = 0.5) (11)										
Order	A_{10}	A_9	A_6	A_7	A_5	A_8	A_4	A_3	A_1	A_2
Value		0.482	0.494	0.465	0.440	0.459	0.474	0.494	0.465	0.504
Rank	0	1	2	2	3	3	5	6	7	9
Gamma (χ = 0.5) (12)										
Order	A_{10}	A_9	A_6	A_5	A_7	A_8	A_4	A_3	A_1	A_2
Value		0.448	0.523	0.440	0.384	0.404	0.446	0.448	0.441	0.534
Rank	0	1	2	3	3	5	6	7	8	9
Exact criteria with weighting coefficients-real functions (ATOKRIF) (13), (14), (15)										
Linear weighting functions										
Order	A_{10}	A_9	A_6	A_5	A_7	A_8	A_4	A_3	A_1	A_2
Value		0.536	0.676	0.530	0.518	0.504	0.542	0.536	0.541	0.605
Rank	0	1	2	4	5	5	6	7	8	9
Parametric linear functions										
Order	A_{10}	A_9	A_5	A_6	A_4	A_3	A_8	A_7	A_1	A_2
Value		0.578	0.648	0.507	0.538	0.523	0.516	0.555	0.578	0.553
Rank	0	1	2	3	4	5	6	7	8	9
Quadratic weighting functions										
Order	A_{10}	A_9	A_6	A_5	A_7	A_8	A_4	A_3	A_1	A_2
Value		0.535	0.680	0.531	0.515	0.506	0.544	0.536	0.540	0.607
Rank	0	1	2	4	4	5	6	7	8	9
FTNA algorithm										
Order	A_{10}	A_9	A_3	A_6	A_5	A_8	A_4	A_7	A_1	A_2
Value	0.450	0.406	0.319	0.266	0.232	0.204	0.198	0.198	0.159	0.055
Linear combination of private criteria (LCPC)										
Order	A_{10}	A_9	A_6	A_5	A_8	A_7	A_4	A_3	A_1	A_2
Value		0.909	0.830	0.575	0.531	0.497	0.484	0.448	0.386	0.286

Ten of the most cited bidding strategies in literature sources about CDA simulation take part in the conducted experiment. They were developed to be stable as well as adaptive toward the changes that might occur in bidding. For evaluation and comparison of the ten strategies chosen in the experiment, three generally accepted criteria are used.

Matrices for comparison of paired criteria and matrices with agent's attitudes towards the pair strategy-criterion for FTNA algorithm are filled in on the basis of expert evaluations. The total strategy weighting coefficients are generated finding the mean of the weighting coefficients for each strategy in Step 1 of the FTNA. The alternative-criterion evaluations in the comparison matrices in Step 2 of the FTNA are found by filling in the alternative-criterion matrix which the ATOKRI algorithms need in order to work. All the above mentioned steps guarantee that the input data will be identical for the two algorithms.

Analysis of the solutions found shows that both algorithms suggest similar in essence orders of the compared strategies. In the twelve solutions there are exact matches in the top and in the bottom of the ranking: strategies RB and GA are the best, and strategies Snipping and L are the worst. Four of the ATOKRI-solutions (WMean, WGeom, Gamma and Quadratic weighting function) are identical and almost coincide with the LCPC result. There is only one discrepancy and it is in the middle of the order. Furthermore, these four solutions and the FTNA demonstrate coincidence of the places of six out of the ten strategies – GA, RB, ZIP, H_4, Snipping and L. The discrepancies in the results of the other aggregate operators of the ATOKRI are due, in this particular case, to the large number of collisions (some of the strategies are the same rank). Moreover, WMean, WGeom, Gamma and Quadratic weighting function give a result closer to the LCPC compared with that of the FTNA (eight vs. five coincidences). WMaxMin and WMinMax give worse results than the ATOKRI and FTNA ones, which supports the theoretical assumption of its authors that the orders obtained would be incomplete.

The major advantage of the two algorithms over the "classical" methods is their ability to find a solution when there is only fuzzy and inaccurate information about the compared objects.

The basic difference between the compared algorithms is in the subjectivism degree and in the influence of agent's personal attitude towards the compared alternatives and criteria. In this way, the output information about solving the problem in FTNA, is the attitude of the bidders towards the compared alternatives and criteria on the basis of which they have been compared. Since this information is usually private, and frequently subjective, the solutions found with this method vary in a wider range than those found with ATOKRI.

5. Conclusions and future work

The conducted experiment with ten strategies evaluated on the basis of three criteria shows that ATOKRI and FTNA algorithms can be used for solving the task of multicriteria ordering of bidding strategies in an auction when there is no accurate and objective evaluation of the compared alternatives, because the obtained experimental results are similar to those of the LCPC method. Recommendations about future work on this subject are related to the development of methods for generating of matrices for evaluation of paired strategy criteria and automatic filling of the attitude matrices. A procedure will be soon created for preliminary selection of agents-experts, for example through cluster analysis. The collisions in ATOKRI-orders are also to be solved.

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