

SPEA-Based Method for MCDM Convex Integer Problems

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Abstract: *This paper presents a method, based on the Strength Pareto Evolutionary Algorithm (SPEA), designed to solve multi-objective convex integer optimization problems. The proposed method has the aim to overcome some shortcomings of SPEA, as noted in [2]. An interaction phase with the Decision Maker (DM) is also included in the method, so that the search process can be quickly directed to the part of the search space, where the location of a desired non-dominated solution is expected. In this manner good convergence of the method is ensured.*

Keywords: *Evolutionary multi-objective optimization, Multi-Criteria Decision Making (MCDM), convex integer problems.*

I. Introduction

The multi-objective convex integer optimization problem can be stated in the following general form:

- (1) minimize $f(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})]^T$
- (2) subject to: $\mathbf{g}_j(\mathbf{x}) \leq 0, \quad j = 1, 2, \dots, m,$
- (3) $\mathbf{x}_i^{(L)} \leq \mathbf{x}_i \leq \mathbf{x}_i^{(U)}, \quad i = 1, 2, \dots, n,$
- (4) $\mathbf{x} \in \mathbf{Z}^n,$

where $\mathbf{g}_j(\mathbf{x}), j = 1, 2, \dots, m,$ are convex functions.

A solution $\mathbf{x} \in \mathbf{Z}^n$ is a vector of n decision variables: $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$. The value $x_i^{(L)}$ is the known lower bound and the value $x_i^{(U)}$ is correspondingly the upper bound of the variable x_i . The solutions satisfying the constraints (2)-(4) constitute a feasible decision space $\mathbf{V} \subset \mathbf{Z}^n$. The objective functions values (1) constitute a k -dimensional space, called objective space $\mathbf{S} \subset \mathbf{R}^k$. Here we consider the term “solution” as a vector of variables in the decision space and the term “point” as the vector of criteria values in the objectives space. We use the notations “ \mathbf{x} ” for the solutions and “ s ” for the point $s = \mathbf{f}(\mathbf{x}) = (s_1, s_2, \dots, s_k)^T$.

The domination between two solutions is defined as follows (see [1, 2]):

Definition 1. A solution $x^{(1)}$ is said to dominate the solution $x^{(2)}$, if both the following conditions are true:

1. The solution $x^{(1)}$ is no worse than $x^{(2)}$ in all objectives. Thus, the solutions are compared based on their objective function values (i.e. based on the location of the corresponding points $s^{(1)}$ and $s^{(2)}$ on the objectives space).
2. The solution $x^{(1)}$ is strictly better than $x^{(2)}$ in at least one objective.

All points which are not dominated by any other point $s \in \mathbf{S}$ are called the non-dominated points of class one, or simply the non-dominated points. The non-dominated points together make up a front in the objectives space. The points lying on the non-domination front are called Pareto-optimal points (together they constitute the Pareto-optimal front).

The local Pareto-optimal solutions are defined in multi-objective optimization (see [2, 3]) as follows.

Definition 2. If for every member \mathbf{x} in a set Q there exists no solution \mathbf{y} (in the neighborhood of \mathbf{x} such that $\|\mathbf{y} - \mathbf{x}\|_\infty \leq \varepsilon$, where ε is a small positive scalar) dominating any member of the set Q , then solutions belonging to the set Q constitute a local Pareto-optimal set.

There are two basic multi-objective optimization problems:

1. Find a set of Pareto-optimal solutions, which satisfy in the best way the DM’s preferences.
2. Find a set of solutions which are diverse enough to represent the entire range of the Pareto-optimal front.

The evolutionary multi-objective optimization algorithms handle the multi-objective optimization problems in the following manner (see [1]):

Step 1. Find multiple non-dominated points as close to the Pareto-optimal front as possible, with a wide trade-off among objectives.

Step 2. Choose one of the obtained points using higher-level information.

II. Preliminary considerations

The Strength Pareto Evolutionary Algorithm (SPEA) was introduced by Zitzler and Thiele (1998, 1999). This approach was conceived as a way of integrating different evolutionary multi-objective optimization algorithms. SPEA uses an

archive containing non-dominated solutions previously found (the so-called external non-dominated set or external population). At each generation, non-dominated individuals are copied to the external population. For each individual in this external population, a strength value is computed. This strength is similar to the ranking value of MOGA [3], since it is proportional to the number of solutions to which a certain individual dominates. In SPEA, the fitness of each member of the current population is computed according to the strengths of all external non-dominated solutions that dominate it. The fitness assignment process of SPEA considers both closeness to the true Pareto front and uniform distribution of solutions at the same time. Thus, instead of using niches based on distance, Pareto dominance is used to ensure that the solutions are properly distributed along the Pareto front. Although this approach does not require a niche radius, its effectiveness relies on the size of the external non-dominated set. In fact, since the external non-dominated set participates in the selection process of SPEA, if its size grows too large, it might reduce the selection pressure, thus slowing down the search. Because of this, those authors decided to adopt a clustering technique that reduces the number of individuals in the external population (external non-dominated set) so that its size remains below a certain threshold.

There is also a second algorithm by Zitzler and Thiele, which is known as Strength Pareto Evolutionary Algorithm 2 (SPEA2) [4, 6]. It has three main differences with respect to its predecessor SPEA:

- 1) it incorporates a fine-grained fitness assignment strategy which takes into account for each individual the number of individuals that dominate it and the number of individuals by which it is dominated;
- 2) it uses a nearest neighbor density estimation technique which guides the search more efficiently;
- 3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions.

A simulation study was performed on six evolutionary multi-objective optimization algorithms: SPEA, NSGA, VEGA, HPGA, NPGA, FFGA in [7]. The results indicated that the elitism is an important factor in evolutionary multi-objective optimization. On one hand, SPEA clearly outperformed all other considered algorithms, and this was the only method among all considered, that incorporates elitism as a central part of the algorithm. On the other hand, the performance of the other algorithms could be improved significantly when SPEA's elitist strategy is included.

There are some shortcomings of SPEA (see [2]):

- 1) The non-dominated sorting of the whole population is not used for assigning fitness and for this reason the fitness values do not favor all non-dominated solutions in the same way equally. This feature is dependent on the exact population and densities of solutions in the search space.
- 2) In SPEA fitness assignment, an external solution which dominates more solutions gets a worse fitness. This assignment is justified when all dominated solutions are concentrated near the dominating solution. Since in most cases this is not true, the crowding effect should come only from the clustering procedure.

Otherwise, this fitness assignment may provide a wrong selection pressure for the non-dominated solutions.

3) The number of objectives as a convergence factor is considered in [5]. The results showed that the performance of SPEA2 deteriorates substantially as the number of objectives increases.

To overcome these shortcomings we propose in our SPEA-based multi-objective optimization algorithm the following:

- We include a two-phase procedure for accelerated approaching the whole population to the Pareto front. In this manner it is ensured that the dominated solutions are close to the dominating solutions from the external population and the SPEA fitness assignment becomes justified.

- We include an interaction step, where a Decision Maker (DM) sets a reference point f^r in the objectives space. The DM has the possibility to change his preferences periodically and to replace the former reference point by a new one. This step ensures the convergence of the method to the satisfactory non-dominated solution.

- We use the current reference point to rearrange the clusters in ascending order according to their Euclidean distance to f^r in the objectives space. Then we define a direction vector between the solution from nearest cluster to f^r and the solution from second in its distance to f^r cluster, and make a step along this direction for all solutions in the current population, taking into account the constraints (2)-(4) of the problem, so that the new generated solutions remain feasible. In this way we move the whole population in direction to the reference point and accelerate the search process. In case the point from the new obtained nearest to f^r cluster after this movement has shorter distance to the reference point, than the previous nearest to f^r cluster, the search process continues without interaction with the DM. Otherwise, the closest point to DM's preferences is already found and it is presented to the DM for evaluation. Another possibility is to present the series of computed non-dominated solutions according to the improving direction to the f^r . If the DM is satisfied with one of the computed points he/she stops the search procedure. Otherwise, the DM sets new preferences and the search process continues.

The new proposed steps and phases overcome the shortcomings of SPEA. The obtained new evolutionary method uses a close enough to the Pareto front population and has quickly convergence to the desired non-dominated solution.

III. The SPEA-based method

First we describe a procedure for accelerated approaching.

Let us denote the internal population by $P = \{x^i, i = 1, \dots, N\}$, and the external population by $\bar{P} = \{\bar{x}^i, i=1, \dots, \bar{N}\}$.

Two-phase procedure for accelerated approaching of internal population P to the Pareto front.

Phase 1

1. For each $\bar{x}^j \in \bar{P}$:

1a) Find the most distant solution $x^{i(j)} \in P$;

1b) Set $l = 0$, $x^{i(j)}_l = \bar{x}^j$, σ – precision (sufficiently small positive scalar);

1c) $l = l + 1$.

1d) Calculate the solution $x^{i(j)}_l = x^{i(j)}_{l-1} + \alpha(\bar{x}^j - x^{i(j)}_l)$, where α is a positive scalar. In case $x^{i(j)}_l$ violates some constraint from the system (2)-(4), go to 1e), otherwise go to 1c).

1e) Reduce α twice, check if $\alpha < \sigma$. If “Yes” – go to 1f), otherwise set $x^{i(j)}_l = x^{i(j)}_{l-1}$ and go to 1c).

1f) Round off the obtained solution $x^{i(j)}_l$ to the nearest integer solution. If it is infeasible, find a feasible solution in its neighborhood and set $x^{i(j)}_l$ equal to it.

2. Copy all the obtained $x^{i(j)}$ solutions to \bar{P} .

Phase 2

1. Calculate the mean vector p of all vectors $x^{i(j)} - x^i$ from Phase 1 as

$$p = \frac{1}{\bar{N}} \sum_i (x^{i(j)} - x^i).$$

2. Replace each solution $x^i \in P$ in the internal population by the nearest integer solution to $x^i = x^i + \mu p$, where μ is a positive scalar parameter, depending on the problem properties. By means of this parameter the population is moved close to the Pareto frontier, remaining still feasible.

3. Round off the obtained x^i to the nearest integer solution. If it is infeasible, find a feasible solution in its neighborhood and set x^i equal to it.

SPEA-based method

Step 1. Set $itlim$ – the iterations limit. Set $icount = 1$ – number of the current iteration.

Step 2. Create an initial population P of size N by random generation of N integer n -dimensional vectors with uniform distribution around the Tchebycheff center X_t of the feasible domain.

Create an empty external population \bar{P}_0 of size \bar{N} . (It is recommended that $\bar{N} = 0.2N$, and $N = \min\{kn, 2k\}$, but in the interval [10; 100]. Here k is the number of objectives.

Step 3. Compute the points $s^i = \{f_1(x^i), f_2(x^i), \dots, f_k(x^i)\}$ for each solution $x^i \in P_0$. Find the current approximate non-dominated points and save the corresponding x^i -vectors in \bar{P}_0 .

Step 4. Perform the **Two-phase procedure** for accelerated approaching P to the Pareto front.

Step 5. DM sets a reference point f^r in the objectives space. Set initial vector d_{\min} , which components are big positive numbers and $h = 0$ – iteration counter.

Step 6. Perform SPEA evolution generation [2, p. 251] including a clustering technique to reduce the number of individuals in the external population \bar{P} .

Note. In the clustering technique we work with the nearest point according to the DM's references instead of using clusters' centroids as it is in SPEA.

Step 7. Arrange the clusters $C_i^h, i = 1, \dots, s(h)$, in an ascending order according to their distances to the reference point f^r . Let the cluster with minimal distance d_h to f^r be denoted by C_1^h , and the cluster on the second place in this ascending order – by C_2^h .

Let the corresponding solutions be $x^{1(h)} \in C_1^h$ and solution $x^{2(h)} \in C_2^h$.

If $d_h < d_{\min}$, then set $d_{\min} = d_h$, set $h = h+1$ and go to Step 8,
otherwise set $d = d_{h-1}$, $z = C_1^{h-1}$, $x = x^{1(h-1)}$ and go to Step 9.

Step 8. Moving Calculate the improving vector $q = x^{1(h-1)} - x^{2(h-1)}$ and $x^i = x^i + q$.

Replace each solution $x^i \in P$ in the internal population by the nearest integer solution. Go to Step 6.

Step 9. The DM evaluates f^r, z (or all solutions $\{z\}$ from internal iterations $\{h\}$) and if he/she is satisfied by current solution z (or several of them) then go to Step 10, otherwise set $\text{icount} = \text{icount} + 1$ and if $\text{icount} > \text{itlim}$, then ask the DM to choose to go to Step 10 or to Step 5.

Step 10. End.

IV. Illustrative example

We consider the following illustrative example:

$$\begin{aligned} \min \quad & f_1 = 1/(x_1+1), \\ \min \quad & f_2 = 1/(x_2+1), \end{aligned}$$

subject to:

$$\begin{aligned} x_1^2 + 100 \cdot x_2^2 &\leq 10^6, \\ 0 &\leq x_1 \leq 1000, \\ 0 &\leq x_2 \leq 100, \\ x_1, x_2 &\in \mathbb{Z}. \end{aligned}$$

At Step 2 we create the following initial population P_0 :

$$\begin{aligned} x^1 &= (400; 40), \quad x^2 = (500; 40), \quad x^3 = (600; 40), \\ x^4 &= (400; 50), \quad x^5 = (470; 50), \quad x^6 = (540; 50), \\ x^7 &= (600; 50), \quad x^8 = (400; 60), \quad x^9 = (500; 60), \\ x^{10} &= (600; 60); \end{aligned}$$

At Step 3 we obtain:

$$\begin{aligned}
s^1 &= (0.002494; 0.02439), & s^2 &= (0.001996; 0.02439), \\
s^3 &= (0.001664; 0.02439), & s^4 &= (0.002494; 0.01961), \\
s^5 &= (0.002123; 0.01961), & s^6 &= (0.001848; 0.01961), \\
s^7 &= (0.001664; 0.01961), & s^8 &= (0.002494; 0.01639), \\
s^9 &= (0.001996; 0.01639), & s^{10} &= (0.001664; 0.01639).
\end{aligned}$$

There is only one current approximate non-dominated point s^{10} . The corresponding $x^{10} = (600; 60)$ is deleted from P_0 and is included in \bar{P}_0 .

At Step 4 we perform the **Two-phase procedure**:

Phase 1

1a) The most distant solution $x^{i(j)} \in P$ for x^{10} is $x^{1(10)} = (400; 40)$.
1f) The obtained $x^{1(10)}$, rounded off to the nearest integer solution is $x^{1(10)} = (704; 71)$. It dominates the approximate non-dominated solution in \bar{P}_0 , and for this reason we replace x^{10} by $x^{1(10)}$ in \bar{P}_0 .

Phase 2

1. The obtained mean vector $p = (304, 31)$.
2. For this example we choose $\mu = 0.5$. Replace each solution $x^i \in P$ in the internal population by the nearest integer solution to $x^i = x^i + 0.5p$.
3. Round off the obtained x^i to the nearest integer solution. If it is infeasible, find a feasible solution in its neighborhood and set x^i equal to it. The obtained new internal population is P_0 :

$$\begin{aligned}
x^1 &= (552; 55), & x^2 &= (652; 55), & x^3 &= (752; 55), \\
x^4 &= (552; 65), & x^5 &= (622; 65), & x^6 &= (692; 65), \\
x^7 &= (752; 65), & x^8 &= (552; 75), & x^9 &= (652; 75).
\end{aligned}$$

The corresponding points in the objectives space are:

$$\begin{aligned}
s^1 &= (0.001808; 0.01786), & s^2 &= (0.001531; 0.01786), \\
s^3 &= (0.001328; 0.01786), & s^4 &= (0.001808; 0.01515), \\
s^5 &= (0.001605; 0.01515), & s^6 &= (0.001443; 0.01515), \\
s^7 &= (0.001328; 0.01515), & s^8 &= (0.001808; 0.01316), \\
s^9 &= (0.001531; 0.01316).
\end{aligned}$$

At Step 5 DM sets a reference point $f^r = (0.0049751; 0.0066225)$. It corresponds to the solution $x^r = (200, 150)$.

At Step 6 there are two new non-dominated solutions: x^7 and x^9 . After the clustering procedure the solutions (650, 75) and (704, 71) remain in \bar{P} and delete the solution x^7 . Two new solutions are generated: (661, 75) and (552, 71).

At Step 7 the cluster located closest to the reference point is that one containing the solution (650, 75).

At Step 8 we obtain the improving vector $q = (-54, 4)$. The obtained new population P is the following:

$$\begin{aligned}
 x^1 &= (498; 59), & x^2 &= (598; 59), & x^3 &= (698; 59), \\
 x^4 &= (498; 69), & x^5 &= (568; 69), & x^6 &= (638; 69), \\
 x^7 &= (698; 69), & x^8 &= (498; 79), & x^9 &= (607; 79), \\
 x^{10} &= (498; 75).
 \end{aligned}$$

And the process continues further to Step 6.

The feasible domain is presented on Fig. 1. The objectives space is presented on Fig. 2.

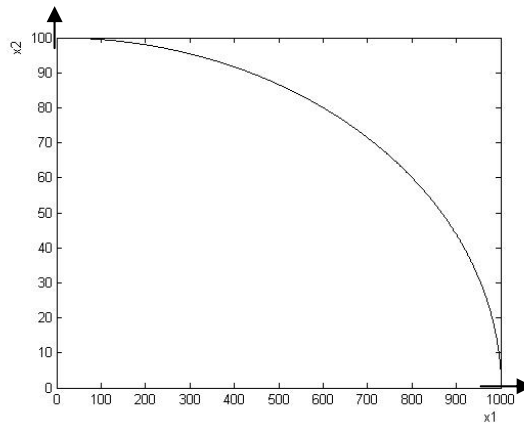


Fig. 1. The feasible domain in the variables space

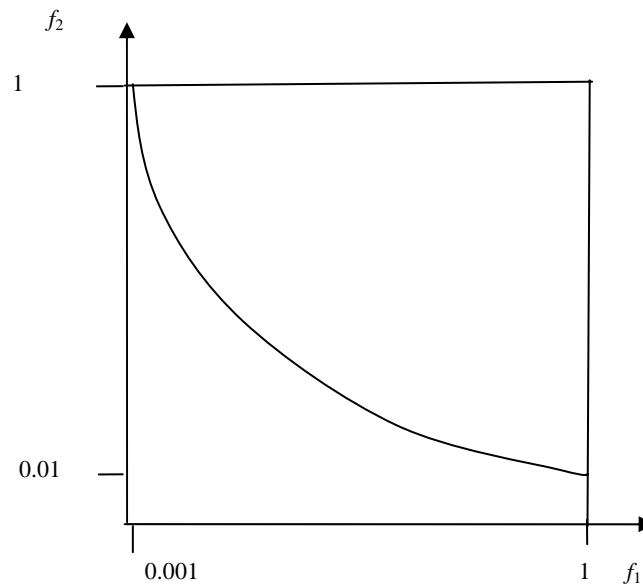


Fig. 2. The objectives space, $f_1(\min) = 0.000999$, $f_2(\min) = 0.0099$

V. Conclusion

The basic characteristics of the presented here evolutionary method for solving MCDM integer problems can be summarized as follows:

- it is elitist MCDM evolutionary method;
- it is an interactive method in contrast to SPEA method;
- the step of generation of new population is the same as in SPEA method;
- the step of clusterization is enriched/complemented by an arrangement according to the DM's preferences;
- accelerated approaching to the Pareto frontier.

In contrast to SPEA method which finds an approximation of efficient frontier we include an interaction step to solve the problem for best satisfactory solution.

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