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Multicriteria Decision Making – Achievements and Directions for Future Research at IIT-BAS

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Abstract: *Multicriteria decision making is one of the rapidly advancing directions in "Operations Research" scientific discipline during the last years. The paper presents the most significant research developments in this direction, accomplished at the Institute of Information Technologies at Bulgarian Academy of Sciences (IIT-BAS). They include the creation of efficient scalarizing problems and interactive algorithms for solving problems of multicriteria optimization and multicriteria analysis.*

Keywords*: Multicriteria optimization, multicriteria analysis, scalarizing problems, interactive algorithms.*

1. Introduction

Multicriteria decision making is evaluated as the fifth in order (among fifteen in number) priority research directions of the scientific discipline "Operations Research" [14]. It is one of the most dynamically developing directions in the last decades.

The problems for multicriteria decision add can be divided in two separate classes depending on their formal statement. In the first class of problems a finite set of explicitly set constraints in the form of functions define an infinite number of feasible alternatives. These problems are called continuous problems for multicriteria decision add or problems of Multicriteria Optimization (MO). In the second class of problems a finite number of alternatives are explicitly given in a tabular form. These problems are called discrete problems for multicriteria decision add or problems of Multicriteria Analysis (MA). A lot of real life problems in management may be formulated as problems of multicriteria optimization or multicriteria analysis.

Several criteria (objective functions) are simultaneously optimized in MO and MA problems in the feasible set of solutions (alternatives). In the general case there does not exist one alternative, optimizing all the criteria. But there exists a set of alternatives, differing by the following characteristic: each improvement in the value of one criterion leads to deterioration in the value of at least one other criterion. This set of alternatives is called a set of non-dominated alternatives or Pareto Optimal (PO) set. Each alternative from this set can be a solution of the multicriteria problem. However, in practice it is necessary to choose one alternative as a final solution of the problem. In order to choose the most preferred solution, additional information, set by the so called Decision Maker (DM) is necessary. The information, which the DM sets, reflects his/her global preferences related to the quality of the alternative sought.

The work in the Institute of Information Technologies at Bulgarian Academy of Sciences (IIT-BAS) in multicriteria decision making area is natural continuation of the many years' activity in the area of integer, combinatorial and network single-criterion optimization of the team members. Prof. Dr Vassil Vassilev was the founding force and the leader of the research and applied developments in the area of MO and MA. During the last two decades new efficient scalarizing problems, MO algorithms and interactive "optimizationally motivated" algorithms for MA have been developed, which lie in the basis of the software systems created. The results achieved are presented in prestigious scientific journals and reported at international conferences.

The next chapters of the paper discuss the developments with the most considerable scientific and research contribution to the area of multicriteria decision making, carried out at IIT-BAS.

2. Research investigations in the area of multicriteria optimization

The research investigations at IIT-BAS in MO area are mainly connected with the application of the scalarizing approach to solving linear, linear integer, convex nonlinear, nonlinear integer and flow multicriteria problems.

The general statement of MO problem is as follows:

$$\text{“max”}\{f_k(x),\ k\in K\}$$

under the constraints

$$g_i(x)\le b_i,\quad i\in M,$$
$$0\le x_j\le d_j,\quad j\in N,$$

$x=(x_1,...,x_n)$ is an n-dimensional vector, the values of which are the searched solution of the problem; $f(x)=(f_1(x),...,f_k(x))^{\mathrm{T}}$ is a vector of the objective functions or the criteria vector; $f(X)=Z$ denotes the image of the set of the feasible solutions X in the criteria space. The type of the functions of the criteria $f_k(x)$, $k \in K$, and of the constraints $g_i(x)$, $i \in M$, with respect to the variables x_j, $j \in N$, as well as the type of these variables, define the type of MO problems. The term scalarization means transformation of the multicriteria optimization problem in one or several single-criterion optimization problems. The basis for such transformation consists in the fact that each PO solution of the multicriteria optimization problem can be obtained as an optimal solution of the scalarizing problem. The scalarizing problems differ depending on the information they require from the DM about his/her preferences in connection with the compromise solution sought. The scalarizing problems are placed in the basis of the most widely used methods for solving MO problems – the interactive methods, where the generation of PO solutions is obtained, using single criterion optimization.

The first developments related to scalarizing problems and interactive algorithms for MO are connected with the experience gained and the results obtained during the development of efficient approximate algorithms, designed for NP problems of single-criterion optimization. When interactive algorithms are developed for solving multicriteria nonlinear and integer (linear and nonlinear) optimization problems, it is obligatory to take in mind the time, necessary for the solution of the scalarizing problems. If this time is too long, the DM may give up the study of different solutions. The use of efficient approximate algorithms in solving scalarizing problems can increase considerably the qualities of MO interactive algorithms. Another significant problem in the development of such algorithms is to what extent the information, required from the DM, reflects his/her preferences and could direct the search towards the most preferred solution.

2.1. Scalarizing problems of the reference directions

The first results in the area of the scalarizing approach, applied for solution of MO problems, are connected with the formulated problems of the reference directions (RD). RD scalarizing problems [9, 10, 16, 28], are the basis of some interactive methods, designed with the purpose to solve linear and nonlinear integer multicriteria problems. On the basis of the criteria values in the current solution f_k, the DM sets the desired (acceptable) levels of the criteria in the reference point $\bar{z}$. Depending on these values, the criteria are divided into three groups – $K^{\geq}$, $K^{\leq}$ and $K^{=}$, at that $k \in K^{\geq}$, if improvement of these criteria $\bar{z}_k \geq f_k$ is searched for; $k \in K^{\leq}$ if deterioration is acceptable $\bar{z}_k \leq f_k$ and $k \in K^{=}$; if the goal is to preserve the value reached $\bar{z}_k = f_k$. Different types of scalarizing problems of this type can be formulated. One of the formulations of the scalarizing problem of the Reference Directions (RD) has the form (RD1):

To maximize

(1) $$S(x) = \min_{k \in K^{\geq}} \left(\frac{f_k(x) - f_k}{\bar{z}_k - f_k} \right)$$

under the constraints

(2) $$f_k(x) \geq \bar{z}_k + \alpha(\bar{z}_k - f_k), \quad k \in K^{\leq},$$

(3) $$f_k = \bar{z}_k, \quad k \in K^{=},$$

(4) $$x \in X,$$

where α is a parameter, $\alpha > -1$.

The optimal solution of this scalarizing problem is a weak PO solution of MO problem. In the general case it is an optimization problem with a non-differentiable objective function. Every concrete scalarizing problem RD1 can be reduced to an equivalent optimization problem with a differentiable objective function at the expense of additional variables and constraints. The equivalence of each pair of optimization problems is with respect to the obtained values of the criteria (objective functions) and of the main variables.

The objective functions of RD scalarizing problems maximize the minimal difference between the values of the criteria with indices $k \in K^{\geq}$ in the solution sought and in the current solution f_k. In this way a solution is sought, which is located as further as possible from the current solution found. The values of the criteria with indices $k \in K^{\geq}$ increase, whereas the values of the criteria with indices $k \in K^{\leq}$ can diminish, while searching for a new solution along the reference direction, defined by the reference point $\bar{z}_k$ and the current solution found f_k. The current solution of the multicriteria problem is a feasible solution of RD problems. This simplifies the exact, as well as the approximate methods for solving integer variants of the problem, because they start with a known initial feasible solution. Depending on parameter α, when searching for a new solution along the reference direction, two strategies can be realized: The first strategy is of the type "great profits-great losses", while the second one is of the type "not great profits-small losses".

2.2. Scalarizing problems of the modified reference point

The scalarizing problems of the Modified Reference Point (MRP) are used in some methods [18, 19, 30], designed to solve convex nonlinear integer multicriteria problems. On the basis of the values of the criteria in the current solution, the DM divides the criteria into three groups – $K^{\geq}$, $K^{\leq}$ and $K^{=}$, and determines the desired or acceptable levels of the criteria at the reference point $\bar{z}$, at that $\bar{z}_k = f_k$ for $k \in K^{=}$. Different types of scalarizing problems of this type can be formulated. A such problem, aiding the discovery of PO solutions, has the form given below.

To minimize

(5) $$S(x) = \max\left(\max_{k \in K^{\geq}} \left(\frac{\bar{z}_k - f_k(x)}{\bar{z}_k - f_k} \right), \max_{k \in K^{\leq}} \left(\frac{f_k - f_k(x)}{f_k - \bar{z}_k} \right) \right) + \\ + \rho \left(\sum_{k \in K^{\geq}} \left(\frac{\bar{z}_k - f_k(x)}{\bar{z}_k - f_k} \right) + \sum_{k \in K^{\leq}} \left(\frac{f_k - f_k(x)}{f_k - \bar{z}_k} \right) - \sum_{k \in K^{=}} f_k(x) \right)$$

under the constraints

(6) $$f_k(x) \geq f_k,\ k \in K,^{=}$$

(7) $$x \in X,$$

where ρ is a small positive number.

MRP scalarizing problems have three features, enabling to a high extent the overcoming of the computational difficulties, connected with their solution as integer programming problems or as nonlinear programming problems. On the other hand, they allow the decrease of DM's loading when comparing the new solutions. The first feature is connected with the fact that the current preferred solution (found at the previous iteration) of the multicriteria problem is a feasible solution of MRP problems. This facilitates both the exact and approximate methods for MRP solutions, since they start with a known initial feasible solution. The second feature is connected with the property, that the feasible area of MRP problems is a part of the feasible area of the multicriteria problem, in contrast to the feasible areas of the scalarizing problems of the reference points, which coincide with the feasible area of the multicriteria problem. Depending on the parameters values, the feasible area of MRP problems can be comparatively narrow and the feasible solutions found by exact or approximate methods, can be located very near to PO surface of the multicriteria problem. The obtaining and use of such approximate PO solutions can decrease considerably the time, during which the DM expects new solutions for evaluation. The dialogue with the DM can be improved on the account of decrease in the quality of the solutions obtained in the criteria space. The third feature is connected with DM's possibility to realize search strategies of the type "not great profits – small losses". This is due to the fact that with the help of MRP problems, an optimal solution is sought, which minimizes Chebyshev's distance from the feasible criteria up to the current reference point, the components of which are equal to the criteria values, intended by the DM, that are improved, and to the current values of the criteria, that are deteriorated. The obtained weak PO or PO solution and the current solution are comparatively close and the DM can make his choice more easily. This is also in force when the scalarizing problem is solved approximately and more feasible solutions are obtained, that are comparatively near to the current solution and in between as well. In other words, the influence of the so called "limited comparability" of weak PO or PO solutions is decreased. RD scalarizing problems have the same property, but in them it depends on the parameter α and its realization requires higher qualification of the DM concerning the scalarizing problem applied.

2.3. Classification-oriented scalarizing problems

RD and MPR scalarizing problems are particularly appropriate for the solution of MO integer problems, because they enable the diminishment of the computational difficulties, related to their solving, and also the requirements towards the DM in comparison and evaluation of the new solutions obtained. As for the information, required from the DM in the search for new solutions, these scalarizing problems are comparatively close to the scalarizing problems of the reference point [33]. In Classification-Oriented Scalarizing Problems (COSP) the DM can present his/her local preferences not only by desired and acceptable levels, but also by desired and acceptable directions and intervals of alteration of the values of the separate criteria [23, 31]. In this way the DM can describe his/her local preferences with greater flexibility, accuracy and certainty. Depending on these preferences, at each iteration the set of the criteria can be divided into seven or less than seven classes, denoted as $K^{>}$, $K^{\geq}$, $K^{=}$, $K^{<}$, $K^{\leq}$, $K^{><}$ and K^{0}. Every criterion $f_k(x)$, $k \in K$, could enter into one of these classes, as follows:

- $k \in K^{>}$, if the DM wishes the criterion $f_k(x)$ to be improved;
- $k \in K^{\geq}$, if the DM wishes the criterion $f_k(x)$ to be improved by a desired value $\Delta_k, \Delta_k > 0$;
- $k \in K^{=}$, if the DM wishes the current value of the criterion $f_k(x)$; not to be deteriorated;
- $k \in K^{<}$, if the DM agrees the criterion $f_k(x)$ to be deteriorated;
- $k \in K^{\leq}$, if the DM agrees the criterion $f_k(x)$ to be deteriorated at most by an acceptable value $\delta_k > 0$;
- $k \in K^{><}$, if the DM wishes the criterion $f_k(x)$ not to be altered out of the bounds for a given interval, defined as: $f_k - t_k^{-} \leq f_k(x) \leq f_k + t_k^{+}$;
- $k \in K^{0}$, if the DM wishes the criterion $f_k(x)$ to be freely altered.

In order to obtain a solution, better than the current PO solution of MO problem, on the basis of the implicit classification of the criteria, done by the DM, the following COSP scalarizing problem may be used:

To minimize

(8) $$S(x) = \max\Big(\max_{k \in K^{\geq}} \left(\bar{z}_k - f_k(x) \right) / G_k,\ \max_{k \in K^{\leq}} \left(\hat{z}_k - f_k(x) \right) / G_k,\ \max_{k \in K^{<}} \left(f_k - f_k((x) / G_k \right) +$$

$$+ \max_{k \in K^{>}} \left(f_k - f_k(x) \right) / G_k + \rho \Big(\sum_{k \in K^{\geq}} \left(\bar{z}_k - f_k(x) \right) + \sum_{k \in K \leq} \left(\bar{z}_k - f_k(x) \right) + \sum_{k \in K^{<}} \left(f_k - f_k(x) \right) +$$

$$+ \sum_{k \in K^{>}} \left(f_k - f_k(x) \right) - \sum_{k \in K^{=} \cup K^{0} \cup K^{><}} f_k(x) \Big),$$

under the constraints

(9) $$f_k(x) \geq f_k,\ k \in K^{>} \cup K^{=},$$

(10) $$f_k(x) \geq f_k - \delta_k,\ k \in K^{\leq},$$

(11) $$f_k(x) \geq f_k - t_k^{-},\ k \in K^{><},$$

(12) $$f_k(x) \leq f_k + t_k^{+},\ k \in K^{><},$$

(13) $$x \in X,$$

where f_k, $k \in K$, is the value of the criterion $f_k(x)$ in the current preferred PO solution;

$\overline{z}_k = f_k + \Delta_k$, $k \in K^{\geq}$, is the desired level for the criterion $f_k(x)$;

$\overline{z}_k = f_k - \delta_k$, $k \in K^{\leq}$, is the acceptable level for the criterion $f_k(x)$;

G_k, $k \in K$, is a scaling coefficient, determined as follows:

$$G_k = \overline{z}_k - f_k,\ k \in K^{\geq} \cup K^{\leq},$$

$$G_k = f_k - z_k^*,\ k \in K^{>} \cup K^{<};$$

ρ is a small positive number.

Usually COSP scalarizing problems are optimization problems with non-differentiable objective functions. Every scalarizing problem COSP can be reduced to an equivalent optimization problem with a differentiable objective function at the expense of additional variables and constraints. The equivalence of each pair of optimization problems is proved in relation to the obtained values of the criteria (objective functions) and the basic variables.

The more significant characteristics of COSP problems, connected with the possibilities to improve the dialogue with the DM, are the following:

- COSP give more freedom to the DM to express his/her local preferences in the search for a better (weak) PO solution. The DM can describe his/her local preferences with the help of desired and acceptable levels of the values of a part or of all the criteria, and also with the help of desired and acceptable directions and intervals of change in the values of a part of the criteria. In this way the DM can describe his/her local preferences with greater flexibility, accuracy and certainty.
- The current preferred solution of the multicriteria problem is a feasible solution of the current COSP scalarizing problem, which decreases considerably its solution time.
- The feasible solutions of COSP scalarizing problem are located near the PO solutions of the multicriteria problem. However, the more the criteria the DM wants to be freely improved or agrees to be freely deteriorated are, the more the quality falls. The narrow feasible area of COSP scalarizing problems enables the successful use of approximate single-criterion algorithms, which is particularly important, when these problems are integer.

2.4. Generalized scalarizing problems

The long experience gained during the development and study of scalarizing problems in the solution of MO problems, has supplied the opportunity to define the so-called generalized scalarizing problems GENWS and GENS [20, 34]. They are a generalization of the scalarizing problems of the weighted sum, of ε-constraints problems, of the reference point problems, of the reference direction problems and of the classification-oriented problems. The generalized scalarizing problems GENWS and GENS combine to a great extent their qualities as well. In order to obtain a weak PO solution, starting directly or indirectly from the current weak PO solution, the following generalized scalarizing problem GENWS could be applied as follows.

To minimize

(14) $$S(x) = \max\left(\max_{k\in K^{\geq}}\left(F_k^1 - f_k(x)\right)G_k^1 \ R_1 \max_{k\in K^{\leq}}\left(F_k^2 - f_k(x)\right)G_k^2 \ R_2\right.$$

$$\left.\max_{k\in K^{<}}\left(F_k^3 - f_k(x)\right)G_k^3 \ R_3 \max_{k\in K^{>}}\left(F_k^4 - f_k(x)\right)G_k^4\right) + \sum_{k\in K^0}\left(F_k^5 - f_k(x)\right)G_k^5 ,$$

under the constraints

(15) $$f_k(x) \geq f_k, \ k \in K^{>} \cup K^{=},$$

(16) $$f_k(x) \geq f_k - D_k, \ k \in K^{\leq},$$

(17) $$f_k(x) \geq f_k - t_k^-, \ k \in K^{><},$$

(18) $$f_k(x) \leq f_k + t_k^+, \ k \in K^{><},$$

(19) $$x \in X,$$

where:

- K is the set of all the criteria;
- $G_k^1, G_k^2, G_k^3, G_k^4, G_k^5$ are scaling coefficients;
- $F_k^1, F_k^2, F_k^3, F_k^4, F_k^5$ are parameters, associated with aspiration, current and other levels of the criteria values;
- R_1, R_2, R_3 are equal to the arithmetic operation "+" or to a divider "," ;
- D_k is the value, with which the DM agrees the criterion with an index $k \in K$ ($0 < D_k < \infty$) to be deteriorated;
- t_k^- and t_k^+ are the lower and upper bounds of the acceptable (for the DM) interval of alteration of the value of the criterion with an index $k \in K^{><}$;
- f_k is the value of the criterion with an index $k \in K$ in the current solution obtained;
- $K^{\geq}$ is the set of the criteria, about which the DM wishes their current values to be improved to levels F_k^1, set by him ;
- $K^{>}$ is the set of criteria, about which the DM wants their current values to be improved;
- $K^{\leq}$ is a set of criteria, about which the DM agrees their current values to be deteriorated to acceptable levels F_k^2, set by him, but not more than defined values $f_k - D_k$ ($F_k^2 \geq f_k - D_k$);
- $K^{<}$ is the set of the criteria, about which the DM agrees their current values to be deteriorated;
- $K^{=}$ is the set of the criteria, about which the DM wants their current values not to be deteriorated;
- $K^{><}$ is the set of the criteria, about which the DM agrees their values to be altered in given intervals;
- K^0 is the set of the criteria, about which the DM does not set explicit preferences about the alteration of their values.

GENWS scalarizing problem has got sense, when $K^{\geq} \neq \varnothing$, and/or $K^{>} \neq \varnothing$, or $K^{0} = K$. That is why we accept that $K^{\geq}$ and/or $K^{>} \neq \varnothing$ or $K^{0} = K$. In the general case the constraints (15)-(18) define a subset $X^{1} \in X$, which contains weak PO solutions.

In order to obtain PO solutions, GENS scalarizing problem can be used, instead of GENWS scalarizing problem, having the following form:

To minimize

$$T(x) = \max \Big(\max_{k\in K^{\geq}} \left(F_k^1 - f_k(x)\right) G_k^1 \; R_1 \max_{k\in K^{\leq}} \left(F_k^2 - f_k(x)\right) G_k^2 \; R_2 \tag{20}$$

$$\max_{k\in K^{<}} \left(F_k^3 - f_k(x)\right) G_k^3 \quad R_3 \max_{k\in K^{>}} \left(F_k^4 - f_k(x)\right) G_k^4 \Big) + \sum_{k\in K^{0}} \left(F_k^5 - f_k(x)\right) G_k^5 +$$

$$+ \rho \Bigg(\sum_{k\in K^{\geq}} \left(F_k^1 - f_k(x)\right) G_k^1 + \sum_{k\in K^{\leq}} \left(F_k^2 - f_k(x)\right) G_k^2 + \sum_{k\in K^{<}} \left(F_k^3 - f_k(x)\right) G_k^3 +$$

$$+ \sum_{k\in K^{>}} \left(F_k^4 - f_k(x)\right) G_k^4 - \sum_{k\in K^{=}\cup K^{><}} f_k(x) G_k^6 \Bigg)$$

under constraints (15)-(19) and ρ is a small positive number.

In the general case GENWS and GENS scalarizing problems are optimization problems with a non-differentiable objective function. Every scalarizing problem GENWS or GENS (with determined values of R_1, R_2, R_3) can be reduced to an equivalent optimization problem with a differentiable objective function at the expense of additional variables and constraints. The equivalence of each pair of optimization problems is with respect to the obtained values of the criteria (objective functions) and the basic variables. For different values of R_1, R_2, R_3 different types of equivalent problems can be obtained.

Each one of the known scalarizing problems, such as the scalarizing problem of the weighted sum, the scalarizing problem of ε-constraints, Chebyshev's scalarizing problem, the scalarizing problems of the reference point, the scalarizing problems of the reference direction and classification-oriented scalarizing problems can be obtained for a given combination of the parameters through GENWS or GENS scalarizing problems. For the first five types of scalarizing problems, the values of the criteria in the current solution obtained are not directly included in the scalarizing problem parameters, although based on these values the DM sets the values of the parameters of the scalarizing problem. In the classification-oriented scalarizing problems, the values of the criteria in the current solution obtained are parameters of the scalarizing problem. In the scalarizing problems of the weighted sum, the DM sets his/her preferences with the help of the values of the criteria weights, and in the scalarizing problems of ε-constraints – by selection of one function for maximization and definition of the lower limit of alteration of the remaining criteria. The reference point is determined by the aspiration levels of the criteria, which levels the DM wishes or agrees to be obtained in the new solution.

These aspiration levels of the criteria are parameters of the scalarizing problems of the reference point. The values of the criteria in the current solution found can also be parameters of the scalarizing problem. Defining desired or acceptable alterations in the values of the criteria in the current solution obtained, the DM indirectly classifies the separate criteria into different groups.

For example, in order to generate the scalarizing problem of the weighted sum by GENWS scalarizing problem, the following substitutions are made:

$K^{\geq} = K^{\leq} = K^{<} = K^{>} = K^{=} = K^{><} =$Ø,

$K^0 = K$,

$F_k^5 = 0$,

$G_k^5 = \omega_k$ and $\sum_{k \in K} G_k^5 = 1$.

The famous scalarizing problem NIMBUS [8] can be obtained from GENWS scalarizing problem through the following substitutions:

$K^{<} = K^{><} =$ Ø,

$R_3 =$ “,” ,

$F_k^1 = \bar{z}_k$,

$F_k^4 = z_k^*$,

$D_k = \Delta_k$,

$G_k^2 = G_k^3 = 0$,

$G_k^1 = G_k^4 = \dfrac{1}{\left|z_k^*\right|}$.

Altering the parameters of the scalarizing problem GENWS or GENS, a lot of new scalarizing problems with different characteristics could be obtained. The qualities of every scalarizing problem are determined by the possibilities, provided to the DM to set his/her preferences, by the capacities for overcoming the computational difficulties in solving certain types of NP optimization problems, as well as by the quality of the obtained solutions.

2.5. Interactive methods for solving multicriteria optimization problems

The interactive methods are among the most widely spread methods for solving MO problems. Every iteration of such method consists of two phases: a computational phase and a dialogue phase. During the computational phase one or more (weak) PO solutions are generated with the help of a scalarizing problem. During the dialogue phase these weak PO or PO solutions are presented to the DM for evaluation. In case the DM does not approve any of these solutions as a final solution (the most preferred solution) of the initial multicriteria problem, then to improve these solutions he/she provides information, concerning his/her local preferences. This information is used to formulate a new scalarizing problem that is solved at the next iteration.

The quality of every interactive method depends to a high extent on the quality of the dialogue with the DM. On its hand, the quality of the dialogue with the DM is determined by the type of the information he sets for improvement of a local preferred (weak) PO solution, by the time of the scalarizing problems solution, by the possibilities for DM's learning with regard to the multicriteria problem being solved, by the type and number of the new weak PO or PO solutions, compared with the current preferred solution.

When solving linear and convex nonlinear MO problems, linear and convex nonlinear programming problems are used as scalarizing problems. These problems are easy solvable problems. That is why in the interactive methods for solving multicriteria linear and convex nonlinear problems, the solution time of the scalarizing problems does not play such a significant role. The attention is drawn to the type of the information, which the DM can present for improvement of the local preferred PO solution. The possibilities for DM's learning during the solution time of linear and convex nonlinear multicriteria problems is the other important feature of the interactive methods. Besides enabling DM's freedom to move in PO criteria set, these possibilities are also expressed in the discovery of more than one PO solution during the computational phase. These solutions are presented for evaluation to the DM. In the comparison and evaluation of more than two PO solutions, in particular when they differ significantly one from another and the criteria number is big, the DM could encounter considerable difficulties in the selection of a current (final) preferred PO solution [4, 6].

In solving integer and non-convex nonlinear MO problems, integer and non-convex nonlinear programming problems are used as scalarizing problems. These problems are of the type of NP problems [2]. The exact algorithms for their solution have exponential complexity. Even the finding of a feasible integer solution is so difficult, as finding an optimal solution. That is why when developing interactive methods, solving integer and non-convex nonlinear problems, it is necessary to take into account the time for scalarizing problems solution. In case this time is too big, the dialogue with the DM, even though user friendly, might not take place (if the DM refuses to wait too long for the scalarizing problem solution).

On the basis of the scalarizing problems of the Reference Direction (RD), the interactive methods RD-IM [9, 16, 28] are developed, with the purpose to solve linear, nonlinear and linear integer MO problems. These interactive methods are learning oriented methods. This means that the DM can freely seek the final or the most preferred solution of the initial multicriteria problem from the sets of PO, weak PO or approximate weak PO solutions. For this purpose, during DM's learning phase, the DM must receive a notion about these sets, about the feasible ranges of criteria alteration, about some general relations between the changes of the separate criteria. That is why, in the development of the interactive method for solving MO integer problems, three different strategies are used to search for new solutions for evaluation. The first strategy, called integer strategy, is the search of each iteration of PO or weak PO integer solution by exact solving the corresponding integer scalarizing problems. The second strategy, called approximate integer strategy, searches at some iterations approximate weak PO integer solutions by approximate solution of the corresponding scalarizing problems. During the learning phase, and in problems of large dimension, up to the mere end, only approximate weak PO integer solutions are sought. The third strategy, called mixed strategy, accomplishes during most of the iterations a search for continuous PO solutions by solving continuous scalarizing problems, at that periodically an integer PO, an integer weak PO or an approximate integer weak PO solution is sought, which is close to the current continuous PO solution found. The search for continuous PO solutions is a solution of MO continuous problem. The notions integer PO solution, integer weak PO solution or approximate integer weak PO solution mean such a solution of MO integer problem in the criteria space, the solution of which in the variables space, is an integer solution.

On the basis of the scalarizing problems of the Modified Reference Directions (MRP), the interactive methods [18, 30] have been developed for solving linear, nonlinear and convex nonlinear integer PO problems. The scalarizing problems MRP are particularly appropriate for including in the interactive methods for solving linear integer and convex nonlinear integer MO problems due to their three features: a known feasible initial integer solution, narrow feasible area, independent on parameters (the feasible integer solutions of this problem lie near to PO front) and a possibility for slight changes in the criteria values.

On the basis of COSP scalarizing problems, some interactive methods [23, 31] have been developed for solving linear, nonlinear and linear integer MO problems. COSP scalarizing problems belong to the classification-oriented scalarizing problems and the DM can describe his/her local preferences with the help of desired and acceptable levels of a part or of all the criteria (as in RD and MRP problems), as well as with the help of desired and acceptable directions and intervals of alteration in the values of a part or all criteria. COSP scalarizing problems indicate two more properties (like RD and MRP scalarizing problems), connected with the fact that the current preferred solution of the multicriteria problem is a feasible solution of the new scalarizing problem and the feasible solutions of COSP scalarizing problems in the criteria space are located close to the PO surface of the corresponding multicriteria problem. COSP scalarizing problems enable the expansion of the information, with the help of which the DM can set his/her local preferences. This information expansion, however, leads to enlargement of the feasible set of criteria alteration in the criteria space and of the variables in the variables space. Hence, the feasible solutions of MRP problems are found nearer to PO surface, than the feasible solutions of COSP problems. In relation to this, when solving integer MO problems with large dimensions and when to reduce the time for new solutions obtaining, the scalarizing problems must be solved approximately, it is better to use MRP scalarizing problems, than COSP scalarizing problems. On the other hand, MRP problems have the least in number variables and constraints and from a computational viewpoint they prove to be the most appropriate for the case considered.

The main features of the interactive MO methods developed may be presented, as follows:

- a possibility to widen the information, with the help of which the DM can set his/her local preferences;
- a possibility to obtain continuous solutions and approximate integer solutions, which decreases the waiting time of the DM in solving integer multicriteria problems;
- decrease of the number of the integer scalarizing problems being solved;
- a possibility for relatively rapid DM's learning with regard to the multicriteria problems being solved, presenting at each iteration more weak PO or PO solutions for evaluation, and DM's freedom also to move freely over the whole area of these solutions.

A generalized interactive MO method is developed with the help of GENWS and GENS scalarizing problems. The interactive methods above discussed are representatives of the most widely spread interactive methods – the methods of the reference point, the methods of the reference direction and the classification-oriented methods. GENWS-IM generalized interactive method is with variable scalarization and parameterization. It is a generalization of the interactive methods described – RD-IM, MRP-IM and COSP-IM, as well as of a large part of some well known interactive methods. This generalization is with respect to the classes of the solved problems, the type of the given preferences, the number and type of the scalarizing problems used, the strategies used to seek new PO solutions.

MOIP software system has been developed [21, 29] on the basis of RD-IM interactive methods and intended to support the solution of MO linear integer problems. MRP-IM interactive method is realized in MONIP research program [17], intended to solve MO convex non-linear integer problems.

MKO-1 and MKO-2 software systems are designed to support the solution of MO linear and linear integer problems. They include COSP-IM classification-oriented interactive methods [25-27].

GENWS-IM interactive method is a good basis for the development of a software system with improved interface with the DM, both related to the class of MO problems being solved, and also to the possibilities to set his/her preferences. GENWS-IM interactive method could become the basis for creating an intelligent system, supporting MO problems solution.

3. Research developments in the area of multicriteria analysis

When solving MA problems, the DM plays a significant role. His/her global and local preferences define the final (the most preferred) solution of the respective multicriteria problem. Depending on the way of DM's preferences information acquisition and processing, and whether it is accepted that there exists a limit in DM's possibilities to do comparison among the alternatives, a large part of the methods for solving MA problems can be divided into three separate classes [32]:

- methods, in which DM's preferences are aggregated as a result of the synthesis of one generalized criterion (approach of the multi-attribute utility theory);

- methods, in which DM's global preferences are aggregated as a result of the synthesis of one or several generalized relations of the preferences among the alternatives (outranking approach);
- methods, in which DM's local preferences are accumulated iteratively after direct and indirect comparisons between two or more alternatives (interactive approach).

The outranking methods and the methods, based on the theory of multi-attribute utility, are traditional methods for solving a wide class of MA problems. At least so far, these methods have not any rival alternative when solving problems with a large number of criteria and a comparatively small number of alternatives.

The problems with a large number of alternatives and a comparatively small number of qualitative criteria, in which the DM is not able to evaluate simultaneously all alternatives, are similar to MO problems. Some interactive methods have been developed with the purpose to solve this type of problems. The interactive methods (VIMDA method [5], method of the aspiration levels [7], InterQuad method [15], LBS method [3], CBIM method [11] are "optimization motivated" and belong to the methods from the third class. The contribution of the research developments at IIT-BAS in the area of MA is notably in the design of efficient "optimization motivated" interactive algorithms. They are constructed on the basis of the formulated discrete scalarizing problems.

3.1. Discrete scalarizing problems

The first discrete scalarizing problem, suggested for solution of multicriteria choice problems, is the scalarizing problem of the reference directions [22]. The DM is presented a current preferred set of l ranked alternatives for evaluation and selection, or a current alternative, or the most preferred alternative. The parameter l is defined by the DM and it is quite smaller than the total number of the alternatives. In case the DM is not satisfied, on the basis of the current preferred alternative, he/she determines the desired alteration in the values of the criteria or desired direction of their alteration. DM's preferences are reflected in the formulated discrete scalarizing problem. On the basis of the scalarizing problem solutions, a new currently preferred set of l ranked alternatives is formed. .

The discrete scalarizing problem, called DOSP1 [11], is a discrete analog of the classification-oriented scalarizing problem of MO. It is based on the information, provided by the DM for the desired or acceptable levels, directions and intervals of change of the values of some or of all the criteria with respect to the current preferred alternative. After the solution of DOSP1 scalarizing problem, depending on the value of its objective function, the alternatives are ranked in an ascending order. The first l ranked alternatives form the current preferred set of alternatives, which is displayed to the DM for evaluation and choice of the current preferred alternative.

The current selection of alternatives is generated at each iteration. Let h denotes the index of a current preferred alternative. The following denotations, connected with this alternative, will be introduced:

– I' is the set of alternatives with exception of the alternative with an index h, $I' = I \backslash h$;

– a_{hj} is the value of the criterion with an index $j \in J$, for the current preferred alternative;

– $K_h^{\geq} \bigcup K_h^{>}$ is the set of criteria with indices $j \in J$, about which the DM wants their values to be improved with respect to their values in the current preferred alternative, where:

– $K_h^{\geq}$ is the set of criteria with indices $j \in J$, about which the DM wants their values to be improved with the aspiration value Δ_{hj};

– $K_h^{>}$ is the set of criteria with indices $j \in J$, about which the DM wants their values to be improved in a desired direction;

– $K_h^{\leq} \bigcup K_h^{<}$ is the set of criteria with indices $j \in J$, about which the DM agrees their values to be deteriorated compared to their values in the current preferred alternative, where:

– $K_h^{\leq}$ is the set of criteria with indices $j \in J$, about which the DM agrees their values to be deteriorated by no more than δ_{hj};

– $K_h^{<}$ is the set of criteria with indices $j \in J$, about which the DM agrees their values to be deteriorated in a given direction;

– $K^{\times}$ is the set of criteria with indices $j \in J$, about which the DM wishes their values to be within the interval $(a_{hj} - t_{hj}^{-} \leq a_{hj} \leq a_{hj} + t_{hj}^{+})$, in relation to their values in the current preferred alternative;

– $K^{=}$ is the set of criteria with indices $j \in J$, about which the DM does not wish their values to be deteriorated;

– K^{0} is the set of criteria with indices $j \in J$, towards which the DM is indifferent;

– $\overline{a}_{hj}$ is the desired aspiration value of the criterion with an index $j \in K_h^{\geq}$,

$$\overline{a}_{hj} = a_{hj} + \Delta_{hj}, \quad j \in K_h^{\geq};$$

Λ'_j is the difference between the maximal and minimal value of the criterion with an index j;

$$\Lambda'_j = \max_{i \in I'} a_{ij} - \min_{i \in I'} a_{ij}.$$

DOSP1 scalarizing problem is defined as follows:

$$\min_{i \in I'} S(i,h) = \min_{i \in I'} \{\max[\min_{j \in K_h^{\geq}} (\overline{a}_{hj} - a_{ij}) / \Lambda_j, \max_{j \in K_h^{<} \cup K_h^{\leq}} (a_{hj} - a_{ij}) / \Lambda_j] +$$
$$+ \max_{j \in K_h^{>}} (a_{hj} - a_{ij}) / \Lambda_j\}$$

under the constraints

$$a_{hj} \geq a_{ij}, \; i \in I', \; j \in K_h^{>} \bigcup K_h^{=},$$

$$a_{hj} \geq a_{ij} - \delta_{hj} \ , \ i \in I', \ j \in K_h^{\leq},$$
$$a_{hj} \geq a_{ij} - t_{hj}^{-} \ , \ i \in I', \ j \in K_h^{\times},$$
$$a_{hj} \leq a_{ij} + t_{hj}^{+} \ , \ i \in I', \ j \in K_h^{\times},$$

where

$$\Lambda_j = \begin{cases} \varepsilon \ , & \text{if} \ \ \Lambda'_j \leq \varepsilon, \\ \Lambda'_j, & \text{if} \ \ \Lambda'_j > \varepsilon, \end{cases}$$

ε is a small positive number.

When solving DOSP1 scalarizing problem, the value of $S(i,h)$ is computed for each alternative with an index *i*, satisfying the problem constraints. The objective function $S(i,h)$ is a "modified" Chebyshev's distance between the alternative with an index *i* and the "aspiration" alternative, defined by the problem conditions.

3.2. Interactive methods for solving multicriteria analysis problems

The interactive methods are used to solve MA problems with a large number of alternatives and a small number of qualitative criteria. The advantages of the interactive methods are connected with DM's opportunities to control the process of search for the most preferred alternative, selecting from sets of currently ranked alternatives. In order to obtain the sets of ranked alternatives, discrete optimization scalarizing problems are applied. Weak PO (nondominated) alternatives are found with the help of the discrete classification-oriented problem DOSP1. On its basis, a discrete classification-oriented interactive method CBIM is designed, intended for solving problems of discrete multicriteria choice with a large number of alternatives and a small number of qualitative criteria, and providing the DM the possibility to control the process of seeking for the most preferred alternative selected among sets of currently ranked alternatives. This is a new type of MA methods, expanding DM's possibilities to describe his/her local preferences and for evaluation of the solutions obtained.

The software systems MKA-1 and MKA-2, [24, 27], developed at IIT-BAS, are intended for solving MA problems. AHP weighting method [13], PROMETHEE II outranking method [1], as well as CBIM interactive method [11, 22] are included in MKA-1 system. MKA-1 system operates wit two type of criteria – qualitative and quantitative criteria. MKA-2 system is an extension of MKA-1 system. One outranking method more – ELECTRE III [12], is added to it and the possibility to work with weighting and ranking types of criteria is expanded.

4. Directions of future investigations

Usually the optimization problems, which arise in different spheres of human activity, are multicriteria problems. The dimensions of the real life multicriteria problems, that have to be solved, are continuously increasing. That is why the attempts of many researchers are focused on the development of new more efficient algorithmic approaches for their solution. One of the trends in the future research activities at IIT-BAS is the development of interactive algorithms, which combine the achievements of the classical scalarization and population-based methodology for more efficient solving of some classes of NP multicriteria optimization problems. For example, the development of interactive evolutionary algorithms of integer MO could avoid the multiple solving of integer scalarizing problems, needed by the DM in the classical interactive algorithms for learning the characteristics of the PO set of a given problem. The generation of a set of solutions, approximating the PO front, will enable the DM to gain an idea of the possible alternative values of the objective functions, of some common features of PO solutions, etc. This information will allow the more exact defining of the preferences with the purpose to find a better PO solution. In this way the obtaining of the most preferred solution will be achieved after a smaller number of iterations, which saves time, while DM's confidence in the quality of this solution will be preserved.

The development of an intelligent system, supporting the solution of MO problems, on the basis of GENWS-IM generalized interactive method, is one of the forthcoming scientific-applied activities. This software system will afford the DM, depending on his/her qualification, to set the preferences in the most convenient way. On the basis of the information received, the most appropriate method for solving MO problem will be selected. The development of integrated software systems for multicriteria decision making is also one of the directions for the future research and applied work. They will unite the functions, concerning the better construction and edition of the models of MO and MA problems and the different approaches to their solution.

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