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Towards a Model of the Digital University: Generalized Net Model of Appraisal of Lecturers with Intuitionistic Fuzzy Estimations

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Abstract**:** *A generalized net is used to construct a model which describes the process of a evaluation of a lecturer. The model utilizes the theory of intuitionistic fuzzy sets. The model can be used to simulate some processes related to the professional appraisal of a lecturer.*

Keywords**:** *E-Learning, Generalized Net, Intuitionistic Fuzzy Sets, University.*

1. Introduction

In a series of research, the authors studied some of the most important processes in the functioning of universities ([5,7,8,9,10,11,12]). Generalized Nets (GNs, see

[1, 2]) have been used to describe the process of student assessment [5,11,12]. The evaluations to cope with the varying student backgrounds on different topics are represented in intuitionistic fuzzy form; (for the concept of Intuitionistic Fuzzy Set (IFS, see [3,4]). In [5] the process of evaluation of the problems solved by students is described by Generalized Nets. The paper [9] describes the process of assessment with intuitionistic fuzzy estimations by lecturers of the tasks presented by students. [10] represents a generalized net model of the process from [9]. A generalized net was constructed in [10] to correspond to a model which describes the standardization of the process of assessment by lecturers. In [12] the process of assessment of students' courses is described by GNs.

In the present paper the process of evaluation of lecturers is described by GNs. The appraisal of a lecturer is a function of the average of the students' evaluations from the examinations of the course, averaged evaluation from the students' judgement of the lecturer and the evaluation of the scholarly activity of the lecturer.

2. Determination of the evaluations

Suppose we have m students that have to solve n problems related to a current course, k student's courses, and let us have q (in number) lecturers, $i = 1, 2, ..., m$, $j = 1, 2, ..., n,\ l = 1, 2, ..., k,\ s = 1, 2, ..., q$.

2.1. Determination of the evaluations of the student's solutions

The assessments which correspond to the students' solutions from the different students' courses are represented by intuitionistic fuzzy estimations. They have the form $\langle \mu(s)^l, \nu(s)^l \rangle$, where $\mu(s)^l$ and $\nu(s)^l$ determine the degrees of comprehension and incomprehension of the l-th course of the s-th lecturer, $\mu(s)^l, \nu(s)^l, \mu(s)^l + \nu(s)^l \in [0, 1]$, and:

(1) $$\mu(s)^l = \frac{1}{m} \sum_{i=1}^{m} \mu(s)_l^s,$$

(2) $$\nu(s)^l = \frac{1}{m} \sum_{i=1}^{m} \nu(s)_l^s,$$

where $\mu(s)_l^s$ and $\nu(s)_l^s$ are the intuitionistic fuzzy evaluations of the i-th student solutions of the problems included in the l-th course of the s-th lecturer.

The degree of uncertainty $\pi(s)_l^s = 1 - \mu(s)_l^s - \nu(s)_l^s$ represents those cases where the students did not go in for an examination for the course l.

The intuitionistic fuzzy evaluations of the student's solutions of the problems $\langle \mu(s)_l^s, \nu(s)_l^s \rangle \in [0, 1] \times [0, 1]$ for the l-th course from the s-th lecturer can be obtained, in general, by three ways according [5]:

• The solution of problems is evaluated as valid or non-valid only on the bases of the answer obtained or on the basis of the validity of a complete proof. In this case the evaluation of the i-th student is

(3) $$\langle \mu(s)_l^s, \nu(s)_l^s \rangle = \langle \frac{r}{n}, \frac{f}{n} \rangle$$

where:

r is the number of right solved problems,

f is the number of wrong solved problems.

Therefore, the degree of uncertainty here is determined by the number of the problems which the student had not worked over.

• j-th problem is divided into subproblems and the solution of each subproblem is evaluated independently. Each problem of the i-th student is evaluated by

$$\langle \mu(s)^s_l, \nu(s)^s_l \rangle = \langle \frac{1}{n}. \sum_{j=1}^{n} \frac{z_j}{p_j}, \frac{1}{n}. \sum_{j=1}^{n} \frac{y_j}{p_j} \rangle, \tag{4}$$

where:

z_j is the number of the first correctly solved subproblems that precede a wrong subproblem (if such a one exists),

y_j is the number of the incorrectly solved subproblems that follow a correctly solved subproblem (if such a one exists),

p_j is the number of the subproblems for the j-th problem.

• r-th problem is divided into subproblems and the solution of each unique subproblem is evaluated independently. Each problem of the i-th student is evaluated by

$$\langle \mu(s)^s_l, \nu(s)^s_l \rangle = \langle \frac{1}{n}. \sum_{j=1}^{n} \frac{x_j}{p_j}, \frac{1}{n}. \sum_{j=1}^{n} \frac{y_j}{p_j} \rangle, \tag{5}$$

where

x_j is the number of all correctly solved subproblems (if such exist),

y_j is the number of all incorrectly solved subproblems (if such exist),

p_j is the number of the subproblems for the j-th problem.

2.2. Determination of the evaluations of the students' appraisals of the lecturers

The evaluations corresponding to the appraisals of the lecturers from the student's investigation have the form $\langle \varepsilon(s)^l, \delta(s)^l \rangle$, where $\varepsilon(s)^l$ and $\delta(s)^l$ determine the degrees of approve of and non-approve of the s-th lecturer from the l-th course, $\varepsilon(s)^l, \delta(s)^l$, $\varepsilon(s)^l + \delta(s)^l \in [0, 1]$, and:

$$\varepsilon(s)^l = \frac{1}{m}. \sum_{i=1}^{m} \varepsilon(s)^s_l, \tag{6}$$

$$\delta(s)^l = \frac{1}{m}. \sum_{i=1}^{m} \delta(s)^s_l, \tag{7}$$

where $\varepsilon(s)^s_l$ and $\delta(s)^s_l$ are the intuitionistic fuzzy evaluations of the s-th lecturer for l-th course from i-th student. The couple $\langle \varepsilon(s)^s_l, \delta(s)^s_l \rangle \in [0, 1] \times [0, 1]$ reflects the degree of the acceptance of the lecturer (ε) and the non-acceptance of the lecturer (δ) from the students, and:

$$\varepsilon(s)_l^s = \frac{r(i)_l^s}{p}, \tag{8}$$

$$\delta(s)_l^s = \frac{t(i)_l^s}{p} \tag{9}$$

where:

$r(i)_l^s$ is the number of positive answers for the s-th lecturer form the i-th student for the l-th course,

$t(i)_l^s$ is the number of negative answers for the s-th lecturer form the i-th student for the l-th course,

p is the total number of questions in the investigation.

At the beginning, when still no information has been obtained, all estimations are given the initial values of $\langle 0, 0\rangle$.

The degree of uncertainty $\varphi(s)^l = 1 - \varepsilon(s)^l - \delta(s)^l$ represents those cases where the students did not engage with final opinion about the lecturer.

To illustrate the estimation of the lecturer' acceptance, we will give the following example: a students make 20 estimations for the lecturer. The 10 of the answers is "yes", 5 of the answers is "no" and in the rest 5 cases he abstain from voting. That is why we determine his estimation as $\langle 0.5, 0.25\rangle$.

The calculated estimation of the each lecturer reflects the coefficient of its acceptance.

2.3. Determination of the evaluations of the scholarly activity of the lecturers

Let the s-th lecturer, $s = 1, 2, ..., q$, have w^s papers for the current year. Let

$$W^{\max} = \max(w^1, w^2, ..., w^s) \tag{10}$$

and $W^{\min}$ is the minimal requirement number of the papers for a year. In this case the evaluations of the scientific activity of the s-th lecturer $\langle\theta(s), \sigma(s)\rangle$, where $\theta(s)$ and $\sigma(s)$ corresponding to the rating of the lecturers' scholarly activity, $\theta(s), \sigma(s)$, $\theta(s) + \sigma(s) \in [0, 1]$ are:

$$\theta(s) = \frac{w^s}{W^{\max}}, \tag{11}$$

$$\sigma(s) = \frac{W^{\min} \dot{-} w^s}{W^{\max}}, \tag{12}$$

where operation $\dot{-}$ means:

$$x \dot{-} y = \begin{cases} x - y, & \text{if } x \geq y, \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

The degree of uncertainty $\zeta(s)$ is

$$\zeta(s) = \frac{W^{\max} - w^s - (W^{\min} \div w^s)}{W^{\max}}. \tag{14}$$

2.4. Determination of the lecturers' evaluations

The appraisal of the s-th lecturer $\langle \rho(s), \tau(s) \rangle$ is a function of the averaged student's assessments from the examination of the course, the averaged evaluation from the student's appraisals of the lecturer and the evaluation of the

$$\rho(s) = \frac{\mu(s)^l + \varepsilon(s)^l + \theta(s)}{3}, \tag{15}$$

$$\tau(s) = \frac{\nu(s)^l + \delta(s)^l + \sigma(s)}{3}. \tag{16}$$

The degree of uncertainty is $\psi(s) = 1 - \rho(s) - \tau(s)$.

3. A GN-model

The GN-model for this section (Figure. 1) contains 10 transitions and 33 places, collected in five groups and related to the five types of the tokens that will enter respective types of places:

α- tokens and a-places represent the lecturers and their activities,

β- tokens and b-places represent the courses and connected with them problems,

γ- tokens and c-places represent the investigations and connected with them questions,

φ-tokens and d-places represent the students and their solutions of the problems, and their answers of the investigations,

κ– tokens and k-places represent the criterions for the evaluation of the scientific activity of a lecturer.

For brevity, we shall use the notation α-, β-, γ-, φ- and κ-tokens instead of α_s-, β_l-, γ_i-, φ_t- and κ_v-tokens, where s, l, i, v, t are numerations of the respective tokens, $s = 1, 2, ..., q;\ l = 1, 2, ..., k;\ i = 1, 2, ..., m.$

Initially the α-, β-, γ-, φ- and κ-tokens remain, respectively, in places a_4, b_3, c_3, d_3 and k_2 with initial characteristics: $x_0^\alpha =$ "name and specialty of a lecturer,intuitionistic fuzzy estimation of a lecturer"

x_0^β = "name of the course, text of a problem, theme, level of difficulty",

x_0^γ = "investigation of the lecturers, text of a questions",

x_0^φ = "name, specialty and current evaluations of a student",

x_0^κ = "criterions for the evaluation of the scientific activity of a lecturer".

If we would like the model to be more detailed, the first and the latest characteristics can have, e.g., the following larger forms.

x_0^α = "name, specialty and score of a teacher, variant of assessment of the course", x_0^φ= "name, specialty and current evaluations of a student, name of the student's teacher who will give the problems and/or examine the student, name of the course".

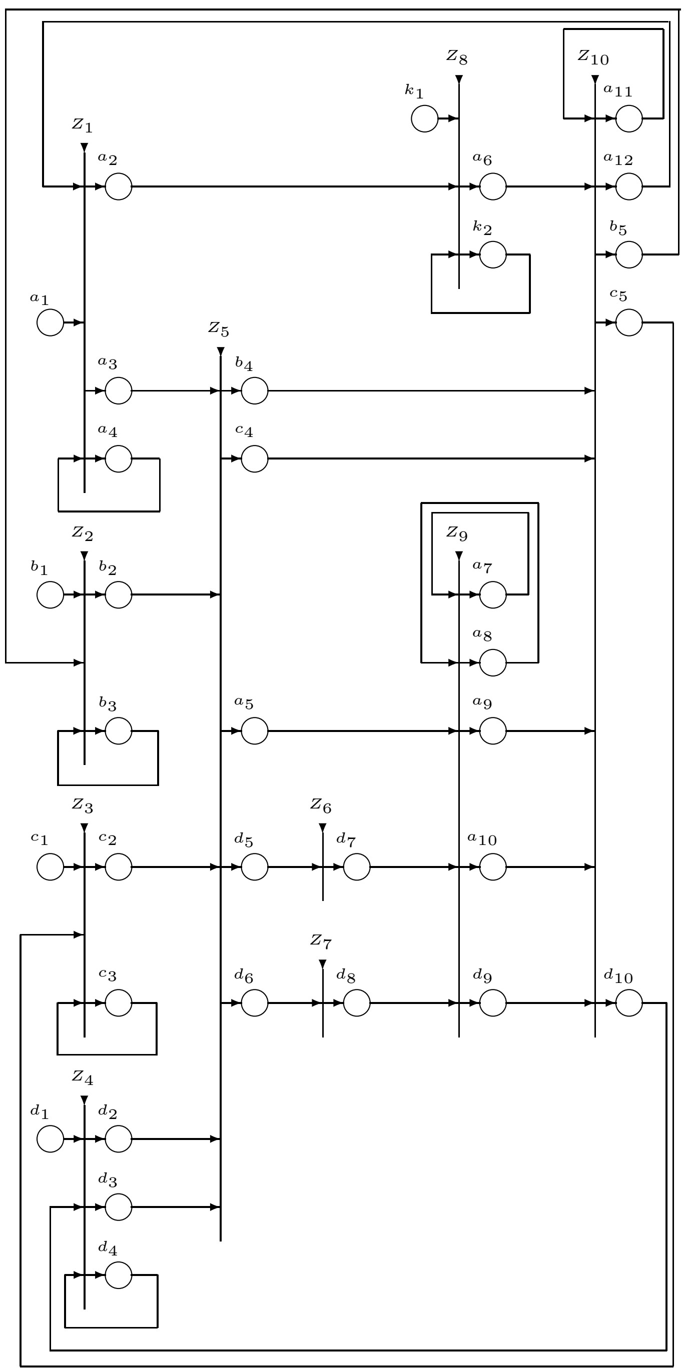

Fig. 1. Generalized net model of the lecturer's assessment processes

All α-tokens, all β-tokens, all γ-tokens, all φ-tokens, and all κ-tokens have equal priorities, but the priority of α-tokens is higher than the priority of β-tokens, that is higher than the priority of γ- and κ-tokens.

Let $x^{\alpha}_{cu}, x^{\beta}_{cu}, x^{\gamma}_{cu}, x^{\varphi}_{cu}$ and x^{κ}_{cu} be the current characteristics of the α-, β-, γ-, φ- and κ-tokens, respectively. The forms of the transitions are the following.

$$Z_1 = \langle \{a_1, a_4, a_{12}\}, \{a_2, a_3, a_4\}, \begin{array}{c|ccc} & a_2 & a_3 & a_4 \\ \hline a_1 & false & false & true \\ a_4 & W^a_{4,2} & W^a_{4,3} & W^a_{4,4} \\ a_{12} & false & false & true \end{array} \rangle,$$

where:
$W^a_{4,2} =$ "The lecturer must examine",
$W^a_{4,3} =$ "The scientific activity of the lecturer must be evaluated",
$W^a_{4,4} = \neg W^a_{4,2} \vee \neg W^a_{4,3}$
and $\neg P$ is the negation of predicate P.

The α-tokens do not obtain any characteristic in place a_4 and they obtain the characteristics:

"list of the problems that the student must solve"

in place a_2 "list of the problems that the student must solve"

in place a_3.

$$Z_2 = \langle \{b_1, b_3, b_5\}, \{b_2, b_3\}, \begin{array}{c|cc} & b_2 & b_3 \\ \hline b_1 & false & true \\ b_3 & W^b_{3,2} & W^b_{3,3} \\ b_5 & false & true \end{array} \rangle,$$

where:
$W^b_{3,2} =$ "The problem is included in x^{α}_{cu}",
$W^b_{3,3} = \neg W^b_{3,2}$.

The β-tokens do not have any characteristic in place b_3 and they take on the characteristic

"current course, texts of the problems that the student must solve"

in place b_2.

$$Z_3 = \langle \{c_1, c_3, c_5\}, \{c_2, c_3\} \begin{array}{c|cc} & c_2 & c_3 \\ \hline c_1 & false & true \\ c_3 & W^c_{3,2} & W^c_{3,3} \\ c_5 & false & true \end{array} \rangle,$$

where:
$W^c_{3,2} =$ "The lecturer must be evaluated by student's investigation", $W^c_{3,3} = \neg W^c_{3,2}$.

The γ-tokens do not obtain any characteristic in place c_3 and they obtain the characteristic

"investigation: list of the questions that the student must fill in"

in place c_2.

$$Z_4 = \langle \{d_1, d_4, d_{10}\}, \{d_2, d_3, d_4\}, \begin{array}{c|ccc} & d_2 & d_3 & d_4 \\ \hline d_1 & false & false & true \\ d_4 & W^d_{4,2} & W^d_{4,3} & W^d_{4,4} \\ d_{10} & false & false & true \end{array} \rangle,$$

where:
$W^d_{4,2} =$ “The student must fill in the investigation for the lecturer”,
$W^d_{4,3} =$ “The student must have examination”,
$W^d_{4,4} = \neg W^d_{4,2} \vee \neg W^d_{4,3}$.

The γ-tokens do not obtain any characteristic in places d_2, d_3 and d_4.

$$Z_5 = \langle \{a_3, b_2, c_2, d_2, d_3\}, \{a_5, b_4, c_4, d_5, d_6\},$$

$$\begin{array}{c|ccccc} & a_5 & b_4 & c_4 & d_5 & d_6 \\ \hline a_3 & true & false & false & false & false \\ b_2 & false & true & false & false & false \\ c_2 & false & false & true & false & false \\ d_2 & false & false & false & true & false \\ d_3 & false & false & false & false & true \end{array} \rangle,$$

The α-, β- and γ-tokens do not have any characteristic in places a_5, b_4, and c_4 respectively, while φ-tokens obtain characteristic respectively:

“investigation, student’s answers for the lecturer of the course”

in place d_5, and

“course, student’s solutions of the problems for the course”

in place d_6.

$$Z_6 = \langle \{d_5\}, \{d_7\}, \begin{array}{c|c} & d_7 \\ \hline d_5 & W^d_{5,7} \end{array} \rangle,$$

where:
$W^d_{5,7} =$ “The evaluations of the student’s answers for the lecturer in the investigation according (8) and (9) are ready”.

The φ-tokens that enter place d_7 obtain characteristic

“investigation, evaluations of the student’s answers for the lecturer”.

$$Z_7 = \langle \{d_6\}, \{d_8\}, \begin{array}{c|c} & d_8 \\ \hline d_6 & W^d_{6,8} \end{array} \rangle,$$

where:
$W^d_{6,8} =$ “The evaluations of the student’s solutions according (3), (4) or (5) are ready”.

The φ-tokens that enter place d_8 obtain characteristic

“course, evaluations of the student’s solutions of the problems for the course”.

$$Z_8 = \langle \{a_2, k_1, k_2\}, \{a_6, k_2\}, \begin{array}{c|cc} & a_6 & k_2 \\ \hline a_2 & false & true \\ k_1 & false & true \\ k_2 & W^a_{2,6} & W^k_{2,2} \end{array} \rangle,$$

where:
$W^a_{2,6} =$ “The evaluations of the scientific activity of the lecturer according (11) and

(12) are ready”,
$W^k_{2,2} = \neg W^a_{2,6}$.

The α-tokens that enter place a_6 obtain characteristic

“evaluations of the scientific activity of the lecturer”.

$$Z_9 = \langle \{a_5, a_7, a_8, d_7, d_8\}, \{a_7, a_8, a_9, a_{10}, d_9\},
\begin{array}{c|ccccc}
 & a_7 & a_8 & a_9 & a_{10} & d_9 \\ \hline
a_5 & W^a_{5,7} & W^a_{5,8} & W^a_{5,9} & W^a_{5,10} & false \\
a_7 & W^a_{7,7} & false & W^a_{7,9} & false & false \\
a_8 & false & W^a_{8,8} & false & W^a_{8,10} & false \\
d_7 & true & false & false & false & true \\
d_8 & false & true & false & false & true
\end{array} \rangle,$$

where:
$W^a_{5,7} = W^a_{7,7} =$ “There are students’ investigation for the current lecturer who must be fill in,”
$W^a_{5,8} = W^a_{8,8} =$ “There are students whose research must be evaluated by the current teacher”,
$W^a_{5,9} = W^a_{7,9} = \neg W^a_{5,7}$,
$W^a_{5,10} = W^a_{8,10} = \neg W^a_{5,8}$.

The α- and φ-tokens do not obtain any characteristic in places a_7, a_8 and d_9.

The α-tokens that enter places a_9 and a_{10} obtain characteristics respectively:

“the averaged evaluations of the student’s answers in the lecturers’ investigation, according (6) and (7)”

in place a_9, and

“the averaged evaluations of the degrees of comprehension and incomprehension of the current course, according (1) and (2)”

in place a_{10}.

$$Z_{10} = \langle \{a_6, a_9, a_{10}, a_{11}, b_4, c_4, d_9\}, \{a_{11}, a_{12}, b_5, c_5, d_{10}\},
\begin{array}{c|ccccc}
 & a_{11} & a_{12} & b_5 & c_5 & d_{10} \\ \hline
a_6 & true & false & false & false & true \\
a_9 & true & false & false & false & true \\
a_{10} & true & false & false & false & true \\
a_{11} & W^a_{11,11} & W^a_{11,12} & false & false & false \\
b_4 & false & false & true & false & true \\
c_4 & false & false & false & true & true \\
d_9 & false & false & false & false & true
\end{array} \rangle,$$

where:

$W^a_{11,11} =$ “There are tokens from places a_6, a_9 and a_{10}”,
$W^a_{11,12} = \neg W^a_{11,11}$.

The α-tokens that enter place a_{11} from places a_6, a_9 and a_{10} merge in new α-token that enter place a_{12} with characteristic

“The evaluation of the s-th lecturer, according (15) and (16).”

4. Conclusions

The GN-model constructed in manner described above offers the opportunity to simulate many of the processes which need to be considered when carrying out summative assessments of the progress of the students and the appraisal of their lecturers. The present model is thus an important element of a more general model to describe the different information flows within a university. As such it complements the pioneering work of Alf Pollard [6] in this field.

References

1. Atanassov, K. Generalized Nets. World Scientific, 1991.

2. Atanassov, K. On Generalized Nets Theory. Sofia, Prof. M. Drinov Academic Publ. House, 2007.

3. Atanassov, K. Intuitionistic Fuzzy Sets. Berlin, Springer Physica-Verlag, 1999.

4. Atanassov, K., Intuitionistic Fuzzy Sets. – Fuzzy Sets and Systems, Vol. **20**, 1986, 87-96 .

5. Melo-Pinto, P., T. Kim, K. Atanassov, E. Sotirova, A. Shannon, M. Krawczak, Generalized Net Model of e-Learning Evaluation with Intuitionistic Fuzzy Estimations. – In: Issues in the Representation and Processing of Uncertain and Imprecise Information, Warszawa, 2005, 241-249.

6. Pollard, A. H. Some Hypothetical Models in Systems of Tertiary Education. – The Australian Journal of Statistics, Vol. **12**, 1970, 78-91.

7. Shannon, A., D. Langova-Orozova, E. Sotirova, I. Petrounias, K. Atanassov, M. Krawczak, P. Melo-Pinto, T. Kim. Generalized Net Modelling of University Processes. – In: KvB Visual Concepts Pty Ltd, Monograph No 7, Sydney, 2005.

8. Shannon, A., K. Atanassov, E. Sotirova, D. Langova-Orozova, M. Krawczak, P. Melo-Pinto, I. Petrounias, T. Kim. – In: Generalized Nets and Information Flow Within a University, Warszawa, 2007.

9. Shannon, A., E. Sotirova, I. Petrounias, K. Atanassov, M.Krawczak, P. Melo-Pinto, T. Kim. Intuitionistic Fuzzy Estimations of Lecturers' Evaluation of Student Work. – In: First International Workshop on Intuitionistic Fuzzy Sets, Generalized Nets & Knowledge Engineering, University of Westminster, London, 6-7 September 2006, 44-47.

10. Shannon, A., E. Sotirova, I. Petrounias, K. Atanassov, M. Krawczak, P. Melo-Pinto, T. Kim. Generalized Net Model of Lecturers' Evaluation of Student Work with Intuitionistic Fuzzy Estimations. – In: Second International Workshop on Intuitionistic Fuzzy Sets, Banska Bystrica, Slovakia, 3 December 2006, Notes on IFS, Vol. **12**, 2006, No 4, 22-28.

11. Shannon, A., E. Sotirova, K. Atanassov, M. Krawczak, P. Melo-Pinto, T. Kim. Generalized Net Model for the Reliability and Standardization of Assessments of Student Problem Solving with Intuitionistic Fuzzy Estimations. Developments in Fuzzy Sets, Generalized Nets and Related Topics – Applications, Vol. **2**, System Research Institute, Polish Academy of Science, 2008, 249-256.

12. Shannon, A., E. Sotirova, K. Atanassov, M. Krawczak, T. Kim. Generalized Net Model of a Student's Course Evaluation with Intuitionistic Fuzzy Estimations. – Advanced Studies in Contemporary Mathematics, Vol. **18**, No 2, 2009, (in Press).