BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **9**, No 2

Sofia • 2009

Intuitionistic Fuzzy Feed Forward Neural Network

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***Abstract*:** *We propose a feed forward neural network using elements of the intuitionistic fuzzy logic. The parameters of these nets use intuitionistic fuzzy characteristics.*

***Keywords*:** *Intuitionistic fuzzy logic, Feed forward neural network.*

1. Introduction

The possibility for combination of ideas of Neural Networks (NNs) and Intuitionistic Fuzzy Logic (IFL) are discussed in [6, 7, 11].

In the present paper we show that the concepts of Feed Forword Neural Networks (FFNNs) and IFL can also be combined. All these types of NNs have or allow applications in the area of Artificial Intelligence.

Artificial Neural Network (ANN)[8, 9], often just called a NN is a mathematical or computational model based on biological neural networks. It consists of an interconnected group of artificial neurons and processes information using a connectionist approach to computation.

The formal description of the ANN is the following:

$$\langle\langle p_1, p_2, ..., p_{n_0}\rangle^{\mathrm{T}}, \{\langle a_1^i, a_2^i, ..., a_{n_i}^i\rangle^{\mathrm{T}} | 1 \le i \le k\},$$

$$\{\langle W_{1,1}^j, W_{1,2}^j, ..., W_{1,n_0}^j; W_{2,1}^j, W_{2,2}^j, ..., W_{2,n_1}^j; ...;$$

$$W^j_{n_{k-1},1}, W^j_{n_{k-1},2}, ..., W^j_{n_{k-1},n_k}; \rangle | 0 \leq j \leq k\},$$

$$\{\langle b^i_1, b^i_2, ..., b^i_{n_i}\rangle^{\mathrm{T}} | 1 \leq i \leq k\}, \{\langle F^i_1, F^i_2, ..., F^i_{n_i}\rangle^{\mathrm{T}} | 1 \leq i \leq k\}\rangle,$$

where:

T – operation "transposition";

k – number of layers of neurons;

n_k – number of neurons in the layer with number k (n_0 is a number of the zero (input) layer);

$p_1, p_2, ..., p_{n_0}$ – input values for nodes from layer 1;

$a^i_1, a^i_2, ..., a^i_{n_i}$ – outputs for the neurons on i-th layer $W^z_{x,y}$ – weight coefficient from the input node x to the neuron numbered with y on layer z;

$b^i_1, b^i_2, ..., b^i_{n_i}$ – bias coefficient for the neurons on i-th layer;

$F^i_1, F^i_2, ..., F^i_{n_i}$ – transfer function for the neurons on i-th layer.

In the present paper we will define a NN that uses intuitionistic fuzzy information and by this reason it is called an Intuitionistic Fuzzy Neural Network (IFNN). In the conclusion we will give a formal definition of an IFNN, which will extend of the above definition.

IFNN, which architecture is such that the neurons can be divided into layers and do not have any feedback, is called Intuitionistic Fuzzy Feed Forward Neural Network (IFFFNN).

A given FFNN can be extended with elements of IFL and in a result we can construct an Intuitionistic Fuzzy FFNN (IFFFNN).

2. Short remark on propositional intuitionistic fuzzy logic

Intuitionistic Fuzzy Logics are defined as extensions of ordinary fuzzy logic. Now there are propositional IFL, predicative IFL, modal IFL, temporal IFL and others. Here, following [2] we shall introduce only some definitions from the propositional IFL that are necessary for research below.

Two real numbers, $\mu(p)$ and $\nu(p)$, are assigned to the proposition p with the following constraint to hold:

$$\mu(p) + \nu(p) \leq 1.$$

They correspond to the degrees of truth and falsity of p.

Let this assignment be provided by an evaluation function V, defined over a set of propositions in such a way that:

$$V(p) = \langle \mu(p),\ \nu(p)\rangle\,.$$

When values $V(p)$ and $V(q)$ of the proposition forms p and q are known, the evaluation function V can be extended also for the operations "conjunction" (two forms: $\&$ and $\wedge$), "disjunction" (two forms: $\vee$ and $\sqcup$) and others, as follows:

$$V(p)\&V(q) = V(p\&q) = \langle min(\mu(p), \mu(q)), max(\nu(p), \nu(q))\rangle\,,$$

$$V(p) \wedge V(q) = V(p \wedge q) = \langle \mu(p).\mu(q), \nu(p) + \nu(q) - \nu(p).\nu(q)\rangle\,,$$

$$V(p) \vee V(q) = V(p \vee q) = \langle max(\mu(p), \mu(q)), min(\nu(p), \nu(q))\rangle\,.$$

$$V(p) \sqcup V(q) = V(p \sqcup q) = \langle \mu(p) + \mu(q) - \mu(p).\mu(q), \nu(p).\nu(q)\rangle\,.$$

3. Intuitionistic fuzzy feed forward neural network

A single-input neuron is shown on Fig. 1. The input p of an IFFFNN is represented by ordered pairs of real numbers from set $[0, 1]$. Intuitionistic Fuzzy Weight (IFW) w of the neuron (also represented by ordered pairs of real numbers from set $[0, 1]$) is multiplied by p to one of the following forms:

• strongly optimistic formula

$$wp \equiv \langle \mu_P, \nu_P \rangle = \langle \mu_w + \mu_p - \mu_w.\mu_p, \nu_w.\nu_p \rangle ;$$

• optimistic formula

$$wp \equiv \langle \mu_P, \nu_P \rangle = \langle \max(\mu_w, \mu_p), \min(\nu_w, \nu_p) \rangle;$$

• average formula

$$wp = \left\langle \frac{\mu_P + \mu_Q}{2}, \frac{\nu_P + \nu_Q}{2} \right\rangle ;$$

• pessimistic formula

$$wp \equiv \langle \mu_Q, \nu_Q \rangle = \langle \min(\mu_w, \mu_p), \max(\nu_w, \nu_p) \rangle ;$$

• strongly pessimistic formula

$$wp \equiv \langle \mu_Q, \nu_Q \rangle = \langle \mu_w.\mu_p, \nu_w + \nu_p - \nu_w.\nu_p \rangle .$$

It is one of the terms that is sent to the summator Σ. The other element that is passed to the summator, is multiplied by an IFL-bias b represented by an ordered pair $\langle \mu_b, \nu_b \rangle$, where μ_b and ν_b are real numbers from set $[0, 1]$ and $\mu_b + \nu_b \leq 1$.

The summator output n has the form

$$n = \left\langle \frac{\mu_p + \mu_Q + \mu_b}{3}, \frac{\nu_p + \nu_Q + \nu_b}{3} \right\rangle = \langle \mu_n, \nu_n \rangle .$$

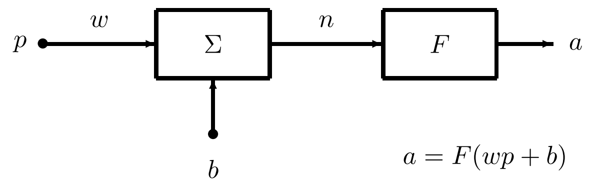

Fig. 1

The summator output n acts as an input for the transfer function F, which produces the neuron's output a. The neuron's output is calculated as

$$a = F(wp + b).$$

In the classical feed forward neural network two types of transferring functions with the form $a = F(n)$ are used: linear and logical sigmoid.

The output of a linear transfer function is equal to the inputs: $a = n$. Here, it will have the form

$$a = F(n) = \langle \mu_n, \nu_n \rangle.$$

The output of a logical sigmoid transfer function is equal to the output in the range $[0, 1]$ according to the expression:

$$a = \frac{1}{1 + e^{-n}}.$$

Now, we construct the couple

$$F_{\text{sigm}} = \left\langle \frac{\varepsilon}{1 + \frac{1}{1 + e^{\mu_n}}}, \frac{\varepsilon}{1 + \frac{1}{1 + e^{\nu_n}}} \right\rangle,$$

where

$$\varepsilon = \frac{4}{e + 3}.$$

Therefore, F_{sigm} is an Intuitionistic Fuzzy Couple (IFC), because:

$$\frac{\varepsilon}{1 + \frac{1}{1 + e^{\mu_n}}} + \frac{\varepsilon}{1 + \frac{1}{1 + e^{\nu_n}}} = \varepsilon \left(\frac{e^{\mu_n}}{1 + e^{\mu_n}} + \frac{e^{\nu_n}}{1 + e^{\nu_n}} \right) =$$

$$= \varepsilon \frac{2e^{\mu_n + \nu_n} + e^{\mu_n} e^{\nu_n}}{e^{\mu_n + \nu_n} + e^{\mu_n} + e^{\nu_n} + 1} = \varepsilon \left(1 + \frac{e^{\mu_n + \nu_n} - 1}{e^{\mu_n + \nu_n} + e^{\mu_n} + e^{\nu_n} + 1} \right)$$

$$\leq \varepsilon \left(1 + \frac{e - 1}{4} \right) = \varepsilon \frac{e + 3}{4} = 1.$$

Let us have a neuron with R inputs. Let them have intuitionistic fuzzy values $\langle \mu_{p_1}, \nu_{p_1} \rangle$, $\langle \mu_{p_2}, \nu_{p_2} \rangle$, ..., $\langle \mu_{p_R}, \nu_{p_R} \rangle$. Let each input p have respective elements

$$p = \langle \langle \mu_{p_1}, \nu_{p_1} \rangle, \langle \mu_{p_2}, \nu_{p_2} \rangle, ..., \langle \mu_{p_R}, \nu_{p_R} \rangle \rangle. \tag{1}$$

with weight coefficient from the IFW-matrix w

$$w = \langle \langle \mu_{w_{1,1}}, \nu_{w_{1,1}} \rangle, \langle \mu_{w_{1,2}}, \nu_{w_{1,2}} \rangle, ..., \langle \mu_{w_{1,R}}, \nu_{w_{1,R}} \rangle \rangle.$$

Thus, the indices in say that weight $\langle \mu_{w_{1,2}}, \nu_{w_{1,2}} \rangle$ represents the connection to the first neuron from the second source.

A neuron with R inputs is shown on Fig. 2.

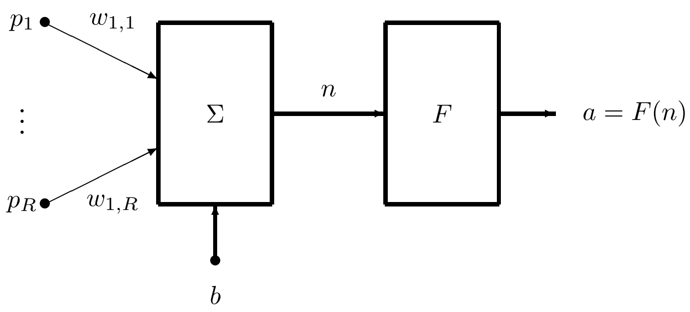

Fig. 2

In this case

$$n = \left\langle \frac{\sum\limits_{i=1}^{R} \frac{\mu_{P_i}+\mu_{Q_i}}{R} + \mu_b}{2}, \frac{\sum\limits_{i=1}^{R} \frac{\nu_{P_i}+\nu_{Q_i}}{R} + \nu_b}{2} \right\rangle = \langle \mu_n, \nu_n \rangle .$$

A single-layer network of S neurons is shown on Fig. 3, but each of its inputs is connected to each of the neurons.

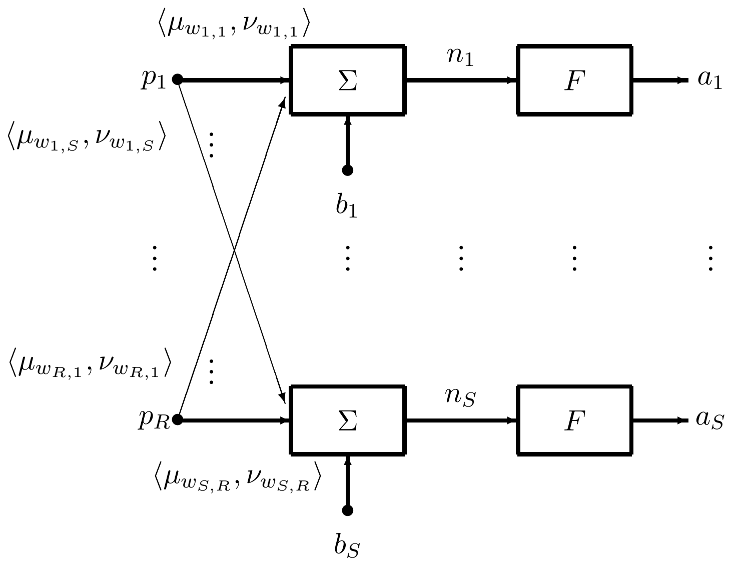

Fig. 3

The layer includes the IFW-matrix of the form

$$W = \left\| \begin{array}{cccc} \langle \mu_{w_{1,1}}, \nu_{w_{1,1}} \rangle & \langle \mu_{w_{1,2}}, \nu_{w_{1,2}} \rangle & \dots & \langle \mu_{w_{1,R}}, \nu_{w_{1,R}} \rangle \\ \langle \mu_{w_{2,1}}, \nu_{w_{2,1}} \rangle & \langle \mu_{w_{2,2}}, \nu_{w_{2,2}} \rangle & \dots & \langle \mu_{w_{2,R}}, \nu_{w_{2,R}} \rangle \\ \dots & \dots & \dots & \dots \\ \langle \mu_{w_{S,1}}, \nu_{w_{S,1}} \rangle & \langle \mu_{w_{S,2}}, \nu_{w_{S,2}} \rangle & \dots & \langle \mu_{w_{S,R}}, \nu_{w_{S,R}} \rangle \end{array} \right\| \tag{2}$$

is summed with the IF bias vector

$$b = \langle\langle\mu_{b_1}, \nu_{b_1}\rangle, \langle\mu_{b_2}, \nu_{b_2}\rangle, ..., \langle\mu_{b_S}, \nu_{b_S}\rangle\rangle. \tag{3}$$

In a result, the transfer function obtains the output vector

$$a = \langle\langle\mu_{a_1}, \nu_{a_1}\rangle, \langle\mu_{a_2}, \nu_{a_2}\rangle, ..., \langle\mu_{a_S}, \nu_{a_S}\rangle\rangle \tag{4}$$

Each element of the input IF-vector p from (1) is connected to each neuron through the IFW-matrix (2). Each neuron has a IF-bias from (3), a summator, a transfer function F and an IF output a from (4).

It is common for the number of inputs to a layer to be different from the number of neurons (i.e., $R \neq S$).

4. Conclusion

The so constructed IFFFNNs are extensions of the FANNs and ANNs. In the notation from first section we can give a formal description of an IFFFNN. It is the following:

$$\langle\langle\langle\mu_{p_1}, \nu_{p_1}\rangle, \langle\mu_{p_2}, \nu_{p_2}\rangle...., \langle\mu_{p_{n_0}}, \nu_{p_{n_0}}\rangle\rangle^{\mathrm{T}}, \{\langle a_1^i, a_2^i, ..., a_{n_i}^i\rangle^{\mathrm{T}} | 1 \leq i \leq k\},$$

$$\{\langle\langle\mu^j_{w_{1,1}}, \nu^j_{w_{1,1}}\rangle, ..., \langle\mu^j_{w_{1,n_0}}, \nu^j_{w_{1,n_0}}\rangle; \langle\mu^j_{w_{2,1}}, \nu^j_{w_{2,1}}\rangle, ..., \langle\mu^j_{w_{2,n_1}}, \nu^j_{w_{2,n_1}}\rangle, ...,$$

$$\langle\mu^j_{w_{n_{k-1},1}}, \nu^j_{w_{n_{k-1},n_k}}\rangle\rangle | 0 \leq j \leq k\},$$

$$\{\langle\langle\mu^i_{b_1}, \nu^i_{b_1}\rangle, \langle\mu^i_{b_S}, \nu^i_{b_S}\rangle\rangle^{\mathrm{T}} | 1 \leq i \leq k\},\ \{\langle F_1^i, F_2^i, ..., F_{n_i}^i\rangle^{\mathrm{T}} | 1 \leq i \leq k\}\rangle,$$

where all parameters are described below.

In a next research the authors will construct a generalized net (see [1, 3]) that is universal for the set of all IFFFNNs. So, we will continue the research from [4, 5, 10, 12-21].

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