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On One of Baczynski-Jayaram's Problems

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***Abstract*:** *Pairs of implications and negations that are solutions to one of Baczynski-Jayaram's problems are constructed.*

***Keywords*:** *Implication, Intuitionistic fuzzy implication, Intuitionistic fuzzy negation, Negation.*

In [6] Michal Baczynski and Balasubramaniam Jayaram formulated some problems related to fuzzy implications I and negations N. Here we give a solution to one of them:

Problem 1.7.1. Give examples of fuzzy implications I such that:

(i) I satisfies only property

(CP) $$I(x, y) = I(N(y), N(x)),$$

(ii) I satisfies only property

(L-CP) $$I(N(x), y) = I(N(y), x),$$

(iii) I satisfies both (CP) and (L-CP), but not

(R-CP) $$I(x, N(y)) = I(y, N(x))$$

with some fuzzy negation N, where $x, y \in [0, 1]$.

We must note that in [6] no example is given. In a series of papers (starting with [1, 3]) 138 Intuitionistic Fuzzy (IF) implications were defined (see Table 1) and some of their basic properties were studied. A part of the research is devoted to implications in IF logic and the rest – to the implications defined over IF Sets (IFSs, see [2, 4, 5]). In both cases, for each its corresponding negation is also constructed. At the moment there are 34 different negations (see Table 2). The relations between the negations and implications are shown on Table 3.

Table 1

$\rightarrow_2$	$\langle x, \overline{\text{sg}}(a-c), d.\text{sg}(a-c)\rangle \vert x \in E\}$
$\rightarrow_3$	$\langle x, 1-(1-c).\text{sg}(a-c))), d.\text{sg}(a-c)\rangle$
$\rightarrow_4$	$\langle x, \max(b,c), \min(a,d)\rangle$
$\rightarrow_5$	$\langle x, \min(1, b+c), \max(0, a+d-1)\rangle$
$\rightarrow_7$	$\langle x, \min(\max(b,c), \max(a,b), \max(c,d)),$ $\max(\min(a,d), \min(a,b), \min(c,d))\rangle$
$\rightarrow_8$	$\langle x, 1-(1-\min(b,c)).\text{sg}(a-c), \max(a,d).\text{sg}(a-c).\text{sg}(d-b)\rangle$
$\rightarrow_{11}$	$\langle x, 1-(1-c).\text{sg}(a-c), d.\text{sg}(a-c).\text{sg}(d-b)\rangle$
$\rightarrow_{12}$	$\langle x, \max(b,c), 1-\max(b,c)\rangle$
$\rightarrow_{13}$	$\langle x, b+c-b.c, a.d\rangle$
$\rightarrow_{14}$	$\langle x, 1-(1-c).\text{sg}(a-c)-d.\overline{\text{sg}}(a-c).\text{sg}(d-b), d.\text{sg}(d-b)\rangle$
$\rightarrow_{15}$	$\langle x, 1-(1-\min(b,c)).\text{sg}(a-c).\text{sg}(d-b)$ $-\min(b,c).\text{sg}(a-c).\text{sg}(d-b),\ 1-(1-\max(a,d)).\text{sg}(\overline{\text{sg}}(a-c)$ $+\overline{\text{sg}}(d-b))-\max(a,d).\overline{\text{sg}}(a-c).\overline{\text{sg}}(d-b)\rangle$
$\rightarrow_{16}$	$\langle x, \max(\overline{\text{sg}}(a), c), \min(\text{sg}(a), d)\rangle$
$\rightarrow_{17}$	$\langle x, \max(b,c), \min(a.b+a^2, d)\rangle$
$\rightarrow_{18}$	$\langle x, \max(b,c), \min(1-b, d)\rangle$
$\rightarrow_{19}$	$\langle x, \max(1-\text{sg}(\text{sg}(a)+\text{sg}(1-b)), c), \min(\text{sg}(1-b), d)\rangle$
$\rightarrow_{20}$	$\langle x, \max(\overline{\text{sg}}(a), \text{sg}(c)), \min(\text{sg}(a), \overline{\text{sg}}(c))\rangle$
$\rightarrow_{21}$	$\langle x, \max(b, c.(c+d)), \min(a.(a+b), d.(c^2+d+c.\nu_B(x)))\rangle$
$\rightarrow_{22}$	$\langle x, \max(b, 1-d), \min(1-b, d)\rangle$
$\rightarrow_{23}$	$\langle x, 1-\min(\text{sg}(1-b), \overline{\text{sg}}(1-d)), \min(\text{sg}(1-b), \overline{\text{sg}}(1-d))\rangle$
$\rightarrow_{24}$	$\langle x, \overline{\text{sg}}(a-c).\overline{\text{sg}}(d-b), \text{sg}(a-c).\text{sg}(d-b)\rangle$
$\rightarrow_{25}$	$\langle x, \max(b, \overline{\text{sg}}(a).\overline{\text{sg}}(1-b), c.\overline{\text{sg}}(d).\overline{\text{sg}}(1-c)), \min(a,d)\rangle$
$\rightarrow_{26}$	$\langle x, \max(\overline{\text{sg}}(1-b), c), \min(\text{sg}(a), d)\rangle$
$\rightarrow_{27}$	$\langle x, \max(\overline{\text{sg}}(1-b), \text{sg}(c)), \min(\text{sg}(a), \overline{\text{sg}}(1-d))\rangle$
$\rightarrow_{28}$	$\langle x, \max(\overline{\text{sg}}(1-b), c), \min(a,d)\rangle$
$\rightarrow_{29}$	$\langle x, \max(\overline{\text{sg}}(1-b), \overline{\text{sg}}(1-c)), \min(a, \overline{\text{sg}}(1-d))\rangle$
$\rightarrow_{30}$	$\langle x, \max(1-a, \min(a, 1-d)), \min(a,d)\rangle$
$\rightarrow_{31}$	$\langle x, \overline{\text{sg}}(a+d-1), d.\text{sg}(a+d-1)\rangle$
$\rightarrow_{32}$	$\langle x, 1-d.\text{sg}(a+d-1), d.\text{sg}(a+d-1)\rangle$
$\rightarrow_{33}$	$\langle x, 1-\min(a,d), \min(a,d)\rangle$
$\rightarrow_{34}$	$\langle x, \min(1, 2-a-d), \max(0, a+d-1)\rangle$
$\rightarrow_{35}$	$\langle x, 1-a.d, a.d\rangle$
$\rightarrow_{36}$	$\langle x, \min(1-\min(a,d), \max(a,(1-a), \max(1-d,d)),$ $\max(\min(a,d), \min(a, 1-a), \min(1-d,d))\rangle$
$\rightarrow_{37}$	$\langle x, 1-\max(a,d).\text{sg}(a+d-1), \max(a,d).\text{sg}(a+d-1)\rangle$
$\rightarrow_{39}$	$\langle x, (1-d).\overline{\text{sg}}(1-a)+\text{sg}(1-a).(\overline{\text{sg}}(d)+(1-a).\text{sg}(d)),$ $d.\overline{\text{sg}}(1-a)+a.\text{sg}(1-a).\text{sg}(d)\rangle$
$\rightarrow_{40}$	$\langle x, 1-\text{sg}(a+d-1), 1-\overline{\text{sg}}(a+d-1)\rangle$
$\rightarrow_{41}$	$\langle x, \max(\overline{\text{sg}}(a), (1-d)), \min(\text{sg}(a), d)\rangle$
$\rightarrow_{42}$	$\langle x, \max(\overline{\text{sg}}(a), \text{sg}(1-d)), \min(\text{sg}(a), \overline{\text{sg}}(1-d))\rangle$

$\rightarrow_{43}$	$\langle x, \max(\overline{\text{sg}}(a), 1-d), \min(\text{sg}(a), d)\rangle$
$\rightarrow_{44}$	$\langle x, \max(\overline{\text{sg}}(a), 1-d), \min(a, d)\rangle$
$\rightarrow_{45}$	$\langle x, \max(\overline{\text{sg}}(a), \overline{\text{sg}}(d)), \min(a, \overline{\text{sg}}(1-d))\rangle$
$\rightarrow_{47}$	$\langle x, \overline{\text{sg}}(1-b-c), (1-c).\text{sg}(1-b-c)\rangle$
$\rightarrow_{48}$	$\langle x, 1-(1-c).\text{sg}(1-b-c), (1-c).\text{sg}(1-b-c)\rangle$
$\rightarrow_{49}$	$\langle x, \min(1, b+c), \max(0, 1-b-c)\rangle$
$\rightarrow_{50}$	$\langle x, b+c-b.c, 1-b-c+b.c\rangle$
$\rightarrow_{51}$	$\langle x, \min(\max(b,c), \max(1-b), b), \max(c, 1-c)),$ $\max(1-\max(b,c), \min(1-b, b), \min(c, 1-c))\rangle$
$\rightarrow_{52}$	$\langle x, 1-(1-\min(b,c)).\text{sg}(1-b-c), 1-\min(b,c).\text{sg}(1-b-c)\rangle$
$\rightarrow_{55}$	$\langle x, 1-\text{sg}(1-b-c), 1-\overline{\text{sg}}(1-b)-c)\rangle$
$\rightarrow_{56}$	$\langle x, \max(\overline{\text{sg}}(1-b), c), \min(\text{sg}(1-b), (1-c))\rangle$
$\rightarrow_{57}$	$\langle x, \max(\overline{\text{sg}}(1-b), \text{sg}(c)), \min(\text{sg}(1-b), \overline{\text{sg}}(c))\rangle$
$\rightarrow_{58}$	$\langle x, \max(\overline{\text{sg}}(1-b), \overline{\text{sg}}(1-c)), 1-\max(b,c)\rangle$
$\rightarrow_{62}$	$\langle x, \overline{\text{sg}}(d-b), a.\text{sg}(d-b)\rangle$
$\rightarrow_{63}$	$\langle x, 1-(1-b).\text{sg}(d-b), a.\text{sg}(d-b)\rangle$
$\rightarrow_{65}$	$\langle x, 1-(1-\min(c,b)).\text{sg}(d-b), \max(d,a).\text{sg}(d-b).\text{sg}(a-c)\rangle$
$\rightarrow_{68}$	$\langle x, 1-(1-b).\text{sg}(d-b), a.\text{sg}(d-b).\text{sg}(a-c)\rangle$
$\rightarrow_{69}$	$\langle x, 1-(1-b).\text{sg}(d-b)-a.\overline{\text{sg}}(d-b).\text{sg}(a-c), a.\text{sg}(a-c)\rangle$
$\rightarrow_{70}$	$\langle x, \max(\overline{\text{sg}}(d), b), \min(\text{sg}(d), a)\rangle$
$\rightarrow_{71}$	$\langle x, \max(c,b), \min(d.c+d^2, a)\rangle$
$\rightarrow_{72}$	$\langle x, \max(c,b), \min(1-c, a)\rangle$
$\rightarrow_{73}$	$\langle x, \max(1-\max(\text{sg}(d), \text{sg}(1-c)), b), \min(\text{sg}(1-c), a)\rangle$
$\rightarrow_{74}$	$\langle x, \max(\overline{\text{sg}}(d), \text{sg}(b)), \min(\text{sg}(d), \overline{\text{sg}}(b))\rangle$
$\rightarrow_{76}$	$\langle x, \max(c, 1-a), \min(1-c, a)\rangle$
$\rightarrow_{77}$	$\langle x, 1-\min(\text{sg}(1-c), \overline{\text{sg}}(1-a)), \min(\text{sg}(1-c), \overline{\text{sg}}(1-a))\rangle$
$\rightarrow_{78}$	$\langle x, \max(\overline{\text{sg}}(1-c), b), \min(\text{sg}(d), a)\rangle$
$\rightarrow_{79}$	$\langle x, \max(\overline{\text{sg}}(1-c), \text{sg}(b)), \min(\text{sg}(d), \overline{\text{sg}}(1-a))\rangle$
$\rightarrow_{81}$	$\langle x, \max(\overline{\text{sg}}(1-c), \overline{\text{sg}}(1-b)), \min(d, \overline{\text{sg}}(1-a))\rangle$
$\rightarrow_{82}$	$\langle x, \max(1-d, \min(d, 1-a)), \min(d, a)\rangle$
$\rightarrow_{83}$	$\langle x, \overline{\text{sg}}(d+a-1), a.\text{sg}(d+a-1)\rangle$
$\rightarrow_{84}$	$\langle x, 1-a.\text{sg}(d+a+1), a.\text{sg}(d+a+1)\rangle$
$\rightarrow_{85}$	$\langle x, 1-d+d^2.(1-a), d.(1-d)+d^2.\rangle$
$\rightarrow_{86}$	$\langle x, (1-a).\overline{\text{sg}}(1-d)+\text{sg}(1-d).\overline{\text{sg}}(a+\min(1-d, \text{sg}(a))),$ $a.\overline{\text{sg}}(1-d)+d.\text{sg}(1-d).\text{sg}(a)\rangle$
$\rightarrow_{87}$	$\langle x, \max(\overline{\text{sg}}(d), (1-a)), \min(\text{sg}(d), a)\rangle$
$\rightarrow_{88}$	$\langle x, \max(\overline{\text{sg}}(d), \text{sg}(1-a)), \min(\text{sg}(d), \overline{\text{sg}}(1-a))\rangle$
$\rightarrow_{89}$	$\langle x, \max(\overline{\text{sg}}(d), (1-a)), \min(d, a)\rangle$
$\rightarrow_{90}$	$\langle x, \max(\overline{\text{sg}}(d), \overline{\text{sg}}(a)), \min(d, \overline{\text{sg}}(1-a))\rangle$
$\rightarrow_{92}$	$\langle x, \overline{\text{sg}}((1-c)-b), \min((1-b), \text{sg}((1-c)-b))\rangle$
$\rightarrow_{93}$	$\langle x, (1-\min((1-b), \text{sg}((1-c)-b))), \min((1-b), \text{sg}((1-c)-b))\rangle$
$\rightarrow_{96}$	$\langle x, \max(\overline{\text{sg}}(1-c), b), \min(\text{sg}(1-c), (1-b))\rangle$
$\rightarrow_{97}$	$\langle x, \max(\overline{\text{sg}}(1-c), \text{sg}(b)), \min(\text{sg}(1-c), \overline{\text{sg}}(b))\rangle$
$\rightarrow_{100}$	$\langle x, \max(\min(b, \text{sg}(a)), c), \min(\min(a, \text{sg}(b)), d)\rangle$
$\rightarrow_{101}$	$\langle x, \max(\min(b, \text{sg}(a)), \min(c, \text{sg}(d))),$ $\min(\min(a, \text{sg}(b)), \min(d, \text{sg}(c)))\rangle$
$\rightarrow_{102}$	$\langle x, \max(b, \min(c, \text{sg}(d))), \min(a, \min(d, \text{sg}(c)))\rangle$

$\rightarrow_{103}$	$\langle x, \max(\min((1-a), \text{sg}(a)), (1-d)), \min(\min(a, \text{sg}(1-a)), d)\rangle$
$\rightarrow_{104}$	$\langle x, \max(\min((1-a), \text{sg}(a)), \min((1-d), \text{sg}(d))),$ $\min(\min(a, \text{sg}(1-a)), \min(d, \text{sg}(1-d)))\rangle$
$\rightarrow_{107}$	$\langle x, \max(\min(b, \text{sg}(1-b)), \min(c, \text{sg}(1-c))),$ $\min(\min((1-b), \text{sg}(b)), \min((1-c), \text{sg}(c)))\rangle$
$\rightarrow_{109}$	$\langle x, b + \min(\overline{\text{sg}}(1-a), c), a.b + \min(\overline{\text{sg}}(1-a), d))\rangle$
$\rightarrow_{110}$	$\langle x, \max(b, c), \min(a.b + \overline{\text{sg}}(1-a), d)\rangle$
$\rightarrow_{111}$	$\langle x, \max(b, c.d + \overline{\text{sg}}(1-c)), \min(a.b + \overline{\text{sg}}(1-a), d.(c.d$ $+\overline{\text{sg}}(1-c))) + \overline{\text{sg}}(1-d)))\rangle$
$\rightarrow_{112}$	$\langle x, b + c - b.c, a.b + \overline{\text{sg}}(1-a).d\rangle$
$\rightarrow_{115}$	$\langle x, 1 - \min(a, d), \min((a.(1-a) + \overline{\text{sg}}(1-a)), d)\rangle$
$\rightarrow_{116}$	$\langle x, \max(1-a, (1-d).d + \overline{\text{sg}}(d)), \min(a.(1-a) + \overline{\text{sg}}(1-a),$ $d.((1-d).d + \overline{\text{sg}}(d)) + \overline{\text{sg}}(1-d))\rangle$
$\rightarrow_{117}$	$\langle x, 1 - a.d, (a.(1-a) + \overline{\text{sg}}(1-a)).d\rangle$
$\rightarrow_{118}$	$\langle x, 1 - a + (1-d).d - (1-a).((1-d).d + \overline{\text{sg}}(d)),$ $(a.(1-a) + \overline{\text{sg}}(1-a)).(d.((1-d).d + \overline{\text{sg}}(d)) + \overline{\text{sg}}(1-d))\rangle$
$\rightarrow_{125}$	$\langle x, \max(c, b), \min(((d.c) + \overline{\text{sg}}(1-d)), a)\rangle$
$\rightarrow_{127}$	$\langle x, ((c+b) - (c.b)), (((d.c) + \overline{\text{sg}}(1-d)).a)\rangle$
$\rightarrow_{129}$	$\langle x, ((1-d) + \min(\overline{\text{sg}}(1-d), (1-a))),$ $((d.(1-d)) + \min(\overline{\text{sg}}(1-d), a))\rangle$
$\rightarrow_{130}$	$\langle x, (1 - \min(d, a)), \min(((d.(1-d)) + \overline{\text{sg}}(1-d)), a)\rangle$
$\rightarrow_{131}$	$\langle x, \max(1-d, (1-a).a + \overline{\text{sg}}(a)),$ $\min(d.(1-d) + \overline{\text{sg}}(1-d), a.((1-a).a + \overline{\text{sg}}(a)) + \overline{\text{sg}}(1-a))\rangle$
$\rightarrow_{132}$	$\langle x, 1 - a.d, (d.(1-d) + \overline{\text{sg}}(1-d)).a\rangle$

Table 2

$\neg_1$	$\langle x, b, a\rangle$
$\neg_2$	$\langle x, \overline{\text{sg}}(a), \text{sg}(a)\rangle$
$\neg_3$	$\langle x, b, a.b + a^2\rangle$
$\neg_4$	$\langle x, b, 1-b\rangle$
$\neg_5$	$\langle x, \overline{\text{sg}}(1-b), \text{sg}(1-b)\rangle$
$\neg_6$	$\langle x, \overline{\text{sg}}(1-b), \text{sg}(a)\rangle$
$\neg_7$	$\langle x, \overline{\text{sg}}(1-b), a\rangle$
$\neg_8$	$\langle x, 1-a, a\rangle$
$\neg_9$	$\langle x, \overline{\text{sg}}(a), a\rangle$
$\neg_{10}$	$\langle x, \overline{\text{sg}}(1-b), 1-b\rangle$
$\neg_{11}$	$\langle x, \text{sg}(b), \overline{\text{sg}}(b)\rangle$
$\neg_{13}$	$\langle x, \text{sg}(1-a), \overline{\text{sg}}(1-a)\rangle$
$\neg_{14}$	$\langle x, \text{sg}(b), \overline{\text{sg}}(1-a)\rangle$
$\neg_{15}$	$\langle x, \overline{\text{sg}}(1-b), \overline{\text{sg}}(1-a)\rangle$
$\neg_{16}$	$\langle x, \overline{\text{sg}}(a), \overline{\text{sg}}(1-a)\rangle$
$\neg_{17}$	$\langle x, \overline{\text{sg}}(1-b), \overline{\text{sg}}(b)\rangle$
$\neg_{18}$	$\langle x, \min(b, \text{sg}(a)), \min(a, \text{sg}(b))\rangle$
$\neg_{19}$	$\langle x, \min(b, \text{sg}(a)), 0\rangle$
$\neg_{20}$	$\langle x, b, 0\rangle$

$\neg_{21}$	$\langle x, \min(1-a, \text{sg}(a)), \min(a, \text{sg}(1-a))\rangle$
$\neg_{22}$	$\langle x, \min((1-a), \text{sg}(a)), 0\rangle$
$\neg_{23}$	$\langle x, 1-a, 0\rangle$
$\neg_{24}$	$\langle x, \min(b, \text{sg}(1-b)), \min(1-b, \text{sg}(b))\rangle$
$\neg_{25}$	$\langle x, \min(b, \text{sg}(1-b)), 0\rangle$
$\neg_{26}$	$\langle x, b, a.b + \overline{\text{sg}}(1-a)\rangle$
$\neg_{27}$	$\langle x, 1-a, a.(1-a) + \overline{\text{sg}}(1-a)\rangle$
$\neg_{28}$	$\langle x, b, (1-b).b + \overline{\text{sg}}(b)\rangle$
$\neg_{30}$	$\langle x, b.a, a.(b.a + \overline{\text{sg}}(1-b)) + \overline{\text{sg}}(1-a)\rangle$
$\neg_{32}$	$\langle x, (1-a).a, a.((1-a).a) + \overline{\text{sg}}(a)) + \overline{\text{sg}}(1-a)\rangle$
$\neg_{34}$	$\langle x, b.(1-b), (1-b).(b.(1-b) + \overline{\text{sg}}(1-b)) + \overline{\text{sg}}(b)\rangle$

Table 3

$\neg_1$	$\to_1, \to_4, \to_5, \to_6, \to_7, \to_{10}, \to_{13}, \to_{61}, \to_{63}, \to_{64}, \to_{66}, \to_{67}, \to_{68}, \to_{69}, \to_{70}, \to_{71}, \to_{72}, \to_{73}, \to_{78}, \to_{80}, \to_{124}, \to_{125}, \to_{127}$
$\neg_2$	$\to_2, \to_3, \to_8, \to_{11}, \to_{16}, \to_{20}, \to_{31}, \to_{32}, \to_{37}, \to_{40}, \to_{41}, \to_{42}$
$\neg_3$	$\to_9, \to_{17}, \to_{21}$
$\neg_4$	$\to_{12}, \to_{18}, \to_{22}, \to_{46}, \to_{49}, \to_{50}, \to_{51}, \to_{53}, \to_{54}, \to_{91}, \to_{93}, \to_{94}, \to_{95}, \to_{96}, \to_{98}, \to_{134}, \to_{135}, \to_{137}$
$\neg_5$	$\to_{14}, \to_{15}, \to_{19}, \to_{23}, \to_{47}, \to_{48}, \to_{52}, \to_{55}, \to_{56}, \to_{57}$
$\neg_6$	$\to_{24}, \to_{26}, \to_{27}, \to_{65}$
$\neg_7$	$\to_{25}, \to_{28}, \to_{29}, \to_{62}$
$\neg_8$	$\to_{30}, \to_{33}, \to_{34}, \to_{35}, \to_{36}, \to_{38}, \to_{39}, \to_{76}, \to_{82}, \to_{84}, \to_{85}, \to_{86}, \to_{87}, \to_{89}, \to_{129}, \to_{130}, \to_{132}$

$\neg_9$	$\to_{43}, \to_{44}, \to_{45}, \to_{83}$
$\neg_{10}$	$\to_{58}, \to_{59}, \to_{60}, \to_{92}$
$\neg_{11}$	$\to_{74}, \to_{97}$
$\neg_{12}$	$\to_{75}$
$\neg_{13}$	$\to_{77}, \to_{88}$
$\neg_{14}$	$\to_{79}$
$\neg_{15}$	$\to_{81}$
$\neg_{16}$	$\to_{90}$
$\neg_{17}$	$\to_{99}$
$\neg_{18}$	$\to_{100}$
$\neg_{19}$	$\to_{101}$
$\neg_{20}$	$\to_{102}, \to_{108}$
$\neg_{21}$	$\to_{103}$

$\neg_{22}$	$\to_{104}$
$\neg_{23}$	$\to_{105}$
$\neg_{24}$	$\to_{106}$
$\neg_{25}$	$\to_{107}$
$\neg_{26}$	$\to_{109}, \to_{110}, \to_{111}, \to_{112}, \to_{113}$
$\neg_{27}$	$\to_{114}, \to_{115}, \to_{116}, \to_{117}, \to_{118}$
$\neg_{28}$	$\to_{119}, \to_{120}, \to_{121}, \to_{122}, \to_{123}$
$\neg_{29}$	$\to_{126}$
$\neg_{30}$	$\to_{128}$
$\neg_{31}$	$\to_{131}$
$\neg_{32}$	$\to_{133}$
$\neg_{33}$	$\to_{136}$
$\neg_{34}$	$\to_{138}$

Here we shall give examples of pairs of implications and negations that satisfy Problem 1.7.1 (ii) and other problems. Let us denote by the pair (m, n) the expression with m-th implication and n-th negation.

First, we shall formulate the following

Theorem 1. The pairs $(4,1)$, $(5,1)$, $(7,1)$, $(12,1)$, $(13,1)$, $(15,1)$, $(24,1)$, $(25,1)$, $(33,1)$, $(34,1)$, $(35,1)$, $(36,1)$, $(37,1)$, $(40,1)$, $(43,1)$, $(49,1)$, $(50,1)$, $(51,1)$, $(52,1)$, $(55,1)$, $(58,1)$, $(101,1)$, $(104,1)$, $(107,1)$, $(20,2)$, $(22,4)$, $(23,5)$, $(27,6)$, $(42,6)$, $(57,6)$, $(76,8)$, $(20,9)$, $(22,10)$, $(74,11)$, $(77,13)$, $(79,14)$,

$(88,14)$, $(97,14)$, $(20,16)$, $(74,17)$, $(101,18)$, $(76,23)$, $(76,27)$ satisfy the three axioms.

Theorem 2. The pairs $(52,7)$, $(55,7)$, $(52,15)$, $(55,15)$, $(88,19)$, $(33,20)$, $(34,20)$, $(35,20)$, $(37,20)$, $(40,20)$, $(43,20)$, $(88,22)$, $(88,25)$ satisfy two axioms and more exactly, they satisfy (L-CP) and (R-CP).

We had not found any pair of implication and negation that are solution of Problem 1.7.1 (iii).

Also, we had not found any pair of implication and negation that satisfy only the first axiom, i.e., we cannot give examples for the case of Problem 1.7.1 (i).

Another result of our search is

Theorem 3. The pairs $(2,2)$, $(3,2)$, $(8,2)$, $(11,2)$, $(16,2)$, $(31,2)$, $(32,2)$, $(37,2)$, $(40,2)$, $(41,2)$, $(42,2)$, $(12,3)$, $(17,3)$, $(49,3)$, $(50,3)$, $(51,3)$, $(52,3)$, $(55,3)$, $(58,3)$, $(107,3)$, $(12,4)$, $(18,4)$, $(49,4)$, $(50,4)$, $(51,4)$, $(52,4)$, $(55,4)$, $(58,4)$, $(107,4)$, $(14,5)$, $(15,5)$, $(19,5)$, $(47,5)$, $(48,5)$, $(52,5)$, $(55,5)$, $(56,5)$, $(57,5)$, $(24,6)$, $(26,6)$, $(31,6)$, $(32,6)$, $(37,6)$, $(40,6)$, $(41,6)$, $(47,6)$, $(48,6)$, $(52,6)$, $(55,6)$, $(56,6)$, $(25,7)$, $(28,7)$, $(33,7)$, $(34,7)$, $(35,7)$, $(36,7)$, $(37,7)$, $(40,7)$, $(43,7)$, $(47,7)$, $(48,7)$, $(56,7)$, $(57,7)$, $(104,7)$, $(33,8)$, $(34,8)$, $(35,8)$, $(36,8)$, $(37,8)$, $(40,8)$, $(43,8)$, $(104,8)$, $(33,9)$, $(34,9)$, $(35,9)$, $(36,9)$, $(37,9)$, $(40,9)$, $(43,9)$, $(104,9)$, $(47,10)$, $(48,10)$, $(52,10)$, $(55,10)$, $(56,10)$, $(57,10)$, $(97,11)$, $(88,13)$, $(47,15)$, $(48,15)$, $(56,15)$, $(57,15)$, $(81,15)$, $(88,15)$, $(88,16)$, $(47,17)$, $(48,17)$, $(52,17)$, $(55,17)$, $(56,17)$, $(57,17)$, $(23,18)$, $(42,18)$, $(100,18)$, $(22,19)$, $(23,19)$, $(31,19)$, $(32,19)$, $(33,19)$, $(34,19)$, $(35,19)$, $(37,19)$, $(39,19)$, $(40,19)$, $(41,19)$, $(42,19)$, $(43,19)$, $(44,19)$, $(45,19)$, $(62,19)$, $(63,19)$, $(65,19)$, $(68,19)$, $(70,19)$, $(74,19)$, $(82,19)$, $(83,19)$, $(84,19)$, $(85,19)$, $(86,19)$, $(87,19)$, $(89,19)$, $(90,19)$, $(100,19)$, $(103,19)$, $(115,19)$, $(116,19)$, $(117,19)$, $(118,19)$, $(129,19)$, $(130,19)$, $(131,19)$, $(132,19)$, $(4,20)$, $(5,20)$, $(12,20)$, $(13,20)$, $(17,20)$, $(18,20)$, $(22,20)$, $(23,20)$, $(25,20)$, $(29,20)$, $(31,20)$, $(32,20)$, $(39,20)$, $(41,20)$, $(42,20)$, $(44,20)$, $(45,20)$, $(49,20)$, $(50,20)$, $(51,20)$, $(52,20)$, $(55,20)$, $(58,20)$, $(62,20)$, $(63,20)$, $(65,20)$, $(68,20)$, $(70,20)$, $(71,20)$, $(74,20)$, $(81,20)$, $(82,20)$, $(83,20)$, $(84,20)$, $(85,20)$, $(86,20)$, $(87,20)$, $(88,20)$, $(89,20)$, $(90,20)$, $(103,20)$, $(107,20)$, $(110,20)$, $(112,20)$, $(115,20)$, $(116,20)$, $(117,20)$, $(118,20)$, $(125,20)$, $(127,20)$, $(129,20)$, $(130,20)$, $(131,20)$, $(132,20)$, $(23,21)$, $(42,21)$, $(104,21)$, $(22,22)$, $(23,22)$, $(31,22)$, $(32,22)$, $(33,22)$, $(34,22)$, $(35,22)$, $(37,22)$, $(39,22)$, $(40,22)$, $(41,22)$, $(42,22)$, $(43,22)$, $(44,22)$, $(45,22)$, $(62,22)$, $(63,22)$, $(65,22)$, $(68,22)$, $(70,22)$, $(74,22)$, $(82,22)$, $(83,22)$, $(84,22)$, $(85,22)$, $(86,22)$, $(87,22)$, $(89,22)$, $(90,22)$, $(103,22)$, $(115,22)$, $(116,22)$, $(117,22)$, $(118,22)$, $(129,22)$, $(130,22)$, $(131,22)$, $(132,22)$, $(2,23)$, $(22,23)$, $(23,23)$, $(24,23)$, $(31,23)$, $(32,23)$, $(33,23)$, $(34,23)$, $(35,23)$, $(37,23)$, $(39,23)$, $(40,23)$, $(41,23)$, $(42,23)$, $(43,23)$, $(44,23)$, $(45,23)$, $(62,23)$, $(63,23)$, $(65,23)$, $(68,23)$, $(70,23)$, $(74,23)$, $(82,23)$, $(83,23)$, $(84,23)$, $(85,23)$, $(86,23)$, $(87,23)$, $(88,23)$, $(89,23)$, $(90,23)$, $(103,23)$, $(115,23)$, $(116,23)$, $(117,23)$, $(118,23)$, $(129,23)$, $(130,23)$, $(131,23)$, $(132,23)$, $(23,24)$, $(42,24)$, $(107,24)$, $(22,25)$, $(23,25)$, $(31,25)$, $(32,25)$, $(33,25)$, $(34,25)$, $(35,25)$, $(37,25)$, $(39,25)$, $(40,25)$, $(41,25)$, $(42,25)$, $(43,25)$, $(44,25)$, $(45,25)$, $(62,25)$, $(63,25)$, $(65,25)$, $(68,25)$, $(70,25)$, $(74,25)$, $(82,25)$, $(83,25)$, $(84,25)$, $(85,25)$, $(86,25)$, $(87,25)$, $(89,25)$, $(90,25)$, $(103,25)$, $(107,25)$, $(115,25)$, $(116,25)$, $(117,25)$, $(118,25)$, $(129,25)$, $(130,25)$, $(131,25)$, $(132,25)$, $(12,26)$, $(107,25)$, $(49,26)$, $(50,26)$, $(51,26)$, $(52,26)$, $(55,26)$, $(58,26)$, $(107,26)$, $(110,26)$, $(112,26)$, $(12,28)$, $(49,28)$, $(50,28)$, $(51,28)$, $(52,28)$, $(55,28)$, $(58,28)$, $(107,28)$ satisfy only the axiom (R-CP).

The most interesting is the following

Theorem 4. The pairs $(57,2)$, $(21,3)$, $(25,3)$, $(33,3)$, $(34,3)$, $(35,3)$, $(36,3)$, $(37,3)$, $(40,3)$, $(43,3)$, $(104,3)$, $(33,4)$, $(34,4)$, $(35,4)$, $(36,4)$, $(37,4)$, $(40,4)$, $(43,4)$, $(104,4)$, $(42,5)$, $(12,7)$, $(29,7)$, $(42,7)$, $(49,7)$, $(50,7)$, $(51,7)$, $(58,7)$, $(107,7)$, $(12,8)$, $(49,8)$, $(50,8)$, $(51,8)$, $(52,8)$, $(55,8)$, $(58,8)$, $(72,8)$, $(107,8)$, $(12,9)$, $(49,9)$, $(50,9)$, $(51,9)$, $(52,9)$, $(55,9)$, $(58,9)$, $(107,9)$, $(42,10)$, $(37,11)$, $(40,11)$, $(62,11)$, $(63,11)$, $(65,11)$, $(68,11)$, $(70,11)$, $(83,11)$, $(84,11)$, $(87,11)$, $(88,11)$, $(15,13)$, $(52,13)$, $(55,13)$, $(69,13)$, $(73,13)$, $(92,13)$, $(93,13)$, $(96,13)$, $(97,13)$, $(24,14)$, $(37,14)$, $(40,14)$, $(52,14)$, $(55,14)$, $(78,14)$, $(83,14)$, $(84,14)$, $(87,14)$, $(92,14)$, $(93,14)$, $(96,14)$, $(29,15)$, $(42,15)$, $(92,15)$, $(93,15)$, $(96,15)$, $(97,15)$, $(52,16)$, $(55,16)$, $(92,16)$, $(93,16)$, $(96,16)$, $(97,16)$, $(42,17)$, $(77,18)$, $(88,18)$, $(102,18)$, $(77,19)$, $(109,19)$, $(36,20)$, $(104,20)$, $(77,21)$, $(88,21)$, $(107,21)$, $(77,22)$, $(109,22)$, $(77,24)$, $(88,24)$, $(104,24)$, $(77,25)$, $(104,25)$, $(109,25)$, $(25,26)$, $(33,26)$, $(34,26)$, $(35,26)$, $(36,26)$, $(37,26)$, $(40,26)$, $(43,26)$, $(104,26)$, $(111,26)$, $(33,28)$, $(34,28)$, $(35,28)$, $(36,28)$, $(37,28)$, $(40,28)$, $(43,28)$, $(104,28)$, $(77,30)$, $(81,30)$, $(88,30)$, $(77,32)$, $(81,32)$, $(88,32)$, $(77,34)$, $(88,34)$ satisfy only Axiom (L-CP).

This theorem gives 135 examples of pairs “implication and negation" that are solutions of Problem 1.7.1 (ii).

Let us call the Problem in its present form (i.e., searching of *some* implication and *some* negation) a “weak problem”. Then, the “strong problem” (this classification is not discussed in [6]) will be related to search of implications and the negations generated by them, which satisfy only (L-CP). Answer of this problem is given by

Theorem 5. The pairs $(21,3),(29,7),(111,26)$ satisfy only Axiom (L-CP).

101 implications and 29 negations participate in some pairs. In Tables 1 and 2 we omit these implications (37 in number) and negations (5 in number) that do not meet in the above assertions.

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