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Indirect Adaptive Wavelet Sliding Mode Control

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Abstract: *An adaptive wavelet sliding mode control algorithm is proposed for a class of continuous time unknown nonlinear systems. In contrast to the existing sliding mode control (SMC) design, where the presence of hitting control may introduce problems to controlled systems, the proposed adaptive wavelet controller takes advantages of SMC control. The chattering action is attenuated and robust performance can be ensured. The stability analysis for the proposed control algorithm is provided. Nonlinear system simulation example is presented to verify the effectiveness of the proposed method.*

Keywords: *Mode control, adaptive control, wavelet approximation, feedback linearization.*

1. Introduction

Recently, the subject of wavelet analysis has attracted much attention from both mathematicians and engineers alike. Wavelets have been applied successfully to multiscale analysis and synthesis, time-frequency signal analysis in signal processing, function approximation, approximation in solving partial differential equations, and so on [1-4]. Wavelets are well suited to depicting functions with local nonlinearities and fast variations because of their intrinsic properties of finite support and self-similarity. As a result, wavelet theory has useful applications in nonlinear control system design.

Feedback linearization techniques for nonlinear control system design have been developed in the last two decades [5, 6]. However, these techniques can only be applied to nonlinear systems whose parameters are known exactly. If the nonlinear system contains unknown or uncertain parameters then the feedback linearization is no longer utilizable. In this situation, the adaptive strategies are used to simplify the problem and to allow a suitable solution. At present, a number of adaptive control design techniques for nonlinear systems based on the feedback linearization can be found in literature [7, 8].

It is well known that the sliding mode control (SMC) method provides a robust controller for nonlinear dynamic systems [9, 10]. However, it inherits a discontinuous control action and hence chattering phenomena will take place when the system operates near the sliding surface. One of the common solutions for eliminating this chattering effect is to introduce a boundary layer neighboring the sliding surface [10, 11]. This method can lead to stable closed loop system without the chattering problem, but there exists a finite steady state error due to the finite steady state gain of the control algorithm.

As the sliding mode control law can separated into two parts i.e. the equivalent control and the switching control [12]. The role of the controller is to schedule these two components under different operating conditions. In order to improve the steady state performance of the adaptive wavelet sliding mode controller, an adaptive wavelet network controller combining the SMC is considered in this paper. The proposed control scheme provides good transient and robust performance. In this paper, it is proved that the closed-loop system is globally stable in the Lyapunov sense and the system output asymptotically with modeling uncertainties and disturbances.

This work is involved by combining the characteristics of wavelet, the technique of feedback linearizations, the adaptive control scheme and the sliding mode control to solve the tracking control design problem for nonlinear systems with bounded unknown or uncertain parameters and external disturbances.

This paper is organized as follows. First, the problem formulation is presented in Section 2. A brief description of wavelet system is included in Section 3. In Section 4, the adaptive wavelet sliding control is proposed. Simulation results for the proposed control concept are shown in Section 5. Finally, the paper is concluded in Section 6.

2. The principle of conventional sliding mode control

Consider a general class of SISO *n*-th order nonlinear systems as follow form [9]

$$x^{(n)} = f(\underline{x}, t) + g(\underline{x}, t)u(t) + d(t), \quad y = x, \tag{1}$$

where *f* and *g* are unknown functions, $\underline{x} \in R^n$ is the state vector of the system which is assumed to be available for measurement, $u \in R$ and $y \in R$ are the input and output of the system, respectively, and $d(t)$ is the unknown external disturbance. It is assumed that $|d(t)| \le D$. It is required that $g(\underline{x}, t) \ne 0$, without loss of generality we assume that $g(\underline{x}, t) > 0$. In the same spirit as with nonlinear control literature [1], these systems are in normal form and have a relative degree equal to *n*. The control objective is to obtain the state $\underline{x}$ for tracking a desired state $\underline{x}_d = (x_d, \dot{x}_d, \cdots, x_d^{(n-1)})$ in the presence of model uncertainties and unknown disturbances.

Define a sliding surface in the space of the error state as

(2) $$s(\underline{x},t) = k_1 e + k_2 \dot{e} + \cdots + k_{n-1} e^{(n-2)} + e^{(n-1)} = -\underline{k}\underline{e},$$

where $\underline{e} = \underline{x}_d - \underline{x} = (e, \dot{e}, \cdots, e^{(n-1)})^{\mathrm{T}} \in R^n$ is the tracking error vector and the coeficients $k_1, k_2, \cdots, k_{n-1}$ are the coeficients of the Hurwitzian polynomial $h(\lambda) = \lambda^{n-1} + k_{n-1}\lambda^{n-2} + \cdots + k_1$.

By means of sliding mode condition $s(\underline{x},t) \cdot \dot{s}(\underline{x},t) \le -\eta |s|$, $\eta > 0$, a sliding mode control law can be derived as

(3) $$u(t) = \frac{1}{g(\underline{x},t)} \left[\sum_{i=1}^{n-1} k_i e^{(i)} - f(\underline{x},t) + x_d^{(n)} - \eta_\Delta \operatorname{sgn}(s) \right],$$

where $\eta_\Delta \ge \eta > 0$.

Due to the fact that system functions *f*, *g* and disturbance *d* are unknown in practical systems, the control law (3) is usually difficult to be obtained.

In the next section we will use wavelet systems to approximate $f(\underline{x},t)$, $g(\underline{x},t)$ and switching-type control law $\eta_\Delta \operatorname{sgn}(s)$, respectively. Moreover an adaptive adjusting law for parameters will be designed.

3. A review of wavelet networks

In this section a brief introduction to wavelet networks is given. We will begin by discussing basic wavelet analysis theory. Consider the closed space $U_i \;\; \forall i \in Z$ with following properties

(4) $$U_i \quad \cdots \subset U_{-1} \subset U_0 \subset U_1 \cdots,$$

(5) $$\cap_{i \in Z} U_i = \{0\},$$

(6) $$U_{i+1} = U_i \oplus W_i \quad \forall i \in Z,$$

(7) $$f(\underline{x}) \in U_i \Leftrightarrow f(2\underline{x}) \in U_{i+1} \quad \forall i \in Z,$$

where *Z* is the set of all integers, $\cap$ is the intersection operator and $\oplus$ is the direct sum, respectively. It is seen that the decomposition of the whole space *S* can be rewritten as follows:

(8) $$S = U_i \oplus W_i \oplus W_{i+1} \oplus \cdots \oplus W_0 \oplus W_1 \oplus \cdots$$

for some $i \in Z$. Let $\phi(\underline{x}) \in S$ be a basic scaling function such that $U_i = \operatorname{span}\{\phi_{ij}(\underline{x})\}$ with $\phi_{ij}(\underline{x}) = 2^{\frac{i}{2}} \phi(2^i \underline{x} - j)$, for all $i, j \in Z$; then, there exists a basic function $\psi(\underline{x}) \in S$ such that $W_i = \operatorname{span}\{\psi_{ij}(\underline{x})\}$ with $\psi_{ij}(\underline{x}) = 2^{\frac{i}{2}} \psi(2^i \underline{x} - j)$, for all $i, j \in Z$. With these descriptions, a function $\psi(\underline{x}) \in S$ is called an ortogonal wavelet if the family $\{\psi_{ij}\}$, defined as above, is an orthonormal basis of *S*; that is [13]

(9) $$\langle \psi_{ij}, \psi_{kl} \rangle = \delta_{ik}\delta_{jl}, \ \forall \ i, j, k, l \in Z .$$

Several kinds of wavelet bases have been successfully developed and widely applied in many different areas, such as time-frequency signal analysis in signal processing, function approximation, approximation in solving partial differential equations and so on. Further development of new families of wavelet bases continues to receive considerable attention from researches.

Now, consider a function $f(\underline{x})$ is *S*. It is obvious that $f(\underline{x})$ can be rewritten as [13, 14]

(10) $$f(\underline{x}) = \sum_i \sum_j \theta_{ij} \psi_{ij}(\underline{x})$$

where

(11) $$\theta_{ij} = \int_{-\infty}^{\infty} f(x) \psi_{ij}(x) dx$$

for all $i, j \in Z$. The above expression of $f(\underline{x})$ is called a wavelet series expansion of the function $f(\underline{x})$.

Based on the wavelet series expansion, a wavelet network of the form

(12) $$\hat{f}(\underline{x}, \underline{\theta}) = \sum_{i=M_1}^{M_2} \sum_{j=N_1}^{N_2} \theta_{ij} \psi_{ij}(\underline{x}) = \underline{\theta}^{\mathrm{T}} W(\underline{x})$$

can be constructed to approximate a nonlinear function $f(\underline{x})$ in *S*, for some integers M_1, M_2, N_1 and N_2 where

(13) $$\underline{\theta} = \left[\theta_{M_1 N_1} \cdots \theta_{M_1 N_2} \cdots \theta_{M_2 N_1} \cdots \theta_{M_2 N_2} \right]^{\mathrm{T}}$$

and

(14) $$W(x) = \left[\psi_{M_1 N_1}(x) \ldots \psi_{M_1 N_2}(x) \ldots \psi_{M_2 N_1}(x) \ldots \psi_{M_2 N_2}(x) \right]^{\mathrm{T}} .$$

This wavelet network represents an alternative to a neural network approximation.

If $\epsilon(M_1, M_2, N_1, N_2) = f(x) - \hat{f}(\underline{x}, \underline{\theta})$ is the approximation error, then it is easy to show that for arbitrary constant $\varepsilon \geq 0$ there exist some constants $M_1, M_2, N_1, N_2 \in Z$ such that $\left\| \epsilon(M_1, M_2, N_1, N_2) \right\|_2 \leq \varepsilon$, for all c in compact set $X \subset R$. This means that the wavelet network $\hat{f}(\underline{x}, \underline{\theta})$ can approximate $f(\underline{x})$ to any desired accurancy.

In the case of a function $f(\underline{x})$ defined on $X \subset R^n$ with $\underline{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}}$, the proposed wavelet network $\hat{f}(\underline{x}, \underline{\theta})$ can not be applied directly because $\hat{f}(\underline{x}, \underline{\theta})$ is defined on $X \subset R$, not on $X \subset R^n$. We must first make a minor modification by replacing the wavelet bases in (12) by $\psi_{ij}\left(\underline{c}^{\mathrm{T}}, \underline{x}\right)=\psi_{ij}\left(\sum_{i=1}^{n} c_i x_i\right)$ with some weighting constants c_i.

Then the modified wavelet network becomes

(15) $$\hat{f}\left(\underline{x}, \underline{\theta}\right)=\sum_{i=M_1}^{M_2}\sum_{j=N_1}^{N_2}\theta_{ij}\psi_{ij}\left(\underline{c}^{\mathrm{T}}\underline{x}\right)=\underline{\theta}^{\mathrm{T}}W\left(\underline{c}^{\mathrm{T}}\underline{x}\right),$$

where

(16) $$\underline{\theta}=\left[\theta_{M_1N_1}\cdots\theta_{M_1N_2}\cdots\theta_{M_2N_1}\cdots\theta_{M_2N_2}\right]^{\mathrm{T}}$$

and

(17) $$W\left(\underline{c}^{\mathrm{T}}\underline{x}\right)=\left[\psi_{M_1N_1}\left(\underline{c}^{\mathrm{T}}\underline{x}\right)\cdots\psi_{M_1N_2}\left(\underline{c}^{\mathrm{T}}\underline{x}\right)\cdots\psi_{M_2N_1}\left(\underline{c}^{\mathrm{T}}\underline{x}\right)\cdots\psi_{M_2N_2}\left(\underline{c}^{\mathrm{T}}\underline{x}\right)\right]^{\mathrm{T}}.$$

Note that this modified wavelet network is composed of four layers. The first layer is the input layer with available input vector $\underline{x}=[x_1, x_2,\cdots, x_n]^{\mathrm{T}}$. A weighting summer $\underline{c}^{\mathrm{T}}\underline{x}$ is given in the second layer. The third layer is composed of the wavelet bases. The output layer is a weighted combination ot the wavelets.

Remark 1. Making an appropriate choice of wavelet basis is an important task in constructing wavelet networks. Many wavelet functions have been reported in the literatures. This simplest one is the Haar function. The best one may be the Daubechies function. Which one is the most suitable basis in practical applications depends on the design specifications.

Remark 2. The constants M_1, M_2, N_1 and N_2 in wavelet networks are closely related to the approximation error $\epsilon\left(M_1, M_2, N_1, N_2\right)$. Although, the wavelet network has been shown to be one of the universal approximators, the question of how to decide M_1, M_2, N_1 and N_2 for a given accurancy ε is still unanswered. In general, the selection of M_1, M_2, N_1 and N_2 may be made by taking advantage of an expert's or operator's knowledge and experience.

4. Design of an indirect adaptive wavelet sliding mode control

First, we consider the control of nonlinear system (1). In order to derive the sliding mode control law (3), we use wavelet system $\hat{f}\left(\underline{x}\,|\,\underline{\theta}_f\right)$ to approximate system $f\left(\underline{x}, t\right)$, wavelet system $\hat{g}\left(\underline{x}\,|\,\underline{\theta}_g\right)$ to approximate system $g\left(\underline{x}, t\right)$ as well as the switching control term $\eta_\Delta\,\mathrm{sgn}\left(s\right)$.

Thus the resulting control law will be

(18) $$u\left(t\right)=\frac{1}{\hat{g}\left(\underline{x}, t\right)}\left[-\hat{f}\left(\underline{x}, t\right)+\sum_{i=1}^{n-1}k_i e\left(i\right)+x_d^{(n)}-\hat{h}\left(s\right)\right],$$

(19) $$\hat{f}\left(\underline{x}\,|\,\underline{\theta}_f\right)=\underline{\theta}_f^{\mathrm{T}}W_f\left(\underline{c}_f^{\mathrm{T}}\underline{x}\right),$$

(20) $$\hat{g}\left(\underline{x} \mid \underline{\theta}_g\right)=\underline{\theta}_g^{\mathrm{T}} W_g\left(\underline{c}_g^{\mathrm{T}} \underline{x}\right),$$

(21) $$\hat{h}\left(s \mid \underline{\theta}_h\right)=\underline{\theta}_h^{\mathrm{T}} W_h\left(\underline{c}_h^{\mathrm{T}} s\right)$$

set $\left|\hat{h}\left(s \mid \underline{\theta}_h\right)\right|=D+\eta_\Delta+\omega_{\max}$ when $s \geq \Phi$ in the adaptation process. The scheme is similar to boundary layer when $s \geq \Phi$ and the control is replaced by wavelet switching when $s<\Phi$.

Here the parameters in $W_f\left(\underline{c}_f^{\mathrm{T}} \underline{x}\right)$, $W_g\left(\underline{c}_g^{\mathrm{T}} \underline{x}\right)$ and $W_h\left(\underline{c}_h^{\mathrm{T}} s\right)$ are supposed to be fixed, while the parameters $\underline{\theta}_f^{\mathrm{T}}$, $\underline{\theta}_g^{\mathrm{T}}$ and $\underline{\theta}_h^{\mathrm{T}}$ are free to be designed by adaptive law.

Theorem 1. Consider the control problem of the nonlinear system (1). If control (18) is applied, $\hat{f}$, $\hat{g}$ and $\hat{h}$ are given by (19)-(21) and the parameters vector $\underline{\theta}_f$, $\underline{\theta}_g$ and $\underline{\theta}_h$ are adjusted by the following adaptive law (22)-(24). Then the closed-loop system signals will be bounded and the tracking error will converge to zero asymptotically.

(22) $$\dot{\underline{\theta}}_f=\gamma_1 s W_f\left(\underline{c}_f^{\mathrm{T}} \underline{x}\right),$$

(23) $$\dot{\underline{\theta}}_g=\gamma_2 s W_g\left(\underline{c}_g^{\mathrm{T}} \underline{x}\right),$$

(24) $$\dot{\underline{\theta}}_h=\gamma_3 s \phi(s).$$

P r o o f. Define the optimal parameters of wavelet systems

(25) $$\underline{\theta}_f^*=\arg \min_{\underline{\theta}_f \in \Omega_f}\left(\sup_{\underline{x} \in R^n}\left|\hat{f}\left(\underline{x} \mid \underline{\theta}_f\right)-f(\underline{x}, t)\right|\right),$$

(26) $$\underline{\theta}_g^*=\arg \min_{\underline{\theta}_g \in \Omega_g}\left(\sup_{\underline{x} \in R^n}\left|\hat{g}\left(\underline{x} \mid \underline{\theta}_g\right)-g(\underline{x}, t)\right|\right),$$

(27) $$\underline{\theta}_h^*=\arg \min_{\underline{\theta}_h \in \Omega_h}\left(\sup_{s \in R^n}\left|\hat{h}\left(s \mid \underline{\theta}_h\right)-u_{sw}\right|\right),$$

where Ω_f, Ω_g and Ω_h are constraint sets for $\underline{\theta}_f$, $\underline{\theta}_g$ and $\underline{\theta}_h$, respectively. Meanwhile, define the minimum approximation error as

(28) $$\omega=f(\underline{x}, t)-\hat{f}\left(\underline{x} \mid \underline{\theta}_f^*\right)+\left(g(\underline{x}, t)-\hat{g}\left(\underline{x} \mid \underline{\theta}_g^*\right)\right) u .$$

Then we have

$$\dot{s}=\sum_{i=1}^{n-1}k_i e^{(i)}+x^{(n)}-x_d^{(n)}=$$
$$=\sum_{i=1}^{n-1}k_i e^{(i)}+f\left(\underline{x},t\right)+g\left(\underline{x},t\right)u\left(t\right)+d\left(t\right)-x_d^{(n)}=$$
$$=\sum_{i=1}^{n-1}k_i e^{(i)}+f\left(\underline{x},t\right)-\hat{f}\left(\underline{x},\underline{\theta}_f\right)+\left(g\left(\underline{x},t\right)-\hat{g}\left(\underline{x},\underline{\theta}_g\right)\right)u-$$
$$-\sum_{i=1}^{n-1}k_i e^{(i)}+x_d^{(n)}\hat{h}\left(s\,|\,\underline{\theta}_h\right)+d\left(t\right)-x_d^{(n)}= \tag{29}$$
$$=\hat{f}\left(\underline{x}\,|\,\underline{\theta}_f^*\right)-\hat{f}\left(\underline{x}\,|\,\underline{\theta}_f\right)+\left(g\left(\underline{x}\,|\,\underline{\theta}_g^*\right)-\hat{g}\left(\underline{x}\,|\,\underline{\theta}_g\right)\right)u+$$
$$+\hat{h}\left(s\,|\,\underline{\theta}_h^*\right)-\hat{h}\left(s\,|\,\underline{\theta}_h\right)+d\left(t\right)+\omega-\hat{h}\left(s\,|\,\underline{\theta}_h^*\right)=$$
$$=\underline{\varphi}_f^{\mathrm{T}}W_f\left(\underline{c}_f^{\mathrm{T}}\underline{x}\right)+\underline{\varphi}_g^{\mathrm{T}}W_g\left(\underline{c}_g^{\mathrm{T}}\underline{x}\right)+\underline{\varphi}_h\phi(s)+d\left(t\right)+\omega-\hat{h}\left(s\,|\,\underline{\theta}_h^*\right),$$

where $\underline{\varphi}_f=\underline{\theta}_f-\underline{\theta}_f^*$, $\underline{\varphi}_g=\underline{\theta}_g-\underline{\theta}_g^*$ and $\underline{\varphi}_h=\underline{\theta}_h-\underline{\theta}_h^*$.

Now we consider the Lyapunov candidate

$$V=\frac{1}{2}s^2+\frac{1}{2\gamma_1}\underline{\varphi}_f^{\mathrm{T}}\underline{\varphi}_f+\frac{1}{2\gamma_2}\underline{\varphi}_g^{\mathrm{T}}\underline{\varphi}_g+\frac{1}{2\gamma_3}\underline{\varphi}_h^{\mathrm{T}}\underline{\varphi}_h, \tag{30}$$

where γ_1, γ_2 and γ_3 are positive constants.

The time derivation of V along the error trajectory (30) is

$$\dot{V}=s\dot{s}+\frac{1}{\gamma_1}\underline{\varphi}_f^{\mathrm{T}}\underline{\dot{\varphi}}_f+\frac{1}{\gamma_2}\underline{\varphi}_g^{\mathrm{T}}\underline{\dot{\varphi}}_g+\frac{1}{\gamma_3}\underline{\varphi}_h^{\mathrm{T}}\underline{\dot{\varphi}}_h=$$
$$=s\left(\underline{\varphi}_f^{\mathrm{T}}W_f\left(\underline{c}_f^{\mathrm{T}}\underline{x}\right)+u\underline{\varphi}_g^{\mathrm{T}}W_g\left(\underline{c}_g^{\mathrm{T}}\underline{x}\right)+\underline{\varphi}_h^{\mathrm{T}}W_h\left(\underline{c}_h^{\mathrm{T}}s\right)+\omega+d\left(t\right)-\hat{h}\left(s\,|\,\underline{\theta}_h^*\right)\right)+$$
$$+\frac{1}{\gamma_1}\underline{\varphi}_f^{\mathrm{T}}\underline{\dot{\varphi}}_f+\frac{1}{\gamma_2}\underline{\varphi}_g^{\mathrm{T}}\underline{\dot{\varphi}}_g+\frac{1}{\gamma_3}\underline{\varphi}_h^{\mathrm{T}}\underline{\dot{\varphi}}_h=$$
$$=s\underline{\varphi}_f^{\mathrm{T}}W_f\left(\underline{c}_f^{\mathrm{T}}\underline{x}\right)+\frac{1}{\gamma_1}\underline{\varphi}_f^{\mathrm{T}}\underline{\dot{\varphi}}_f+us\underline{\varphi}_g^{\mathrm{T}}W_g\left(\underline{c}_g^{\mathrm{T}}s\right)+\frac{1}{\gamma_2}\underline{\varphi}_g^{\mathrm{T}}\underline{\dot{\varphi}}_g+s\underline{\varphi}_h^{\mathrm{T}}W_h\left(\underline{c}_h^{\mathrm{T}}s\right)+\frac{1}{\gamma_3}\underline{\varphi}_h^{\mathrm{T}}\underline{\dot{\varphi}}_h-$$
$$-s\hat{h}\left(s\,|\,\underline{\theta}_h^*\right)+s\omega+sd\left(t\right)\leq \tag{31}$$
$$\leq s\underline{\varphi}_f^{\mathrm{T}}W_f\left(\underline{c}_f^{\mathrm{T}}\underline{x}\right)+\frac{1}{\gamma_1}\underline{\varphi}_f^{\mathrm{T}}\underline{\dot{\varphi}}_f+su\underline{\varphi}_g^{\mathrm{T}}W_g\left(\underline{c}_g^{\mathrm{T}}s\right)+\frac{1}{\gamma_2}\underline{\varphi}_g^{\mathrm{T}}\underline{\dot{\varphi}}_g+sW_h\left(\underline{c}_h^{\mathrm{T}}s\right)+\frac{1}{\gamma_3}\underline{\varphi}_h^{\mathrm{T}}\underline{\dot{\varphi}}_h-$$
$$-s\left(D+\eta_\Delta\right)\mathrm{sgn}(s)+sd\left(t\right)+s\omega<$$
$$<\frac{1}{\gamma_1}\underline{\varphi}_f^{\mathrm{T}}\left(\gamma_1 sW_f\left(\underline{c}_f^{\mathrm{T}}\underline{x}\right)+\underline{\dot{\varphi}}_f\right)+\frac{1}{\gamma_2}\underline{\varphi}_g^{\mathrm{T}}\left(u\gamma_2 sW_g\left(\underline{c}_g^{\mathrm{T}}\underline{x}\right)+\underline{\dot{\varphi}}_g\right)+\frac{1}{\gamma_3}\underline{\varphi}_h^{\mathrm{T}}\left(\gamma_3 sW_h\left(\underline{c}_h^{\mathrm{T}}s\right)+\underline{\dot{\varphi}}_h\right)-$$
$$-|s|\eta_\Delta+s\omega,$$

where $\underline{\dot{\varphi}}_f=-\underline{\dot{\theta}}_f$, $\underline{\dot{\varphi}}_g=-\underline{\dot{\theta}}_g$ and $\underline{\dot{\varphi}}_h=-\underline{\dot{\theta}}_h$. Substitute (22)-(24) into (31), then we have

$$\dot{V}<s\omega-|s|\eta_\Delta \tag{32}$$

since ω is the minimum aproximation error, (32) is the best we can obtain. Therefore, all signals in the system are bounded. Obviously, if $\underline{e}(0)$ is bounded, then $\underline{e}(t)$ is also bounded fo all t. Since the reference signal $\underline{x}_d$ is bounded, then the system states $\underline{x}(t)$ is bounded as well. To complete the proof and establish asymptotic convergence of the tracking error, we need proving that $s \to 0$ as $t \to \infty$. Assume that $|s| \le \eta$, then equation (32) can be rewritten as

(33) $$\dot{V} \le |s||\omega| - |s|\eta \le \eta_s |\omega| - |s|\eta .$$

From the universal approximation theorem, it can be expect that the term $s\omega$ should be very small if not equal to zero in the adaptive wavelet system. So we have

(34) $$\dot{V} \le 0 .$$

Integrating both sides of (33), we have

(35) $$\int_0^t |s| d\tau \le \frac{1}{\eta_\Delta}\left(|V(0)| + |V(t)|\right) + \frac{\eta_s}{\eta_\Delta}\int_0^t |\omega| d\tau .$$

If $\omega \in L_1$ then from (32) we have $s \in L_1$. From (32), we know that s is bounded, so we have $s \in L_\infty$. Because we have proved that all the variables on the right-hand side of (31) are bounded, we have $\dot{s} \in L_\infty$. Using the Corollary of Barbalat's lemma [15], we have $\lim_{t\to\infty} |s(t)| = 0$.

Therefore $\lim_{t\to\infty} |\bar{e}(t)| = 0$.

5. Simulation example

5.1. Example 1

The above described adaptive wavelet control algorithm will now be evaluated using the inverted pendulum system depicted in Fig. 1.

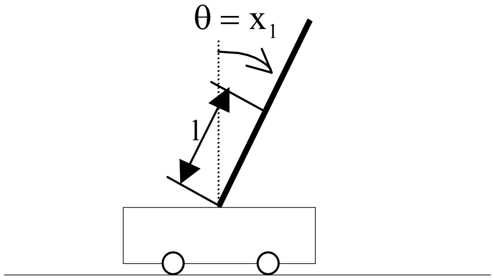

Fig. 1. The inverted pendulum system

Let $x_1 = \theta$ and $x_2 = \dot{\theta}$. The dynamic equation of the inverted pendulum is given by [16]:

$$\dot{x}_1 = x_2,$$

(36)
$$\dot{x}_2 = \frac{g\sin x_1 - \dfrac{mlx_2^2\cos(x_1)\sin(x_1)}{m_c+m}}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2(x_1)}{m_c+m}\right)} + \frac{\dfrac{\cos(x_1)}{m_c+m}}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2(x_1)}{m_c+m}\right)} u_c + d,$$

$$y = x_1,$$

where g is the acceleration due to gravity, m_c denotes the mass of the cart, m is the mass of the pole, l is the half-length of the pole, the force u_c represents the control signal and d is the external disturbance. In simulations following parameter values are used: $m_c = 1\,\text{kg}$, $m = 0.1\,\text{kg}$ and $l = 0.5\,\text{m}$. The reference signal is assumed to be $y_r(t) = (\pi/30)\sin(t)$ and an external disturbance $d(t) = 0.1\sin(t)$.

If we require

(37)
$$|\underline{x}| \le \frac{\pi}{6},\ |u| \le 180$$

and substitute the functions sin(.) and cos(.) by their bounds, we can determine the bounds

(38)
$$f^M(x_1, x_2) = 15.78 + 0.366x_2^2,$$

(39)
$$g^M(x_1, x_2) = 1.46,\ g_m(x_1, x_2) = 1.12,$$

$k_1 = 2$, $k_2 = 1$ and $Q = \text{diag}(10, 10)$ are set. Then the algebraic Riccati equation solution is $P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$ and $\lambda_{\min}(P) = 2.93$. To satisfy the constraint related to $|\underline{x}|$ we choose $M_f = 16$, $M_g = 1.6$, $M_p = 1.6$ and $\varepsilon = 0.48$.

Using the method of trial and errors $\gamma_f = 50$ and $\gamma_g = 1$ are chosen. The MATLAB command "ode45" is used to simulate the overall control system. The pendulum initial position is chosen as far as possible $(\theta(0) = x_1 = \pi/20)$ to emphasize the efficiency of our algorithm.

The Haar wavelets are chosen to be the basis of the wavelet network. The vectors $\underline{c}_f$ and $\underline{c}_g$ are both chosen as $\underline{c}_f = \underline{c}_g = \underline{c} = [1\ \ 1]^T$, and the size of our network is chosen as $M_1 = -2$, $M_2 = 2$, $N_1 = -1$ and $N_2 = 1$. In this example, the wavelet bases for $f(\underline{x})$ and $g(\underline{x})$ are chosen and are the same. Therefore, $W_f(\underline{c}_f^T \underline{x}) = W_g(\underline{c}_g^T \underline{x}) = W(\underline{c}^T \underline{x})$. So we need 15 parameters to estimate the nonlinear functions $f(\underline{x})$ and $g(\underline{x})$, respectively. the initial conditions $x_1(0) = 0.2$ and $x_2(0) = 0.2$ are selected.

Choose the sliding surface as $s = \dot{e} + k_1 e$, where $k_1 = 3$.

Obtain the adaptive wavelet network sliding mode control law as

(40) $$u = \frac{1}{\underline{\theta}_g^T W(x_1 + x_2)} \left[-\underline{\theta}_f^{\mathrm{T}} W(x_1 + x_2) - \frac{\pi}{30} \sin(t) + e + 2\dot{e} - \underline{\theta}_p^{\mathrm{T}} (\dot{e} + k_1 e) \right],$$

(41) $$\dot{\theta}_f = 0.1(5e + 5\dot{e}) s W(x_1 + x_2),$$

(42) $$\dot{\theta}_g = 0.01(5e + 5\dot{e}) s W(x_1 + x_2) u,$$

(43) $$\dot{\theta}_p = 0.1(5e + 5\dot{e}) s W(x_1 + x_2).$$

The tracking performance of both cases for a sinusoidal trajectory is illustrated in Fig. 2.

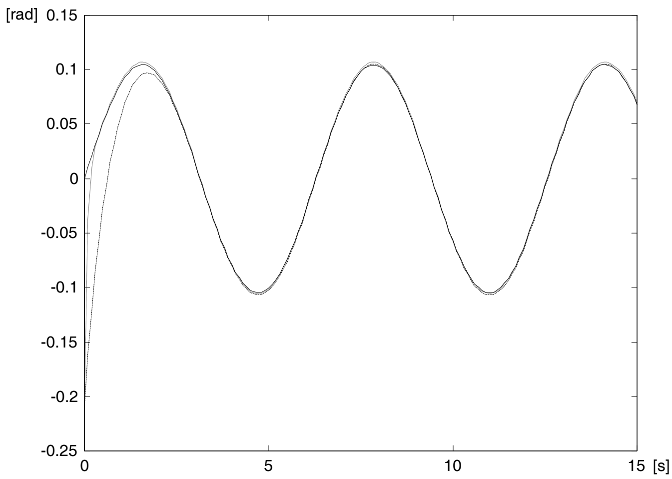

Fig. 2. The state x_1 in case 1(dashed line), in case 2 (dotted line) and desired value $y_r(t)$ (solid line) for $\underline{x}(\underline{0}) = (\pi/12, 0)^{\mathrm{T}}$

5.2. Example 2

In this example, we apply the adaptive wavelet controller to the system

(44) $$y'' + \frac{1}{0.25 + y} y' + 1.7y - 0.5u = 0.$$

The reference model is assumed to be

(45) $$M(s) = \frac{1}{s^2 + 2s + 1}$$

and the reference signal is the square periodic signal of magnitude 1.5 and frequency 0.01 Hz.

We choose $P = \begin{bmatrix} 50 & 30 \\ 30 & 20 \end{bmatrix}$, $k_1 = 2$, $k_2 = 1$, and $\lambda_{\min}(P) = 1.52$. To satisfy the constraint related to $|\underline{x}|$ we choose $M_f = 20$, $M_g = 2.1$, $M_p = 2$ and $\varepsilon = 0.25$.

At 200th second of simulation the system (44) was switched to another system

$$y''' + 5y'' + \left[\frac{1}{(0.25+y)^2} - 1.7\right]y' + y - 5u = 0. \tag{46}$$

All initial states have been set to zero $y(0) = y'(0) = y''(0) = y'''(0) = 0$.

As it can be seen from Fig. 3, the simulation results confirm good adaptation capability of the proposed control system. The system dynamic changes are in particular manifested by changes of control input signal (Fig. 4).

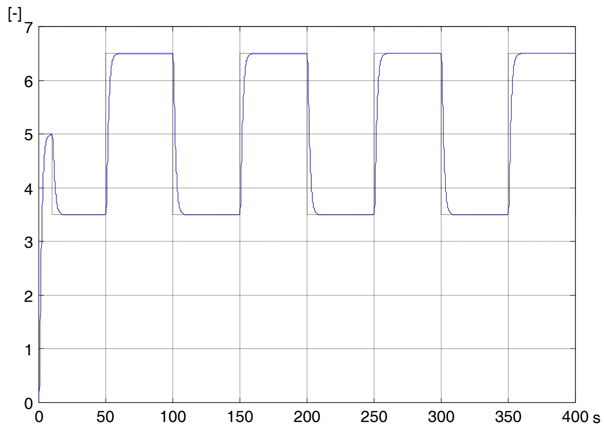

Fig. 3. The state x_1 (solid line), its desired reference model value $y_m(t)$ (dotted line) and reference signal (dashed line)

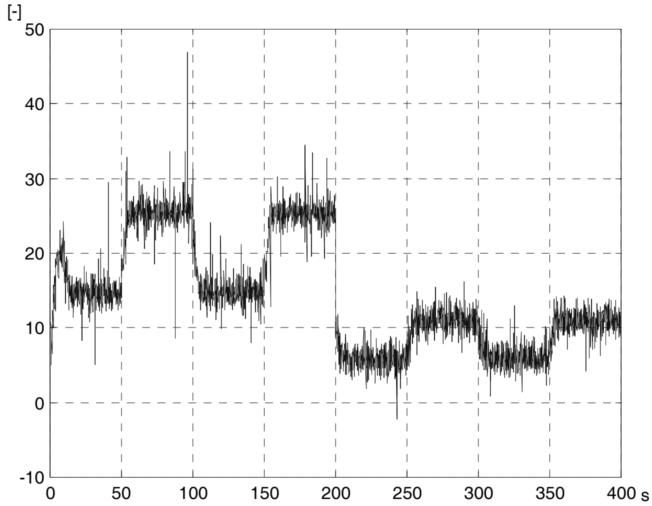

Fig. 4. Control signal

6. Conclusions

In this paper, an adaptive wavelet sliding control algorithm has been proposed for a class of unknown nonlinear systems. We introduced the wavelet sliding mode control and proposed the robust control using the adaptive control strategy. The drawback of chattering in sliding mode control is avoided zero steady tracking error can be ensured. The closed loop system is stable in the sense of Lyapunov. Finally, the proposed method has been applied to control the inverted pendulum to track a reference trajectory. The simulation results show that the adaptive controller can achieve desired performance.

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