

An Integrated Software System for Optimization and Decision Support *MultiOptima**

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Abstract: *The paper presents a multicriteria decision support system, called MultiOptima. The system consists of three independent parts – MKA-2 system, MKO-2 system and LIOP-1 system and it is intended to support the decision maker in modelling and solving different problems of multicriteria analysis and linear and linear integer problems of single and multicriteria optimization. The class of the problems solved, the structure, the used methods, the operation with the interface modules for input data entry and the information about decision maker's local preferences, the operation with the interface modules for visualization of the current and final solutions for the three systems MKA-2, MKO-2 and LIOP-1 are discussed.*

Keywords: *multicriteria decision support system, multicriteria analysis, multicriteria optimization, linear programming.*

1. Introduction

Decision making problems are non-formalized or weak formalized problems, the solution of which requires the participation of the so called decision maker (DM). The solutions obtained are to a great extent subjective and depend on DM's preferences. The decision making problems are divided into three main classes: multicriteria decision making problems, decision making problems under risk and decision making problems under uncertainty. Different problems of planning, control, analysis and monitoring in economy, transport, industrial production,

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education, ecology and other spheres may be reduced to multicriteria decision making problems. The multicriteria decision making problems can be divided in two sub-classes depending on their formal statement. In the first class a finite number of explicitly set constraints in the form of functions defines an infinite number of feasible alternatives. These problems are called continuous multicriteria decision making problems or problems of multicriteria optimization. In the second class a finite number of alternatives is explicitly given in a tabular form. These problems are called discrete multicriteria decision making problems or problems of multicriteria analysis. In multicriteria analysis and multicriteria optimization problems several criteria are simultaneously optimized in the feasible set of alternatives. In the general case there does not exist one alternative, which optimizes all the criteria. There is a set of alternatives however, characterized by the following: each improvement in the value of one criterion leads to deterioration in the value of at least one other criterion. This set of alternatives is called a set of the non-dominating or Pareto optimal alternatives (solutions). Each alternative in this set could be a solution of the multicriteria decision making problem. In order to select one alternative for the final solution of the problem being solved, it is necessary to have additional information set by the decision maker. There exists a subclass of linear multicriteria optimization problems when number of criteria is equal of one. These problems are called problems of single criteria optimization or problems of linear programming. Optimal solutions of these problems don't depend on DM's preferences.

The multicriteria analysis problems can be divided into three types: problems of multicriteria choice, problems of multicriteria ranking and problems of multicriteria sorting. Many real life problems in management practice may be formulated as problems of choice, ranking or sorting of resources, strategies, projects, offers, policies, credits, products, innovations, designs, costs, profits, portfolios, etc. (Belton (1993), Beinat and Nijkamp (1998), Holbourn (1998), Paschetta and Tsoukiàs (2000), Kelley et al. (2002), Ananda and Herath (2003), Srdjevic et al. (2004), Mustajoki et al. (2004)). The multicriteria optimization problems are only problems of multicriteria choice. Many real-life problems in planning, control and industrial production may be formulated as problems of multicriteria choice (Korhonen (1998), Joos (1999), Rajesh et al. (2001), Ehrgott and Ryan (2002), Thibault et al. (2002), Hämäläinen et al. (2003), Kaleta et al. (2003), Vera et al. (2003)). Various real-live problems can be modeled as linear and linear integer programming problems (Vanderbei (1996)). For example these might be problems at levels like planning, designing and managing of companies, organization and institution activity. The most shown up problems among them are the problems for expenses reducing; optimization of a route, recipes and technologies; improving of materials supply and production capacities loading, etc.

Different methods have been developed to solve multicriteria analysis problems. A great number of the methods developed up to now, can be grouped in three separate classes (Vincke (1992)). The first class of methods (Dyer (2004)) includes the multiattribute utility (value) theory methods (value tradeoff method,

UTA method, MACBETH method, direct weighting method, AHP weighting methods). There are differences in the way in which the DM's global preferences are aggregated in the two subclasses of these methods. In the first one a generalized functional criterion is directly synthesized, whereas in the second subclass (weighting methods) it could be said that such a criterion (additive form) is indirectly synthesized. The two subclasses of multiattribute utility theory methods are based on the assumption that there does not exist limited comparability among the alternatives. The methods that form the second class of methods are the outranking methods (ELECTRE methods (Figueira et al. 2005)), PROMETHEE methods (Brans and Mareschal (2005)) etc.) and they are based on the assumption that there exists limited comparability among the alternatives. In these methods one (or several outranking relation(s)) are first built to aggregate DM's global preferences, after which this outranking relation is used to assist the DM in solving the multiple criteria decision analysis problem. In most of the outranking methods it is assumed that the DM selects to specify some preference information about inter- and intra-criteria. While the inter-criteria information is expressed in the form of weights and veto thresholds, the intra-criteria information is usually expressed in the form of indifference and preference thresholds. The interactive algorithms (RNIM method (Narula et al. (2003) etc.) belong to the methods of the third group. They are "optimizationally motivated" and are oriented to solve multicriteria analysis problems with a large number of alternatives and a small number of criteria.

There are two main approaches in solving multicriteria optimization problems: the scalarizing approach (Miettinen (2003), Korhonen (2005)) and the approximation approach (Ehrgott and Wiecek (2005)) The major representatives of the scalarizing approach are the interactive algorithms. Multicriteria optimization problems is treated in these algorithms as a decision making problem and the emphasis is put on the real participation of the DM in the process of its solution. The interactive methods are the most developed and widespread due to their basic advantages – a small part of the Pareto optimal solutions must be generated and evaluated by the DM; in the process of solving the multicriteria problem, the DM is able to learn with respect to the problem being solved; the DM can change his/her preferences in the process of problem solution; the DM feels more confident in his/her preferences concerning the final solution.

The interactive methods of the reference point (direction) and the classification-oriented interactive methods (Miettinen (1999)) are the most widely spread interactive methods for solving multicriteria optimization problems. Though the interactive methods of the reference point are still dominating, the classification-oriented interactive methods enable the better solution of some main problems in the dialogue with the DM, relating to his/her preferences defining, and also concerning the time of waiting for new non-dominated solutions that are evaluated and selected. The generalized interactive algorithm GENWS-IM is an interactive algorithm (Vassileva (2006)) with variable scalarization and parametrization. It is a generalization of a large part of the multicriteria optimization interactive algorithms developed up to now. This generalization is

with respect to the classes of the problems solved, the type of the defined preferences, the number and type of the applied scalarizing problems, the strategies used in the search for new Pareto optimal solutions. Starting from the current (weak) Pareto optimal solution, the generalized scalarizing problem GENWS (Vassileva (2006)) may be used. Altering some parameters of this generalized scalarizing problem GENWS the following known scalarizing problems can be obtained: the scalarizing problem of the weighted sum; the scalarizing problem of ε -constraints; the scalarizing problem STEM; the scalarizing problem STOM; the scalarizing problem of the reference point; the scalarizing problem GUESS; the scalarizing problem of modified reference point; the scalarizing problem of the external reference direction; the classification-oriented scalarizing problem NIMBUS and the classification-oriented scalarizing problem DALDI. On the basis of the generalized scalarizing problem GENWS, a generalized interactive (Vassileva (2006)) method GENWS-IM with variable scalarizations and parameterization could be designed, having the following characteristics: the DM may set his/her preferences with the help of the criteria weights, ε -constraints, desired and acceptable levels of change of the criteria values, desired and acceptable levels, directions and intervals of alteration in the criteria values, etc.; during the process of the multicriteria problems solving, the DM may change the way of presenting his/her preferences. Starting from one and the same current Pareto optimal solution and applying different scalarizing problems (with respective alteration of GENWS), the DM may obtain different new Pareto optimal solutions at a given iteration, and this opportunity is especially useful in education and in comparison of different scalarizing problems.

A variety of methods to approximate the set of Pareto optimal solutions of different types have been proposed (Ehrgott and Wiecek (2005)). A big majority of the methods are iterative and produce points or objects approximating this set. Some methods are exactly equipped with theoretical proofs for correctness and optimality while some other methods are heuristic and often theoretically unsupported. The main representatives of the heuristic methods are the multicriteria genetic (evolutionary) methods (Deb (2001)). The multicriteria optimization problem is treated in these methods rather as a vector optimization problem, than as a decision making problem and the stress is placed on the determination of a subset of potential Pareto optimal solutions, which approximates well enough the whole Pareto optimal set. The solutions obtained with the help of the genetic method, are near Pareto optimal solutions. Besides this, during the process of defining the approximating set, the DM is isolated and he/she is provided with a large set of solutions for evaluation and choice towards the end (this is a comparatively hard problem of multicriteria analysis).

Different methods are created for solving linear programming problems. The most wide spread are the simplex methods (Vanderbei (1996)) and the interior point methods (Yinyu (1997)). Both make progressively improving series of intermediate solutions, until a solution is reached that satisfies the conditions for an optimum. The simplex method can be viewed as moving from extreme boundary point to another one along the boundary of the feasible region. Searching for better

solutions, the interior-point methods visit points within the interior of the feasible region.

Different methods are also created for solving of linear integer programming problems. Among the exact methods the more common ones are the methods from “branch-and-bound” type (Wolsey (1998)) and the methods for logical programming with constraints or constraint programming (Hooker (2000)). Branch-and-bound methods search for an optimal integer solution by solving a sequence of linear programming problems (the so called relaxation problems) that are obtained from the initial integer problems by shrinking the integer variables boundaries of change. In general the methods differ from each other by the ways of defining that sequence and also by the ways of efficient solving of each linear programming problem. The methods of constraint programming do not solve a sequence of linear programming problems. They are based on the results of logical inference research in Artificial Intelligence. The advantage of these methods is the possibility of searching in the tree of possible solutions for combinatorial problems, which are hard (at the cost of many additional variables) to formulate as linear integer programming problems.

The developed systems supporting the solution of multicriteria analysis and multicriteria optimization problems may be classified in three groups: commercial, research or teaching and experimental (for new methods testing). Sometimes it is difficult to make a clear distinction between these groups. In another point of view these systems can be divided in two classes – software systems with general purpose and problem-oriented software systems. The general-purpose software systems aid the solution of different multicriteria analysis and multicriteria optimization problems by different decision makers (DMs). The problem-oriented multicriteria analysis systems serve to support the solution of one or several types of specific multicriteria analysis problems and very often are included in other information-control systems.

The following general-purpose software systems (Weistroffer et al. (2005)) developed – VIMDA, Expert Choice, PROMCALC and GAIA, ELECTRE III-IV, MACBETH, VIP, Decision Lab, Web-HIPRE, MultiChoice and KnowCube aid the solution of different multicriteria problems. Four interesting problem-oriented software systems for multicriteria analysis are the following systems: the FINCLAS system for financial classification problems (Zopounidis and Doumpos (1999)), the Agland Decision system – for agricultural property (Parsons (2002)), the DESYRE system – for rehabilitation of contaminated sites (Carlson et al. (2004)) and the MultCSync system – for incorporating multiple criteria in conservation planning (Moffett et al. (2004)).

Some well-known general-purpose software systems (Weistroffer et al. (2005)), which solve problems of multicriteria optimization, are the systems VIG, DIDAS, DINAS, MOLP-16, LBS, SOMMIX, MOIP, WWW-NIMBUS, MOLIP, NLPJOB and MOMILP. The multicriteria DSS system for river water-quality planning (Lotov et al. (1997)) and the ADELAIS system for portfolio selection (Zopounidis et al. (1998)) are two attractive problem-oriented multicriteria optimization systems. In the class of multicriteria optimization software systems

must also be included software systems, which implement different multicriteria evolutionary methods (algorithms). Four of them are the following ones: the NSGM system (Srinivas and Deb (1994)), the MOSES system (Coello and Christiansen (1999)), the M-PAES system (Knowles and Corne (2000)), and the MOEA toolbox for MATLAB.

Various types of software systems (Fourer and Goux (2001), Mittelmann and Spellucci (2005)) have been developed for solving linear and linear integer programming problems. These systems may be separated in professional and experimental systems (depending on the available tools and on the size of the problems being solved); in modeling systems and program solvers (depending on the user interface quality); and in local and WEB based systems (depending on the available access), etc. The greater part of the professional (commercial) systems (CPLEX, LINDO, HI-PLEX, Xpress-MP, and others) are too expensive, but they are intended to solve problems of hundred of thousands continuous variables and constraints, and thousands integer variables and constraints. Program solvers are with a specific (compact) interface, whilst the modeling systems have input and output modules of broad possibilities and some of the systems include several program solvers. The WEB based systems are oriented to remote solving of problems and their development in the last years is quite intensive.

The present paper describes some basic elements of the decision support system *MultiOptima*, which consists of separate parts (the systems MKA-2, MKO-2 and LIOP-1) and which is designed to support decision makers in solving different multicriteria analysis, multicriteria optimization and linear and integer linear programming problems. The class of the problems solved, the system structure, the operation with the interface modules for input data entry and the information about DM's local preferences, the operation with the interface modules for visualization of the current and final solutions are discussed in the paper, as well as the help information, given in a digital and graphical form.

2. The MKA-2 system description

The MKA-2 system, which is the first part of the system *MultiOptima*, operates under MS Windows operating system. The MKA-2 system is designed to support DM in solving different multicriteria analysis problems. The multicriteria analysis problem may be described by a decision matrix A ($n \times k$), which can be defined as given in Table 1, where:

a_i denotes an alternative with an index $i, i=1, \dots, n$;

k_j (.) denotes a criterion with an index $j, j=1, \dots, k$.

The evaluation of the i -th alternative with respect to all the criteria is given by the row vector $(a_{i1}, a_{i2}, \dots, a_{ik})$. The evaluation of all the alternatives with respect to j -th criterion is given by the column vector $(a_{1j}, a_{2j}, \dots, a_{nj})^T$.

Table 1. Decision matrix

$a_i \backslash k_j$	$k_1(\cdot)$	$k_2(\cdot)$...	$k_j(\cdot)$...	$k_k(\cdot)$
a_1	a_{11}	a_{12}	...	a_{1j}	...	a_{1k}
a_2	a_{21}	a_{22}	...	a_{2j}	...	a_{2k}
...
a_i	a_{i1}	a_{i2}	...	a_{ij}	...	a_{ik}
..
a_n	a_{n1}	a_{n2}	...	a_{nj}	...	a_{nk}

The MKA-2 system consists of internal-system modules, four solving modules and interface modules. It is realized in MS Windows environment, including the standard for this operating system user interface elements. The internal-system modules contain all global definitions of variables, functions and procedures of general purpose. The object possibilities of Visual Basic are utilized in MKA-2 system, creating several classes with respect to internal-system structures. They are the following: a class for messages, which encapsulates the output of error messages, dynamic context help information and logging events in the debug window, localization and identification of errors occurring during the system operation; a class matrix with some specific procedures, necessary for AHP method; a class for storing the information specific for the criteria in ELECTRE III and PROMETHEE II methods and a class for storing elements of the CBIM interactive method history. MKA-2 handles files with “*.mka” extension. Standard operations for creating, editing, loading and saving of files are implemented. The MKA-2 files contain input data and data related to the process and the results from solving multicriteria analysis problems.

The solving modules realize four methods – AHP method, ELECTRE III method, PROMETHEE II method and CBIM method and procedures for transformation of qualitative, ranking and weighting criteria into quantitative criteria. AHP method is one of the most widely spread weighting methods. Pair-wise criteria comparison is used in this method to set DM’s preferences. On this basis a pair-wise comparison matrix is constructed. The estimates of the weights can be found by normalizing the eigenvector corresponding to the largest eigenvalue of this matrix. ELECTRE III is one of the most often used outranking methods. It is based on an outranking relation, characterized by the definition of an outranking degree $S(a, b)$ associated with each ordered pair (a, b) of alternatives, representing the more or less great outranking credibility of a over b . There are two matrices to be evaluated: the concordance and the discordance matrix. The concordance matrix gives an assessment of agreement that one alternative is better than other one. It requires two type thresholds – indifference and preference thresholds. The discordance matrix gives an assessment of disagreement that one alternative is better than the other one. That matrix requires additional threshold, called veto threshold, which allows the outranking relation to be rejected. In order to obtain the degree of credibility of outranking, there follows the combining the two measures from the concordance and discordance matrix. This degree is thus equal to the concordance index where no criterion is discordant or where no veto

threshold is used, in the opposite case. The concordance index is lowered in function of the importance of the discordance. The obtained credibility matrix is essential for generating two distillation orders that show whether one alternative outranks the other or if such an alternative is incomparable to the other. In order final ranking to be obtained the two orders are combined. PROMETHEE II method is the second of the most often used outranking methods. In this method the intensity of the preference of one alternative over another alternative regarding each criterion is measured in terms of the so-called preference function. On the basis of two type thresholds – indifference and preference thresholds six types of preference functions are used in the method. The method provides a complete ranking of the alternatives through a pair-wise dominance comparison of net positive and net negative outranking flows. RNIM method (Narula et al. (2003)) is a representative of the interactive methods and is appropriate for solving multicriteria analysis problems with a large number of alternatives and a small number of criteria. The DM can provide desired or acceptable levels, directions and intervals of changes in the values of the criteria at any iteration. On the basis of this information, the method proposed enables the use of discrete optimization scalarizing problems, with the help of which the DM has the possibility for a more systematic and successful screening of the alternatives set.

The interface modules ensure the interaction between MKA-2 system, the DM and the operating system. This interaction includes the entry of the data for the multicriteria analysis problems; the entry of specific information for every method; the entry of information about DM's preferences; the visualization of the current results and the final result; the graphical presentation of the solutions; the printing out, reading and storing of files; the multi-language support, etc. The editing module enables entering, alteration and storing of quantitative, qualitative, ranking and weighting criteria. The interface preference modules aid the DM in the entry of criteria pair-wise comparison information, inter- and intra-criteria information and information about the desired or acceptable levels, directions and intervals of change in the values of the criteria. The current and final results and the parameters for the separate methods selected by the DM are presented digitally and graphically with the help of visual interface modules. The input/output interface modules enable the reading and storing in files, the printing of the current and final results obtained, as well as the printing of the information, given by the DM. The solution process of a multicriteria problem can be interrupted at any stage and activated from the place of its interruption at any time. MKA-2 system has comparatively rich printing functions – every piece of the data (entered or computed) may be printed. In this way, the entire process of decision making is documented – you can review the input data of the multicriteria problem, the DM's preferences entered, the current values obtained, and the final result also, which on its turn can be printed out in the form of values or graphics. The rest of the interface modules realize a dynamic help, multi-language maintenance, etc.

Fig. 1 shows the initial information entered for the real multicriteria analysis problem, concerning to the choice of a building site for an European electric power station (Mladineo et al. (1987)). Fig. 2 shows a window with information about

the pair-wise comparison of the criteria. This is information about DM's preferences in operation with AHP method. Fig. 3 presents a window with information about DM's preferences in operation with PROMETHEE II method. Fig. 4 shows the final result, obtained for six countries when solving the ranking problem by ELECTRE III method.

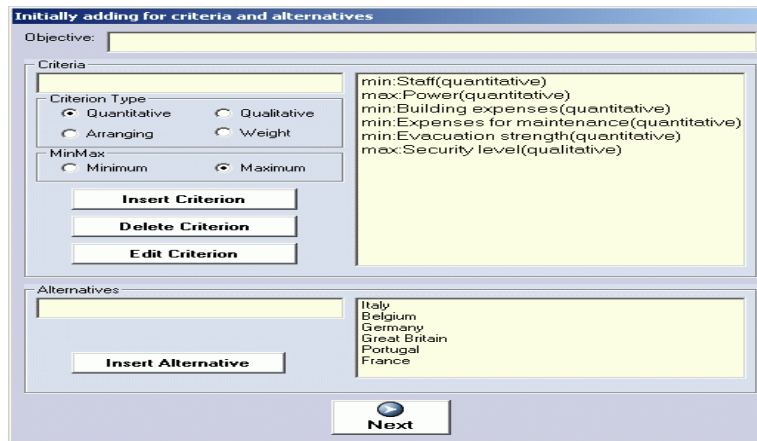


Fig. 1

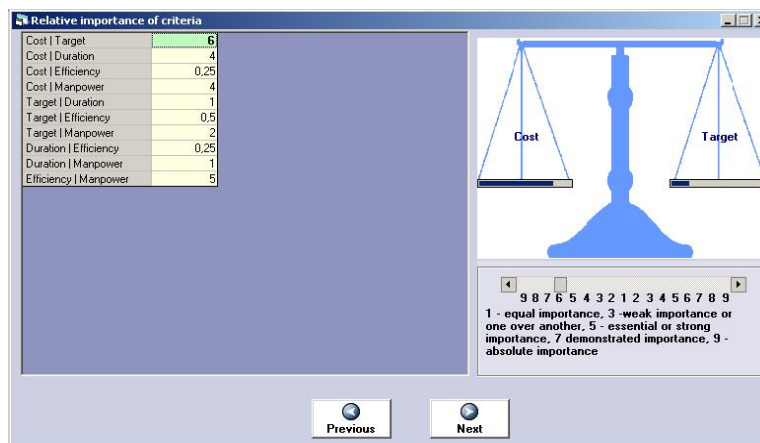


Fig. 2

3. The MKO-2 system description

The MKO-2 software system, which is the second part of the system *MultiOptima*, is developed on the basis of the generalized interactive algorithm GENWS-IM (Vassileva (2005), Vassileva (2006), Vassileva et al. (2007)). The MKO-2 system operates under MS Windows operating system and it is designed to aid the solution of linear and linear integer problems for multicriteria optimization.

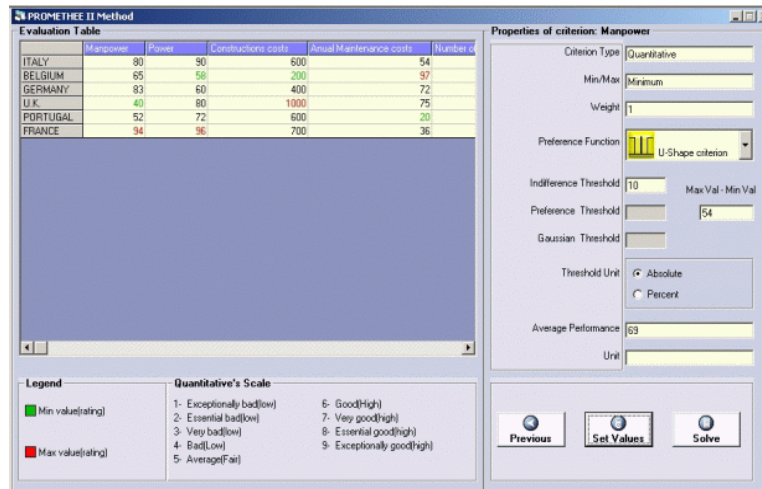


Fig. 3

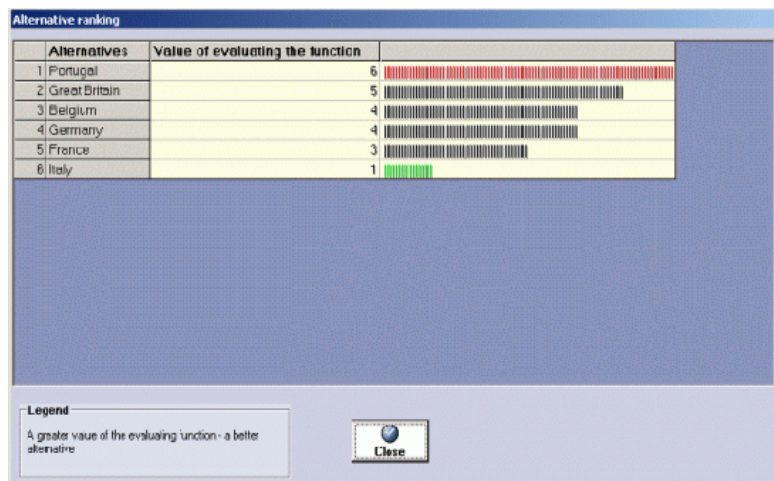


Fig. 4

The linear and linear integer multicriteria optimization problem may be described as follows.

To optimize simultaneously the criteria

$$\{f_k(x), k \in K\}$$

under the constraints:

$$\sum_{j \in N} a_{ij} x_j \leq b_i, i \in M,$$

$$0 \leq x_j \leq d_j, j \in N,$$

$$x_j - \text{integers}, j \in N'; N' \subset N,$$

where

$f_k(x)$, $k \in K$, are linear criteria of the type: $f_k(x) = \sum_{j \in N} c_j^k x_j$;

$x = (x_1, \dots, x_j, \dots, x_n)^T$ is the variables vector;

$f(x) = (f_1(x), \dots, f_k(x), \dots, f_p(x))^T$ is the vector of the criteria;

$K = \{1, 2, \dots, p\}$, $M = \{1, 2, \dots, m\}$, $N = \{1, 2, \dots, n\}$ and $N' = \{1, 2, \dots, n' | n' \leq n\}$ are sets of the indices of the linear criteria, the linear constraints, the variables and the integer variables, respectively.

MKO-2 software system consists of three main groups of modules – a control program, optimization modules and interface modules. The control program is integrated software environment for creation, processing and storing of files associated with MKO-2 system, as well as for linking and execution of different types of software modules. The basic functional possibilities of the control program may be separated in three groups. The first group includes the possibilities to use the applications, menus and system functions being standard for MS Windows - “File”, “Edit”, “View”, “Window”, “Help”, in the environment of MKO-2 system. The second group of functional possibilities encloses the control of the interactions between the modules realizing: creation, modification and storing of files associated with MKO-2 system, which contain input data and data connected with the process of solution of linear and linear integer problems of the multicriteria optimization; interactive solution of the linear and linear integer multicriteria optimization problems entered; localization and identification of the errors occurring during the process of operation with MKO-2 system. The third group of functional possibilities of the control program includes the possibilities for visualization of essential information about the DM and information of the system operation as a whole.

The optimization modules realize the generalized interactive algorithm GENWS-IM, two simplex algorithms solving continuous single-criterion problems (Vanderbei (1996)), an algorithm of “branches and bounds” type for exact solution of linear integer single-criterion problems (Wolsey (1998)) and an algorithm (Vassilev and Genova (1991)) for approximate solution of linear integer single-criterion problems.

The interface modules provide the dialogue between the DM and the system during the entry and correction of the input data of the multicriteria problems solved, during the interactive process of these problems solving and for dynamic numerical and graphical visualization of the main parameters of this process. With the help of an editing module the descriptions of the criteria and constraints are input, altered and stored, and also the type and limits of the variables alteration. Another interface module serves to supply two types of graphic presentation of the information about the values of the criteria at the different steps, as well as the possibilities for their comparison.

One of the main functions of MKO-2 system is to enable the extension of DM’s possibilities to set his/her preferences with the help of criteria weights, ε -constraints, desired and acceptable levels of alteration in the criteria values, desired and acceptable directions of change of the criteria values, desired and

acceptable levels, directions and intervals of alteration of the criteria values. Twelve scalarizing problems are generated in MKO-2 system in order to realize these possibilities. Depending on DM's preferences, these scalarizing problems are automatically generated by the generalized scalarizing problem GENWS with the help of a change in its structure and its parameters.

MKO-2 system presents DM different windows intended for entry and correction of the problem criteria and constraints, as well as for setting his/her preferences. Fig. 5, Fig. 6, Fig. 7 and Fig. 8 show four of these windows. The window presented in Fig. 5 is the basic window of the editor for input data entry, called MKO-2-Editor.

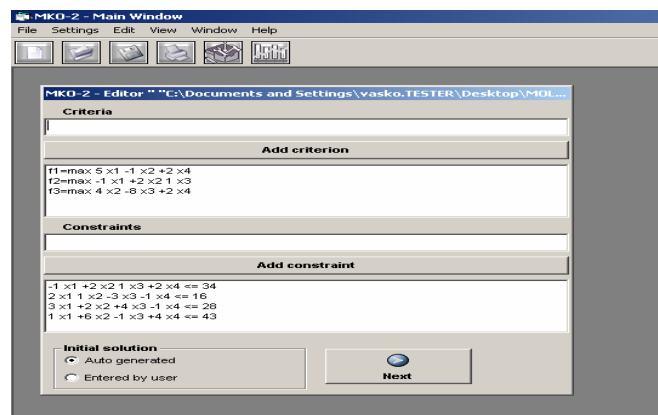


Fig. 5

The window presented in Fig. 6, is designed to identify the type of DM's preferences. The DM may select among five types of preferences and let assume that he/she has selected to set the preferences by the form of desired and acceptable levels by the form of desired and acceptable levels, directions and intervals of alteration in the criteria values (operating with DALDI scalarizing problem). The screen, presented in Fig. 7, shows the setting of a new aspiration level for the value of the third criterion.

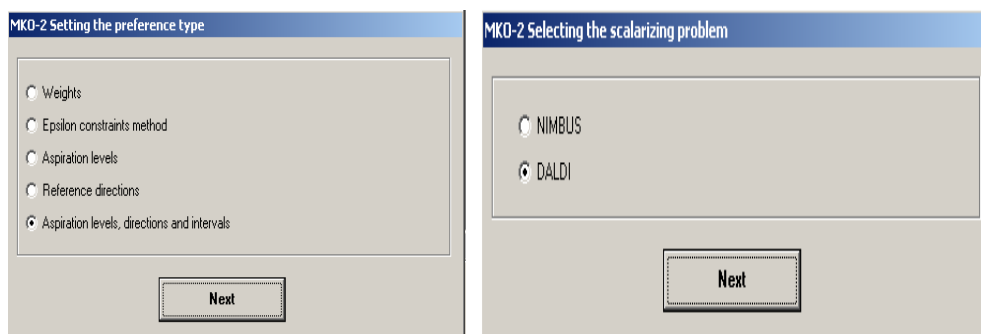


Fig. 6

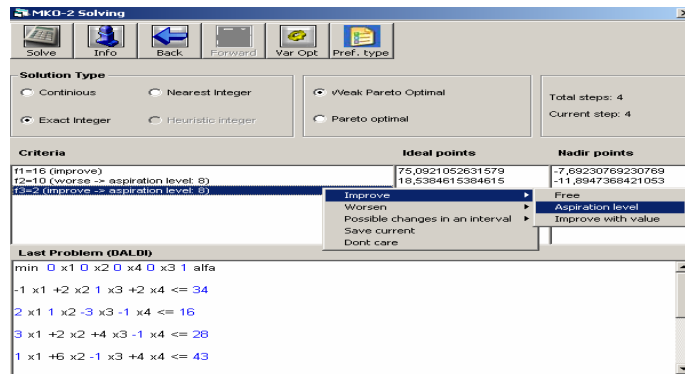


Fig. 7

Choosing “Graphic” command enables the visualization of two types of graphical information about the process of problem solution. For this purpose a window with two types of graphic is opened (Fig. 8). With the help of the upper bar-graphic, visual comparison can be made of the solutions found at two iterations, selected in the fields below it for iterations selection. The lower graphic can trace visually the alteration of the values of the separate criteria at different iterations of the interactive process of search for a better solution. With the help of the selection buttons, the initial and the final iteration of the iterations interval are defined, in which the values of all the criteria are traced.

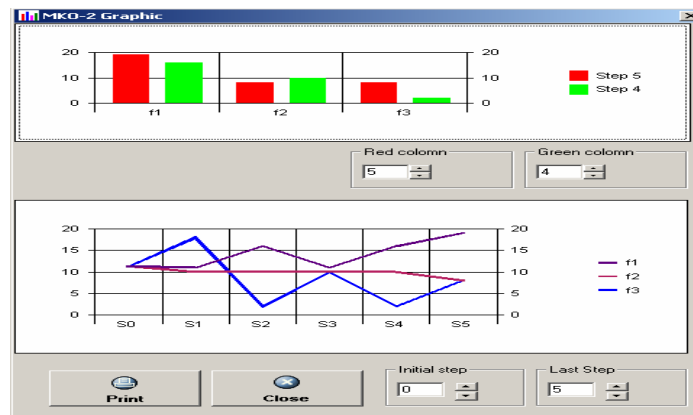


Fig. 8

4. The LIOP-1 system description

The LIOP-1 system, which is the third part of the system *MultiOptima*, operates under MS Windows operating system and is designed to solve a broad class of problems of the linear and integer linear programming.

The formal statement of the Mixed Integer Programming problem (MIP) is the following:

$$F_1 = \sum_{j=1}^n c_j x_j \rightarrow \max \quad (\min)$$

subject to

$$\sum_{j=1}^n a_{ij} x_j \begin{matrix} > \\ = \\ < \end{matrix} b_i, \quad i = 1, \dots, m,$$

$$l_j \leq x_j \leq d_j, \quad j = 1, \dots, n,$$

$$x_j - \text{integers}, \quad j = 1, \dots, n_1,$$

where

$$n_1 \leq n;$$

F_1 is an objective function (criterion);

$x_j, j = 1, \dots, n$ are problem's variables;

$a_{ij}, b_i, i = 1, \dots, m, j = 1, \dots, n$, are respectively coefficients and right side of the

constraint of index i ;

$l_j, u_j, j = 1, \dots, n$, are upper and lower limits of the variable x_j ;

n_1 is the number of the integer variables.

If n_1 is equal to n , the problem is said to be an integer programming problem (IP), while at n_1 equal to zero, the problem is a Linear Programming Problem (LP).

LIOP-1 system is realized in Visual Basic and C programming languages and operates under MS Windows operating system. It consists of three main modules: a control module, optimization modules and interface modules. The control module is an integrated software environment for creating, processing and storing of files associated with the system (with "*.lip" extension), as well as for linking and execution of different types of software modules. The basic functional possibilities of the control module may be separated into three groups. The first group includes the possibilities to use the standard for MS Windows applications menus and system functions – "File", "Edit", "View", "Window", "Help" and others in the system own environment. The second group of functional possibilities includes the control of the interaction between the modules realizing:

- the creating, modification and storing of "*.lip" files associated with the system, which contain input data and data for the intermediate and final results from the solution of the linear and linear integer programming problems;
- solution of the linear and linear integer programming problems entered;
- localization and identification of the errors occurring in the process of operation with LIOP-1 system.

The third group of functional possibilities of the control module consists in the possibilities to visualize significant information about the operation of LIOP-1 system as a whole.

The control module is designed in Multi Document Interface style. There is one main form (window), containing the main menu, and all the other forms (windows) are its "daughter" forms. Several "daughter" forms can be visualized at one and the same time. In order to accomplish communication with the libraries for dynamic linking of different single criterion optimization modules, the independent "CSolver" class is used, which is a data processor. It includes modules for input

data entry, for output data obtaining and for different setups of the type of the variables and the type of the solution required. It works after the black box principle – an optimization problem in certain format and syntax is set at its input as a parameter, syntactic analysis is done, the names and the type of the variables (continuous/integer) are defined, the type of the solution (continuous/integer) is also determined and the optimization problem is transformed in a format, required by the corresponding single criterion optimization module. The information about the solutions obtained is recorded at the output points of the class, where from it can be obtained.

The optimization modules realize two simplex methods of the linear programming and the branch and bounds method of the linear integer programming.

The interface modules guarantee the dialogue with the system during the input and correction of the input data of the problems being solved, during the interactive process of their solution and also during the digital and graphical visualization of the main parameters of this process. With the help of the editing module, the descriptions of the criteria, of the constraints, as well as the type and bounds of alteration of the variables are entered, changed or preserved. Another interface module realizes two types of graphic presentation of the information about the values of the criteria at different steps and the possibilities to compare them. Dynamic information is supplied about the purpose and the way of use of the fields and radio buttons.

LIOP-1 system presents DM different windows intended for entry and correction of the problem objective function and constraints and for visualization of the obtained results. Fig. 9 and Fig. 10 show two of these windows. The window, presented in Fig. 9, is the basic window of the editor for input data entry, called LIOP-1-Editor.

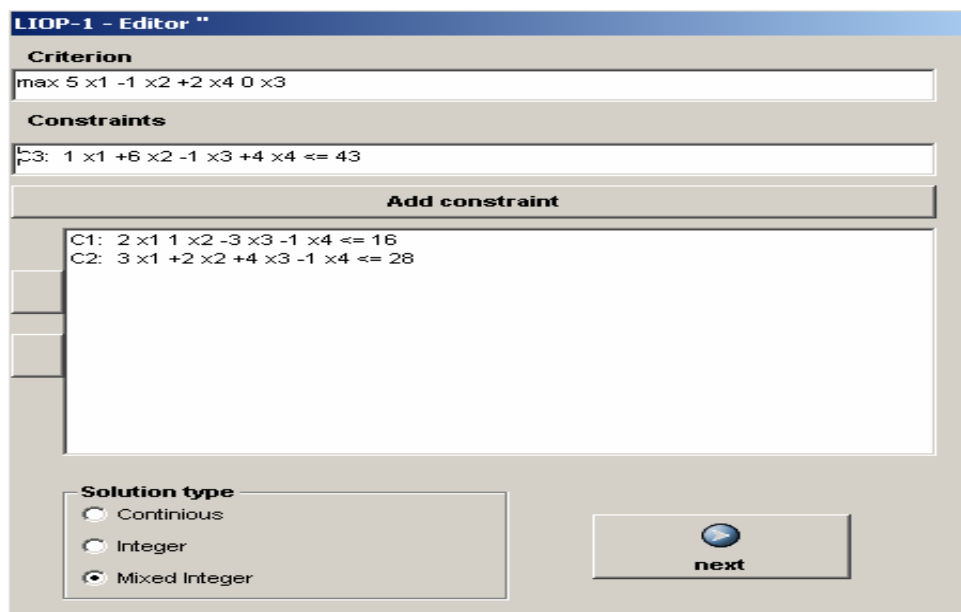


Fig. 9

The obtained results are shown on the “LIOP-1 Solving” window (Fig. 10). The “LIOP-1 Solving” window contains two text fields. In the first field the values of the criterion and variables for the obtained solution are viewed (shown).

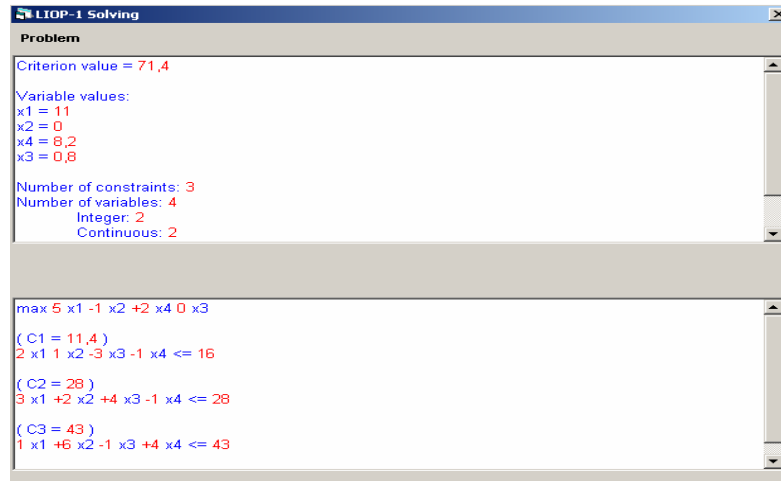


Fig. 10

In this field the iterations' number and some parameters of the solving problem are also given. In the second field the problem being solved is viewed again, but the constraint values for obtained solution are shown in addition. When solving linear integer problems of larger dimension periodically there can be shown messages for possible breaking of solving process (including information for feasible solutions found). The messages' appearing time is fixed in advance, but it can be changed.

The printing of short or detailed information about the problem currently solved is done with the help of “Print” command. A form is opened for selecting the information to be printed. Printing of the solution, the problem being solved, the input variables type and bounds, and the detailed information about the process solution may be selected. Requirements about the quality of printing, the number of copies and the character style can also be set.

5. Conclusion

MultiOptima system is designed to support DM in solving different multicriteria analysis, multicriteria optimization and linear and linear integer programming problems. MKA-2 system is designed to support DM in modeling and solving problems of multicriteria ranking and multicriteria choice. MKO-2 system is designed to model and solve linear and linear integer problems of multicriteria optimization. LIOP-1 system is designed to solve a broad class of problems of the linear and linear integer programming. The user-friendly interface of MKA-2, MKO-2 and LIOP-1 systems facilitates the operation of DMs with different qualification level relating to the multicriteria analysis and optimization methods and software tools. MKA-2, MKO-2 and LIOP-1 systems can be used for education

and for experimental and research problems solving as well. *MultiOptima* system is a local multicriteria decision support system and operates in two languages – Bulgarian and English. A number of Bulgarian universities use the system for the purposes of education and for experimental and research problems solving as well. A number of official organizations and companies use the system for solving real-life multicriteria decision making problems. The future development of the *MultiOptima* system will be realized in two directions. The first direction is connected with the addition of new methods. The second direction refers to web-based versions of the system, enabling distant decision making.

References

1. A n a n d a, J., H e r a t h, G. The Use of the Analytic Hierarchy Process to Incorporate Stakeholder Preferences into Regional Forest Planning. – *Forest Policy and Economics*, **5**, 2003, 13-26.
2. B e i n a t, E., N i j k a m p, P. Multi-Criteria Evaluation in Land-Use Management. Dordrecht, Kluwer Academic Publishers, 1998.
3. B e l t o n, V. Project Planning and Prioritisation in the Social Services – an OR Contribution. – *Journal of the Operational Research Society*, **44**, 1993, 115-124.
4. B r a n s, J., M a r e s c h a l, B. PROMETHEE Methods. – In: *Multiple Criteria Decision Analysis: State of the Art Surveys*. J. Figueira, S. Greco, M. Ehrgott, Eds. London, Springer Verlag, 2005, 163-196.
5. C o e l l o, C., A. C h r i s t i a n s e n. MOSES: A Multiobjective Optimization Tool for Engineering Design. – *Engineering Optimization*, **31**, 1999, No 3, 337-368.
6. D e b, K. Multi-Objective Optimization using Evolutionary Algorithms. Wiley-Interscience Series in Systems and Optimization. Chichester, John Wiley & Sons, 2001.
7. D y e r, J. MAUT-Multiattribute Utility Theory. (2005). – In: *Multiple Criteria Decision Analysis: State of the Art Surveys*. J. Figueira, S. Greco, M. Ehrgott, Eds. London, Springer Verlag, 2004, 265-297.
8. E h r g o t t, M., D. M. R y a n. Constructing Robust Crew Schedules with Bicriteria Optimization. – *Journal of Multicriteria Decision Analysis*, **11**, 2002, No 3, 139-150.
9. E h r g o t t, M., M. W i e c e k. Multiobjective Programming. – In: *Multiple Criteria Decision Analysis: State of the Art Surveys*. J. Figueira, S. Greco, M. Ehrgott, Eds. London, Springer Verlag, 2005, 990-1018.
10. F i g u e i r a, J., V. M o u s s e a u, B. Roy. ELECTRE Methods. – In: *Multiple Criteria Decision Analysis: State of the Art Surveys*. J. Figueira, S. Greco, M. Ehrgott, Eds. London, Springer Verlag, 2005, 133-162.
11. F o u r e r, R., J. P G o u x. Optimization as an Internet Resource. – *INTERFACES*, **31**, 2001, No 2, 130-150.
12. H a m a l a i n e n, J., K. M i e t t i n e n, P. T a r v a i n e n, J. T o i v a n e n. Interactive Solution Approach to a Multiobjective Optimization Problem in Paper Machine Headbox Design. – *Journal of Optimization Theory and Applications*, **116**, 2003, No 2, 265-281.
13. H o l b o u r n, M. Decision Conferencing – a Tool for Budget Allocation. – *Focus on Police Research and Development*, **10**, 1998, 22-3.
14. H o o k e r, J. Logic-Based Methods for Optimization. Wiley-Interscience, 2000.
15. J o o s, H. D. A Methodology for Multi-Objective Design Assessment and Flight Control Synthesis Tuning. – *Aero Science and Technology*, Vol. **3**, 1999, No 3, 161-176.
16. K a l e t a, M., W. O g r y c z a k, E. T o c z y t o w s k i, I. Z o t t o w s k a. On Multiple Criteria Decision Support for Suppliers on the Competitive Electric Power Market. – *Annals of Operations Research*, **121**, 2003, No 1-4, 79-104.

17. K e l l e y, C., J. G a r s o n, J., A. A g g a r w a l, S. S a r k a r. Place Prioritization for Biodiversity Reserve Network Design: A Comparison of the SITES and ResNet Software Packages for Coverage and Efficiency. – *Diversity and Distributions*, **8**, 2002, 297-306.
18. K n o w l e s, J. D., D. W. C o r n e. M-PAES: A Memetic Algorithm for Multiobjective Optimization. – In: Proc. of the Congress on Evolutionary Computation, Vol. **1**, Piscataway, New Jersey, July. IEEE Service Center, 2000, 325-332.
19. K o r h o n e n, P. Multiple Objective Linear Programming in Supporting Forest Management. – In: *Multiple Use of Forests and Other Natural Resources*. F. Hellens, A. Holten, L. Wichmann, Eds. Dordrecht, Kluwer, 1998, 85-95.
20. K o r h o n e n, P. Interactive Methods. – In: *Multiple Criteria Decision Analysis: State of the Art Surveys*. J. Figueira, S. Greco, M. Ehrgott, Eds. London, Springer Verlag. 2005, 642-665.
21. L o t o v, A. V., V. A. B u s h e n k o v, O. L. C h e r n y k h. Multicriteria DSS for River Water-Quality Planning. – *Computer-Aided Civil and Infrastructure Engineering*, **12**, 1997, No 1, 57-67.
22. M l a d i n e o, N., J. M a r g e t a, J. P. B r a n s, B. M a r e s h a l. Multicriteria Ranking of Alternative Locations for Small Scale Hydroplants. – *European Journal of Operational Research*, **31**, 1987, 215-222.
23. M i e t t i n e n, K. *Nonlinear Multiobjective Optimization*. Boston, Kluwer Academic Publishers, 1999.
24. M i e t t i n e n, K. Interactive Nonlinear Multiobjective Procedures. – In: *Multiple Criteria Optimization: State of the Art Annotated Bibliographic Surveys*. M. Ehrgott, X. Gandibleux, Eds. New York, Springer, 2003, 227-276.
25. M i t t e l m a n n, H., P. S p e l l u c c i. *Decision Tree for Optimization Software*, 2005. <http://plato.la.asu.edu/guide.html>
26. M u s t a j o k i, J., R. P. H a m a l a i n e n, M. M a r t t u n e n. Participatory Multicriteria Decision Support with Web-HIPRE: a Case of Lake Regulation Policy. – *Environmental Modelling and Software*, **19**, 2004, 537-547.
27. N a r u l a, S. C., V. V a s s i l e v, K. G e n o v a, M. V a s s i l e v a. A Partition-Based Interactive Method to Solve Discrete Multicriteria Choice Problems. – *Cybernetics and Information Technologies*, **3**, 2003, No 2, 55-66.
28. P a r s o n s, J. Agland Decision Tool: A Multicriteria Decision Support System for Agricultural Property. – In: *iEMSS, Integrated Assessment and Decision Support, Proceedings*, Vol. **3**, 181-187. <http://www.iemss.org/iemss2002/>
29. P a s c h e t t a, E., A. T s o u k i à s. A Real World MCDA Application: Evaluating Software. – *Journal of Multiple Criteria Decision Analysis*, **9**, 2000, 205-226.
30. R a j e s h, J. K., S. K. G u p t a, G. P R a n g a i a h., A. K. R a y. Multi-Objective Optimization of Industrial Hydrogen Plants. – *Chemical Eng. Sci.*, **56**, 2001, 999-1010.
31. S r d j e v i c, B., Y. D. P. M e d e i r o s, A. S. F a r i a. An Objective Multi-Criteria Evaluation of Water Management Scenarios. – *Water Resources Management*, **18**, 2004, 35-54.
32. S r i n i v a s, N., K. D e b.. Multiobjective Optimization using Nondominated Sorting Genetic Algorithms. – *Evolutionary Computation*, **2**, 1994, No 3, 221-248.
33. T h i b a u l t, J., R. L a n o u e t t e, C h. F o n t e i x, L. N. K i s s. Multicriteria Optimization of a High-Yield Pulping Process. – *The Canadian Journal of Chemical Engineering*, **80**, 2002, No 5, 897-902.
34. V a n d e r b e i, R. *Linear Programming: Foundations and Extensions*. Kluwer Academic Publishers, 1996.
35. V a s s i l e v, V., K. G e n o v a. An Algorithm of Integer Feasible Directions for Linear Integer Programming. – *European Journal of Operational Research*, **52**, 1991, 203-214.
36. V a s s i l e v a, M., V. V a s s i l e v, B. S t a y k o v, D. D o c h e v. Generalized Multicriteria Optimization Software System MKO-2. – In: Proc. of the 9th International Conference on Enterprise Information Systems (ICEIS 2007). G. Cordoso et al., Eds. Madeira, Portugal, 2007, 283-289.
37. V a s s i l e v a, M. Generalized Scalarizing Problem of Multicriteria Optimization. – *Compt. Rend. Acad. Bulg. Sci.*, **58**, 2005, No 5, 537-544.

38. V a s s i l e v a, M. Generalized Interactive Algorithm for Multicriteria Optimization. *Problems of Engineering Cybernetics and Robotics*, **56**, 2006, 69-77.
39. V e r a, J., P. d e A t a u r i, M. C a s c a n t e, N. V. T o r r e s. Multicriteria Optimization of Biochemical Systems by Linear Programming: Application to Production of Ethanol by *Saccharomyces Cerevisiae*. – *Biotechnol Bioeng*, **83**, 2003, No 3, 335-343.
40. V i n c k e, P. *Multicriteria Decision-Aid*. New York, John Wiley & Sons, 1992.
41. W e i s t r o f f e r, H., C. S m i t h, S. N a r u l a. Multiple Criteria Decision Support Software. – In: *Multiple Criteria Decision Analysis: State of the Art Surveys*. J. Figueira, S. Greco, and M. Ehrgott, Eds. London, Springer Verlag, 2005, 990-1018.
42. W o l s e y, L. A. *Integer Programming*. Wiley-Interscience, 1998.
43. Y i n y u, Y e. *Interior Point Algorithms: Theory and Analysis*. New York, Wiley & Sons, 1997.
44. Z o p o u n i d i s, C., M. D o u m p o s. Developing a Multicriteria Decision Support System for Financial Classification Problems: The FINCLAS System. – *Optimization Methods and Software*, **8**, 1998, 277-304.
45. Z o p o u n i d i s, C., D. K. D e s p o t i s, I. K a m a r a t o u. Portfolio Selection Using the ADELAIS Multiobjective Linear Programming System. – *Computational Economics*, **11**, 1998, 189-204.