BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **8**, No 2

Sofia • 2008

Generalized Nets Having Arcs with Limited Global Capacities

Krassimir Atanassov

Centre for Biomedical Engineering "Prof. Ivan Daskalov", 1113 Sofia
E-mail: ***krat@bas.bg***

Abstract: *A new generalized net extension is defined and it is proved that this extension is conservative, similarly to all other extensions introduced up to now.*

Keywords: *Generalized net, Operator.*

1. Introduction

The idea of the concept of Generalized Net (GN, see [1, 2]) arises for the first time in 1982. Nowadays there are more than 25 different GN-extensions. For each extension it is proved that it is conservative, i. e, there exists an ordinary GN that represents the way of functioning and the results of the work of the given extension. Here a new GN-extension will be defined for the first time and it will be proved that it is also conservative.

2. On the concept of a generalized net having arcs with limited global capacities

Let us start with some notations:

- $N = \{0, 1, 2, \ldots\} \cup \{\infty\}$;
- $pr_i X$ is the i-th projection of the n-dimensional set, where $n \in N$, $n \geq 1$ and $1 \leq k \leq n$. More generally, for a given n-dimensional set X ($n \geq 2$)

$$pr_{i_1, i_2, \ldots, i_k} X = \prod_{j=1}^{k} pr_{i_j} X$$

$$(1 \leq i_j \leq n,\ 1 \leq j \leq k,\ i_{j'} \leq i_{j''} \text{ for } j' \neq j'');$$

- card(X) is the cardinality of set X.

Now, we shall introduce the definition of a transition of the new type of nets. It is described by a seven-tuple:

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle,$$

where:

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively); for the transition in Fig. 1 these are

$$L' = \{ l'_1, l'_2, \ldots, l'_m \}$$

and

$$L'' = \{ l''_1, l''_2, \ldots, l''_n \};$$

(b) t_1 is the current time-moment of the transition's firing;

(c) t_2 is the current value of the duration of its active state;

(d) r is the transition's *condition* determining which tokens will transfer from the transition's inputs to its outputs. Parameter r has the form of an Index Matrix (IM):

$$r = \begin{array}{c|ccccc} & l''_1 & \ldots & l''_j & \ldots & l''_n \\ \hline l'_1 & & & & & \\ \vdots & & & r_{i,j} & & \\ l'_i & & & (r_{i,j} \text{ – predicate}) & & \\ \vdots & & & (1 \le i \le m,\ 1 \le j \le n) & & \\ l'_m & & & & & \end{array},$$

where $r_{i,j}$ is the predicate which gives the condition for a transfer from the i-th input place to the j-th output place. When $r_{i,j}$ has a truth-value "*true*", then a token from the i-th input place can be transferred to the j-th output place; otherwise, this is impossible;

(e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|ccccc} & l''_1 & \ldots & l''_j & \ldots & l''_n \\ \hline l'_1 & & & & & \\ \vdots & & & m_{i,j} & & \\ l'_i & & & (m_{i,j} \ge 0 \text{ – natural number or } \infty) & & \\ \vdots & & & (1 \le i \le m,\ 1 \le j \le n) & & \\ l'_m & & & & & \end{array};$$

(f) $\square$ is called transition type and it is an object having a form similar to a Boolean expression. It may contain as variables the symbols that serve as labels for a transition's input places, and it is an expression built up from variables and the Boolean connectives $\wedge$ and $\vee$ determining the following conditions:

- $\wedge(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ – every place $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$ must contain at least one token,

➢ $\vee(l_{i_1}, l_{i_2}, ..., l_{i_u})$ – there must be at least one token in all places $l_{i_1}, l_{i_2}, ..., l_{i_u}$, where $\{l_{i_1}, l_{i_2}, ..., l_{i_u}\} \subset L'$.

When the value of a type (calculated as a Boolean expression) is "*true*", the transition can become active, otherwise it cannot.

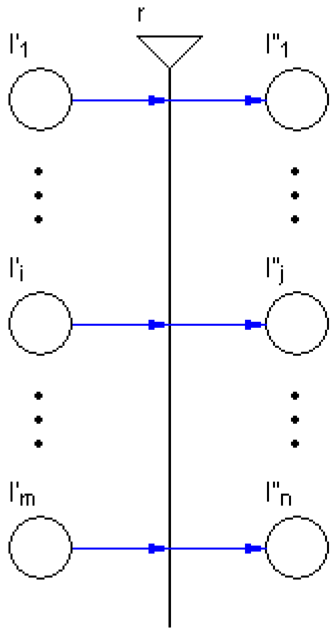

Fig. 1

The ordered four-tuple

$$E = \langle\langle A, \pi_A, \pi_L, c, f, C, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, b\rangle\rangle$$

is called a Generalized Net *having Arcs with Limited Global Capacities (GN-ALGC)* if:

(a) A is a set of transitions (see above);

(b) π_A is a function giving the priorities of the transitions, i.e., $\pi_A: A \to N$;

(c) π_L is a function giving the priorities of the places, i.e., $\pi_L: L \to N$, where

$$L = pr_1 A \cup pr_2 A.$$

Obviously, L is the set of all GN-places;

(d) c is a function giving the capacities of the places, i.e., $c: L \to N$;

(e) f is a function which calculates the truth values of the predicates of the transition's conditions (for the (ordinary) GNs, described in this section, the function f obtains values "false" or "true", or values from the set $\{0, 1\}$. If P is the set of the predicates used in a given model, then we can define f as $f: P \to \{0, 1\}$;

(f) C is a function giving the global limit of the arc capacities, i.e., $C: L \to N;$ here, the expression "*global capacities*" denotes that for a given arc $m_{i,j}$, $C(m_{i,j})$ is the maxiamal number of tokens that can go through the arc during the functioning of the GN;

(g) θ_1 is a function giving the next time-moment for which a given transition Z can be activated, i.e., $\theta_1(t) = t'$, where $pr_3Z = t$, $t' \in [T, T + t^*]$ and $t \le t'$. The value of this function is calculated at the moment when the transition terminates its functioning;

(h) θ_2 is a function giving the duration of the active state of a given transition Z, i.e., $\theta_2(t) = t'$, where $pr_4Z = t \in [T, T + t^*]$ and $t' \ge 0$. The value of this function is calculated at the moment when the transition starts functioning;

(i) K is the set of the GN's tokens. In some cases, it is convenient to consider this set in the form

$$K = \bigcup_{l \in Q^I} K_l\,,$$

where K_l is the set of tokens which enter the net from place l, and Q^I is the set of all input places of the net;

(j) π_K is a function giving the priorities of the tokens, i.e., $\pi_K: K \to N$;

(k) θ_K is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

(l) T is the time-moment when the GN starts functioning. This moment is determined with respect to a fixed (global) time-scale;

(m) t^0 is an elementary time-step, related to the fixed (global) time-scale;

(n) t^* is the duration of the GN functioning;

(o) In all publications on GNs it is defined that X is the set of all initial characteristics that the tokens can receive when they enter the net. Here, for the first time another interpretation of X will be introduced: X is a function which assigns initial characteristics to every token when it enters input places of the net;

(p) Φ is the characteristic function which assigns new characteristics to every token when it makes a transfer from an input to an output place of a given transition;

(q) b is a function giving the maximum number of characteristics a given token can receive, i.e., $b: K \to N$.

For example, if $b(\alpha) = 1$ for any token α, then this token will enter the net with some initial characteristic (marked as its zero-characteristic) and subsequently it will keep only its current characteristic. When $b(\alpha) = \infty$, token α will keep all its characteristics. When $b(\alpha) = k < \infty$, except its zero-characteristic, token α will keep its last k characteristics (characteristics older than the last k will be "forgotten"). Hence, in general, every token α has $b(\alpha)+1$ characteristics when it leaves the net.

When for each arc $m_{i,j}$, of a given GN $C(m_{i,j}) = \infty$, then the GN-ALGC is an ordinary GN. Therefore, a GN-ALGC is an extension of a GN.

Theorem. The functioning and the results of the work of each GN-ALGC can be represented by an ordinary GN.

P r o o f. Let E be a given GN-ALGC. We construct the ordinary GN F with the form

$$F = \langle\langle A^*, \pi_A, \pi^*_L, c, f, \theta_1, \theta_2\rangle, \langle K^*, \pi^*_K, \theta^*_K\rangle, \langle T, t^o, t^*\rangle, \langle X^*, \Phi^*, b\rangle\rangle,$$

where A^* is the set of the F-transitions. Let transition Z^* of F, corresponding to transition Z of E, have the form

$$Z^* = \langle L'^*, L''^*, t_1^*, t_2^*, r^*, M^*, \square^* \rangle,$$

(see Fig. 2) where t_1^* and t_2^* are as the above and

$$L'^* = L' \cup \{l_Z\}$$
$$L''^* = L'' \cup \{l_Z\}$$
$$\square^* = \wedge(\square, l_Z).$$

If

$$r = pr_5\, Z = [L', L'', \{r_{li,lj}\}]$$

has the form of an IM, then

$$r^* = pr_5\, Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{r^*_{\mathrm{li,lj}}\}]$$

where

$$(\forall\ l_i \in L')(\forall\ l_j \in L'')(\ r^*_{li,lj} = r_{li,lj}\ \&\ \text{“}pr_2[x^{\alpha_z}{}_{0,}\ l_i, l_j] \geq pr_2[x^{\alpha_z}{}_{cu,}\ l_i, l_j]\text{”}),$$

where $pr_2[x^{\alpha_z}{}_{k,}\ l_i, l_j]$ denotes the second component in couple “<< l_i, l_j>, $C(l_i, l_j)$>” in k-th characteristic of token α_z and $x^{\omega}{}_{cu}$ is the current characteristic of token ω,

$$(\forall\ l_i \in L')\ (\forall\ l_j \in L'')(\ r^*_{li,lz} = r^*_{lz,lj} = false),$$
$$r^*_{lz,lz} = true.$$

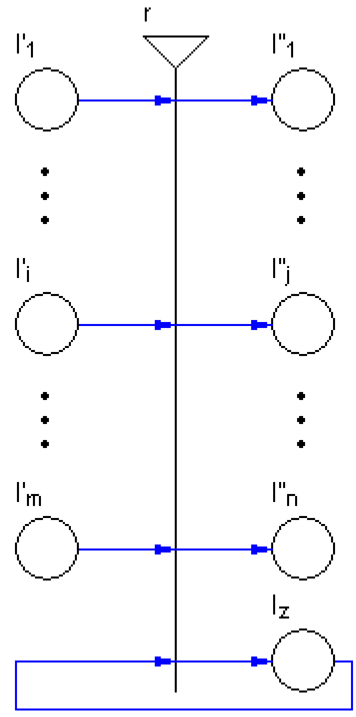

Fig. 2

If

$$M = pr_6\, Z = [L', L'', \{m_{li,lj}\}]$$

has the form of an IM, then

$$M^* = pr_6\, Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{m^*_{li,lj}\}],$$

where

$$(\forall\ l_i \in L')\ (\forall\ l_j \in L'')(\ m^*_{li,lj} = m_{li,lj}),$$
$$(\forall\ l_i \in L')\ (\forall\ l_j \in L'')(\ m^*_{li,lz} = m^*_{lz,lj}\ = 0),$$
$$m^*_{lz,lz} = 1.$$
$$\pi^*_L\ = \pi_L\ \cup\ \pi_{\{lz\,|\,Z\,\in\,A\}}\ ,$$

where the function $\pi_{\{lz\ |\ z\ \in\ A\}}$ determines the priorities of the new places, that are elements of the set $\{l_Z\ |\ Z\ \in\ A\}$ and the priorities of l_Z-places for every transition $Z \in A$ are the minimum among the place priorities of this transition Z;

$$c^*_L\ = c_L\ \cup\ c_{\{lz\,|\,Z\,\in\,A\}}\ ,$$

where function $c_{\{lz\,|\,z\ \in\ A\}}$ satisfies the equality

$$c_{\{lz\,|\,Z\,\in\,A\}}\ (l_Z) = 1$$

for place l_Z;

$$c^*_L\ = c_L\ \cup\ c_{\{lz\,|\,Z\,\in\,A\};}$$
$$K^*\ = K \cup\ \{\ \alpha_z\,|\,Z \in\ A\};$$
$$\theta^*_K = \theta_K\ \cup\ \theta_{\{\ \alpha_z\,|\,Z\,\in\,A\}},$$

where the function $\theta_{\{\ \alpha_z\,|\,Z\,\in\,A\}}$ determines that each α_z-token will stay in its place at the initial time-moment T

$$X^*\ = X \cup\ \{x^{\alpha_z}{}_0\,|\,Z \in\ A\},$$

where $x^{\alpha_z}{}_0$ is the initial α_z-token characteristic and it is

$$\text{“}\{<l_i,l_j>,\ C(l_i,l_j)>\,|\,l_i \in\ L'\ \&\ l_j \in\ L''\}\text{”}.$$
$$X^*\ = X \cup\ \{x^{\alpha_z}{}_0\,|\,Z \in\ A\},$$
$$\Phi^*\ = \Phi \cup\ \Phi_{\{lz\,|\,Z\,\in\,A\}},$$

where the function $\Phi_{\{lz\,|\,Z\,\in\,A\}}$ determines the characteristics of the α_z-tokens in the form

$$\Phi_{\{lz\,|\,Z\,\in\,A\}}\ (\alpha_z) = \text{“}\{<l_i,l_j>,\ \lambda\,(l_i,l_j)>\,|\,l_i \in\ L'\ \&\ l_j \in\ L''\}\text{”},$$

where $\lambda\,(l_i\ ,\ l_j)$ is the number of tokens going through the arc between places l_i and l_j.

Now, we shall prove that both GNs E and F function equally. For this aim we can compare the functioning of one arbitrary transition Z of GN E and its respective transition Z^* from F. Obviously, these transitions will start functioning in one time-moment and will have equal duration of functioning. They will have equal priorities, the respective places of both transitions will have equal capacities and priorities, the arcs will have equal capacities. Let us see the process of tokens transfer for one arbitrary chosen arc in transition Z – let it be between places l_i and l_j and its respective arc between places l^*_i and l^*_j in transition Z^*. Obviously, the arc from Z^* is not of the above mentioned special type. It can be easily seen that in both transitions an equal number of tokens will go through the respective arcs. If at the time of the current transition activation the total number of token transfers is smaller than the fixed limit, all tokens that can go through the respective arcs in the two transitions will realize these transfers. If in the present transition activation the capacity of the arc is filled, other tokens will not have possibility to go through it. The same situation will be observed for the respective arc in tranzition Z. Therefore, both arcs will have equal behaviour. Hence, both transitions have equal behaviour, but they were arbitrary transitions and therefore the behaviour of both GNs will be equal, which proves the Theorem.

In conclusion we shall mention that in a next research we will discuss the optimal way of token transfers through arcs with globally limited capacities. If each token can go through an arc with limited capacity, it is possible, its resource to receive new tokens will finish very quickly and in some cases this can generate difficulties of the GN-functioning. Therefore, it will be important to construct special algorithms determining which tokens to go through the arcs with limited global capacities.

References

1. A t a n a s s o v, K. Generalized Nets. New Jersey, London, World Scientific, Singapore, 1991.
2. A t a n a s s o v , K. On Generalized Nets Theory. Sofia, Prof. M. Drinov Publishing House, 2007.