

A Hybrid Direct Search – Quasi-Newton Method for the Inverse EIT Problem

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Abstract: *This paper presents a hybrid direct search – quasi-Newton method for the inverse nonlinear problem of Electrical Impedance Tomography (EIT) in 2D domain. It belongs to the interior path methods. The Finite Element Method (FEM) is used to solve the forward EIT problem regarding the nodal scalar potentials and current density values. The variational approach is applied to solve the inverse problem. The preliminary knowledge about the inhomogeneities in the domain is used.*

Keywords: *Electrical Impedance Tomography, Finite element method, Variational approach, Direct search, Quasi-Newton methods.*

1. Introduction

The Electrical Impedance Tomography (EIT) is an image reconstruction technique. It is designed to get an image of the electric field inside a studied object. In this way EIT makes non-destructive testing of materials, geophysical explorations such as core sample analysis and investigations of the Earth contamination, as well as biomedical diagnoses like diagnosis of breast cancer, investigation of chest organs and cerebral haemorrhaging (brain stroke). There are different algorithms for detection of flaws in materials. Some of them are presented in [11, 12, 13, 20, 21]. The system, proposed in [13, 21] permits geographically distributed research with remote measurement and data acquisition for eddy current test signals. In EIT technique low-frequency voltages, obtained as a result of injected currents in an inhomogeneous object, are measured by means of electrodes on the boundary of the studied object. Then two joined problems have to be solved: the forward and the inverse problem.

The *forward problem* is to calculate the scalar potentials (voltages), as well as the current density values inside the object, given an approximate conductivity distribution, boundary voltages and currents for known boundary geometry of the studied object.

The *inverse problem* is to receive an adequate estimation of the interior conductivity distribution, based on the calculated (known) scalar potentials and current density values.

At the end EIT gets an image of the electric field inside the object, based on the conductivity distribution in it.

To solve the forward problem the Finite Element Method (FEM) is used (see [23, 26]). The inverse problem – the conductivity recovering – is very complicated, since it involves the inversion of the nonlinear and compact operator (see [27]), mapping the conductivity σ of the studied specimen into the Dirichlet-to-Neumann map. It is known, that the inversion of a compact operator is an ill-posed problem, i.e. it either does not admit a unique solution or its solution does not depend continuously upon the data. Even an operator is invertible, the inverse of a compact operator defined onto an infinite-dimensional space is not continuous. One way to overcome this critical feature is to use a regularization method (see [5, 6]). This is a method for receiving a stable solution for the ill-posed problem, which is obligatory for developing reliable reconstruction algorithms. The regularization scheme uses a family of continuous operators approximating the inverse of the operator, which is to be inverted. Another way to overcome the lack of continuous dependence of the solution upon the data is to use the quasi-solution, obtained by minimization of the error functional $\Psi(\sigma)$, depending on the measured data. The introduction of the error functional $\Psi(\sigma)$ arises naturally, because usually there is available a contamination of the data by noise. The problem of computing the quasi-solution is well posed under suitable conditions, but the minimization of $\Psi(\sigma)$ is difficult, because there are local minima of $\Psi(\sigma)$ as a result of the nonlinearity of the compact operator and because of valleys or plateaus (regions, where the error functional is almost flat), resulting from the ill-posed start of the original problem. In this study the second way is chosen and the inverse EIT problem is converted to an optimization problem.

The computational complexity of the exact optimization methods for such ill-posed problems grows exponentially with the number of the unknown parameters and it depends on that, how much detailed mesh in FEM is chosen. Here it must be taken into account, that the purpose of EIT methods requires the use of finer mesh, in order to achieve a better quality of the reconstructed image. On the one hand the calculation of the first and the second order derivatives of the objective function is a slow procedure for such large size problems. On the other hand the noise in the data and the availability of plateaus of objective function values leads to very slow convergence to the optimal solution. To overcome these shortcomings many hybrid algorithms are proposed (see [1, 2, 7, 15, 16, 19, 22, 25]). Usually they apply a technique, which does not require local gradient information (like genetic algorithms) and hybridize it with a gradient-type method. To solve the inverse EIT problem for biomedical purposes, a new hybrid direct search method is proposed in

this study. It minimizes an error functional and its performance is enhanced alternating direct search and quasi-Newton steps.

The paper is organized as follows: A brief formulation of the problem is presented in Section 2. Section 3 states the mentioned hybrid direct search – quasi-Newton method. Some conclusions are drawn in section 4 and directions for further research are outlined.

2. The problem formulation

A. Experimental setup of the problem in 2D case

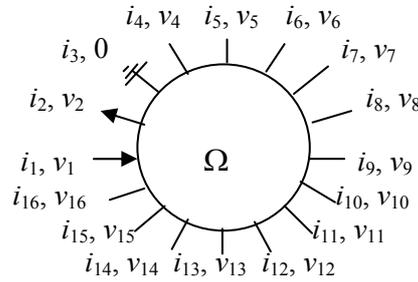


Fig. 1. Experimental setup for the EIT problem

The illustration of the experimental setup for the EIT problem is presented on Fig. 1. To perform measurements on the boundary of the studied 2D object, 16 electrodes are used. The object is considered as inhomogeneous, conducting body having a known overall shape Ω . For simplicity here the domain Ω is chosen to be a circle. It is divided by a uniform triangularization into 256 triangles. This mesh is assumed to be fine enough, so that the FEM numerical calculations are sufficiently accurate. Direct currents i_1 (input current) and i_2 (output current) are applied to the body. The injected current between these two electrodes has a value of 10 mA. The potentials (voltages) are measured between pairs of the other electrodes, where one of the electrodes in each pair is the grounded electrode. Usually the voltage at the injection electrodes cannot be measured reliably and for this reason it is not included in the data set. The measured voltages have values about 1 V. Each electrode can be held to be equi-potential and the contact impedance is neglected. In this case the current field $J(x)$ and the electric field $E(x)$ are constrained by the Kirchhoff's laws:

$$(1) \quad \nabla \cdot J(x) = 0,$$

$$(2) \quad \nabla \times E(x) = 0$$

and by the Ohm's law

$$(3) \quad J(x) = \sigma(x)E(x),$$

where $\sigma(x)$ is the conductivity and $J(x)$ is the current density. The body is assumed to be locally isotropic, so that $\sigma(x)$ is a positive real number. It is assumed that $\sigma = 1 S$ in all triangle elements, except in several chosen neighboring elements, where inhomogeneity is simulated, and where $\sigma = 2 S$.

Since $\nabla \times E = 0$, E has the form

$$(4) \quad E(x) = -\nabla\Phi(x),$$

where $\Phi(x)$ is the scalar potential (the voltage). The equations (1)-(4) are equivalent to the single elliptic equation for $\Phi(x)$:

$$(5) \quad \nabla \cdot (\sigma(x)\nabla\Phi(x)) = 0 \text{ in } \Omega.$$

B. Boundary data and feasibility constraints

The experimental setup consists in injecting a measured current between two electrodes and measurement of the voltage between pairs of other electrodes located on the boundary of the body. This procedure is repeated N times (N is the number of electrodes) clockwise, injecting current between all possible adjacent pairs of electrodes. For the setup in Fig. 1 we have $N = 16$. In case $\sigma(x)$ is known, $\Phi(x)$ and $J(x)$ are completely determined either by the boundary voltage $\Phi|_{\partial\Omega}$, or by the boundary current flux $J.n|_{\partial\Omega}$, where $n(x)$ is the unit outward normal to the boundary of the body $\partial\Omega$.

For the conductivity problem there are two distinct variational principles (see for example [3, 14]): the Dirichlet's principle:

$$(6) \quad \min \int_{\Omega} \sigma(x)|\nabla\Phi(x)|^2 dx \geq P,$$

where P is the power dissipated into heat (the measured power) in the true conductivity medium Ω , and its dual – the Thompson's variational principle, which takes the form

$$(7) \quad \int_{\Omega} \sigma^{-1}(x)|J(x)|^2 dx \geq P.$$

These two constraints allow us to obtain upper and lower bounds on the feasible domain of the space that contains the solution to the inverse problem (for details see [3, 4]).

C. Formulation of the direct problem

The direct EIT problem is decomposed as two quadratic optimization problems: The first one has the form:

$$(8) \quad \min \int_{\Omega} \sigma(x)|\nabla\Phi(x)|^2 dx,$$

subject to:

$$(9) \quad \Phi(x) = V(x) \text{ for } x \in \partial\Omega,$$

where $V(x)$, $x \in \partial\Omega$, are the measured potentials on the boundary of the body.

The second optimization problem has the form:

$$(10) \quad \min \int_{\Omega} \frac{1}{\sigma(x)} |J(x)|^2 dx,$$

subject to:

$$(11) \quad -J(x).n(x) = I(x) \text{ for } x \in \partial\Omega,$$

$$(12) \quad \int_{\partial\Omega} I(x)dx = 0,$$

$$(13) \quad \nabla \cdot J(x) = 0 \text{ for } x \in \Omega,$$

where $J(x)$ are the currents on the boundary $\partial\Omega$ and $n(x)$ is the unit outward normal to the boundary $\partial\Omega$.

The power dissipated into heat in Ω is

$$(14) \quad P = \int_{\partial\Omega} I(x)V(x)dx.$$

The current density $J(x)$ can be expressed by means of the electric vector potential $T(x)$ (see [27]):

$$(15) \quad J(x) = \nabla \times T(x).$$

Hence the second optimization problem can be written in the form :

$$(16) \quad \min \int_{\Omega} \frac{1}{\sigma(x)} |\nabla \times T(x)|^2 dx,$$

subject to:

$$(17) \quad -(\nabla \times T(x)) \cdot n(x) = I(x) \text{ for } x \in \partial\Omega,$$

$$(18) \quad \int_{\partial\Omega} I(x)dx = 0.$$

Starting with initial approximate values for $\sigma(x)$, $x \in \Omega$, we solve the optimization problems (8)-(9) and (16)-(18) by means of the FEM (see for example [23, 26]) and calculate $\Phi(x)$, $T(x)$ and $J(x)$, $x \in \Omega$.

D. Formulation of the inverse problem. A variational approach

The variational approach described in [17, 18] has been adopted here. Using data from N different measurements each time with different current injection pair of electrodes we solve N times the quadratic optimization problems (8)-(9) and (16)-(18). The essence of variational approach is to consider the linear equations (1) and (4) as constraints and to minimize the violation of nonlinear equation (3). So we solve the inverse EIT problem with unknowns $\sigma(x)$, $x \in \Omega$, minimizing the error functional:

$$(19) \quad F = \sum_{i=1}^N \frac{1}{2} \int_{\Omega} |\sigma(x)^{1/2} \nabla \Phi_i(x) + \sigma^{-1/2} J_i(x)|^2 dx,$$

subject to:

$$(20) \quad \Phi_i(x) = V_i(x), \quad -J_i(x) \cdot n(x) = I_i(x), \quad \nabla \cdot J_i(x) = 0, \quad i=1, \dots, N.$$

After expanding the square in (19) we have

$$(21) \quad F = \sum_{i=1}^N \left[\frac{1}{2} \int_{\Omega} \sigma(x) |\nabla \Phi_i(x)|^2 dx + \frac{1}{2} \int_{\Omega} \frac{1}{\sigma(x)} |J_i(x)|^2 dx + \int_{\Omega} J_i(x) \cdot \nabla \Phi_i(x) dx \right].$$

The last term in (21) is irrelevant to the minimization of F seeking $\sigma(x)$, because it is entirely determined by the boundary data. Minimization of the first

term in (21) corresponds to the Dirichlet's variational principle and the minimization of the second term corresponds to the Thompson's variational principle. The expression for $\sigma(x)$, which minimizes F in (21) has the form

$$(22) \quad \sigma(x) = \left(\sum_{i=1}^N |J_i(x)|^2 \right)^{1/2} \left(\sum_{i=1}^N |\nabla \Phi_i(x)|^2 \right)^{-1/2}.$$

3. A hybrid direct search – quasi-Newton method

From the point of view of mathematical programming (either linear or nonlinear) the optimization methods are divided into interior and exterior methods, depending on whether the iterative steps of the correspondent method are made inside or outside the feasible domain (see [8]). For example the least square method (see [28]) is an exterior method, whereas the Kohn and Vogelius method ([18]) is an interior method. Solving the EIT inverse problem both types of methods attempt to converge to a solution on the boundary of the feasible domain, but the exterior methods converge from outside the feasible domain, while the interior methods converge from inside the feasible domain. In [3] it is pointed out that the exterior methods can achieve convergence quickly for data without errors, but the interior methods have the advantage to be insensitive to data errors and perform stable. The disadvantage of interior methods is that they are often slowly converging.

The hybrid direct search algorithm, proposed here belongs to the interior algorithms. To reconstruct the electrical field image we solve the problem:

$$(23) \quad \min \mathbf{G} = \sum_{i=1}^N \left[\int_{\Omega} \sigma(x) |\nabla \Phi_i(x)|^2 dx + \int_{\Omega} \frac{1}{\sigma(x)} |J_i(x)|^2 dx \right]$$

subject to the constraints (20).

The constraints $\sigma(x) \geq 0$, (6) and (7) are used to reduce the step length if it is necessary.

To solve the inverse EIT problem ADI method (see [17]) performs iteratively the following procedure:

1) Using the last computed $\sigma(x)$ and the measured voltages, minimize (8) and (10) over $\Phi_i(x)$ and $J_i(x)$ for $i = 1, \dots, N$.

2) Using the obtained $\Phi_i(x)$ and $J_i(x)$ minimize \mathbf{G} from (23) over $\sigma(x)$, and update $\sigma(x)$ according to (22).

The authors pointed out that ADI method performs stable but very slow. More rapid convergence is achieved by means of a Modified Newton (MN) method (see [17]).

Several successful attempts are known, applying a genetic (see [7, 10]) or an evolutionary hybrid algorithm to solve this ill-posed problem (see for example [1, 2, 7, 15, 16, 20, 22, 25]). In [15] a genetic algorithm is combined with the Davidon-Fletcher-Powell method (see [9]) and with Pareto-optimization. The Powell method is used to enhance a genetic algorithm in [1, 2]. In [16] a genetic algorithm is coupled with Newton-Raphson method and mesh-grouping. In [22] a genetic

algorithm is used, alternating its generations by quasi-Newton steps. An evolutionary algorithm for such problem has been studied in [7]. In all these cases very encouraging results are obtained.

To solve the inverse EIT problem (23), (20) we also propose a hybrid optimization method in order to overcome the slow converging of the interior methods and the instability of the exterior methods when the signal/noise ratio is greater than 1%.

The method, proposed here to solve the EIT inverse problem is based on the Nelder and Mead's method [11], known as the most efficient among the direct search methods. In contrast to the above listed hybrid algorithms, where the number of genetic generations is reduced through a combination with another search method, the proposed method is a direct search method and is fast enough, i. e. comparatively small number of evaluations of the error functional \mathbf{G} from (23) are necessary. Here an alternating direct search steps and quasi-Newton steps approach is proposed in order to evaluate and to use the curvature of \mathbf{G} during the search process, when the direct search has no more success, or near the optimum.

A. Calculation of the Quasi-Newton steps

As pointed out in [17] the classic Newton's method performs well as long as the Hessian of the objective is positive definite. In our case the Hessian of \mathbf{G} may not be positive due to the data noise and taking into account the ill-posedness of the problem. For this reason a quasi-Newton approximation B of the Hessian of \mathbf{G} is calculated by means of Broyden-Fletcher-Goldfarb-Shanno (BFGS) – formula (see for example [9]):

$$(24) \quad B_{k+1} = B_k + \frac{y_k y_k^T}{y_k^T s_k} - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k},$$

where $s_k = \sigma(x)_{k+1} - \sigma(x)_k$, and $y_k = g(x)_{k+1} - g(x)_k$ is the increase of the gradient $g(x)$ of \mathbf{G} at the k -th iteration. The gradient $g(x)$ is evaluated by means of finite differences. The step s_k is calculated, solving the system:

$$(25) \quad B_k \cdot s_k = -g(x)_k.$$

After a given number of direct search iterations the proposed algorithm performs k consecutive quasi-Newton steps, where $k \in (3, 5)$. The initial B is assumed to be identity matrix, i.e. $B_1 = I$. Each new series of quasi-Newton iterations starts using as initial B the last found approximation of B at the previous series of quasi-Newton steps.

B. The new hybrid direct search – quasi-Newton method

The basic idea of the proposed new hybrid direct search method is to use the preliminary knowledge about the inhomogenities in the human body. Hence starting with a given conductivity $\sigma(x)_0$, we can perform consecutive search for optimal conductivity of different small regions of the feasible domain, reducing in this manner the number of unknown parameters and solving smaller equation systems.

Let us assume, that there are t sub-domains or small regions having an inhomogeneity inside them. In case the i -th sub-domain contains M unknown conductivity parameters, the direct search calculations are organized as follows.

Initially a regular simplex in the reduced M -dimensional search space is constructed. The simplex has $M + 1$ vertices. Starting from a known vertex $z^{(0)}$, the other M initial simplex vertices are calculated by means of:

$$(26) \quad z^{(i)} = \begin{cases} z_j^{(0)} + \delta_1 & \text{if } j \neq i, \\ z_j^{(0)} + \delta_2 & \text{if } j = i, \end{cases} \quad \text{for } i, j = 1, 2, \dots, M.$$

$$(27) \quad \delta_1 = \left[\frac{(M+1)^{1/2} + M - 1}{M\sqrt{2}} \right] \alpha,$$

$$(28) \quad \delta_2 = \left[\frac{(M+1)^{1/2} - 1}{M\sqrt{2}} \right] \alpha,$$

where α is a small positive number.

Find the weight center of the M best simplex vertices:

$$(29) \quad C = \frac{1}{M} \sum_{i=1}^M z^{(i)}.$$

Calculate a new simplex vertex reflecting the worst vertex $z^{(\text{worst})}$ towards C :

$$(30) \quad z^{(\text{new})} = C + \lambda(C - z^{(\text{worst})}),$$

where $\lambda = 2$. In case the step is successful a new attempt may be done by $\lambda = 3$. In case the step is not successful two new attempts will be done by $\lambda = 1.25$ and $\lambda = 0.75$, contracting the simplex.

“Pseudo-code” form of the direct search – quasi-Newton method:

```

BEGIN
Initialize  $\sigma(x)_0$ , the number of sub-domains (regions)  $t$  containing inhomogenities,
as well as the iteration limits  $itlim1$  and  $itlim2$ .
For  $ireg = 1, t$ ; do
    Set the initial Hessian approximation  $B_1 = I$ .
    While no stopping criteria are met do
        For  $icount1 = 1, itlim1$ ; do
            Create a regular simplex corresponding
            to the current number
            of unknown parameters  $M_i$ .
            Perform direct search steps.
            Check the stopping criteria.
        EndFor
        For  $icount2 = 1, itlim2$ ; do
            Evaluate the current gradient  $g(x)$  of  $G$  by means of finite
differences.
            Perform quasi-Newton steps.
            Check the stopping criteria.

```

EndFor

EndWhile

EndFor

END

The direct search stops if the contracting of the simplex cannot lead to further improvement of the obtained solutions or when the iteration limit *itlim1* is reached.

The quasi-Newton search stops when the error functional value G cannot be further minimized or when the iteration limit *itlim2* is reached.

The calculation procedure stops when the error functional value G becomes smaller than the prescribed value ε or when the global iteration limit *itlim* is reached.

4. Conclusions and future investigations

From the results presented for the Nelder and Mead's method and for the quasi-Newton method it can be concluded that the proposed new hybrid direct search – quasi-Newton method will be able to perform stable and will find the global optimal solution to the EIT inverse optimization problem comparatively quickly. The new hybrid method has the following good features and advantages:

- Performing an exploration of sub-domains the number of unknown conductivity parameters is drastically reduced and smaller equation systems are solved. Also the gradient evaluation by means of finite differences is facilitated, because the dimensionality of the search space is smaller. In this way the high efficiency of the method is guaranteed.
- The new method performs search in all preliminary known sub-domains, containing inhomogeneities. In this manner the method guarantees an exploration of the whole feasible domain and obtaining the global optimal solution.
- The new method combines the high efficiency of Nelder and Mead's method and the good convergence properties of BFGS-method. Making quasi-Newton steps guarantees that the curvature of the error functional G will be taken into account and the search process will be directed to the global optimum.

The new hybrid method will be tested on a set of test examples with different levels of data noise by means of a MATLAB computational program. The obtained results will be compared with those by other genetic and hybrid evolutionary algorithms solving the inverse EIT problem. The analysis of the results may lead to further refinement of the proposed hybrid method.

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