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CFAR Pulse Detectors in the Presence of Impulse Noise

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Abstract: *In a radar the performance of signal detectors is seriously degraded by arrival of impulse noise that extremely worsens their detectability characteristics. There are many papers, in which different Constant False Alarm Rate (CFAR) detectors have been analyzed in the presence of randomly arriving impulse interference, described as Poisson pulse sequences. In this paper the randomly arriving impulse interference is mathematically described as Binominal pulse sequences. This model of impulse noise is used for numerical analysis of two types of CFAR pulse detectors (CACFAR and EXC CFAR). The detectability of these pulse detectors is numerically analyzed in the presence of Poisson and Binomial impulse noise. Therefore, the proposed detectors can be used in different radar systems and secondary applications of communication technologies.*

Keywords: *Radar signal processing, CFAR processor, binary integration, randomly arriving impulse interference, average decision threshold, detection probability numerical-analysis algorithms.*

1. Introduction

Conventional Cell-Averaging Constant False Alarm Rate (CA CFAR) detectors are very efficient in the case of stationary and homogeneous interference. In such noisy environment the problem of target detection is formulated as detection of a single pulse on the background of Gaussian noise. In a CA CFAR detector, proposed by Finn and Johnson, pulse detection is declared if the signal value exceeds the threshold, which is formed by averaging the samples of the reference window surrounding the test cell [1]. The efficiency of CA CFAR pulse detectors is very sensitive to non-stationary and non-homogeneous background and it extremely degrades in the presence of strong randomly arriving impulse interference (impulse noise) in both the test resolution cell and the reference window [2]. In recent years different approaches have been proposed to improve the detectability of CFAR detectors operating in random impulse noise [3-14]. One of them is the use of ordered statistics for estimating the noise level in the reference window, proposed by Rohling [3]. In Ordered Statistic CFAR (OS CFAR) pulse detectors, the *k*-th ordered sample in the reference window is an estimate of the background level in the test resolution cell. The performance of such an OS CFAR detector in the presence of multipath interference in existing communication networks is evaluated and studied in [4]. Another approach to improve the performance of CFAR detectors in the presence of impulse interference is to excise high-power samples from the reference window before processing by a conventional CA CFAR pulse detector. Goldman used this approach for design of an excision CFAR detector (EXC CFAR) described in [5].

In this paper it is assumed that the samples of the total interference (thermal noise plus impulse noise) are distributed according to the compound exponential law where the weighting coefficients are the probabilities of corrupting and not corrupting the samples by impulse interference. It is also assumed that the samples in the test window are distributed according to the compound exponential law and the target returns fluctuated according to the Swerling II case.

As noted above in [6, 7], the quality of CA CFAR pulse train detectors is analyzed in the presence of impulse interference that arrives randomly in time from a single impulse-noise source. Such impulse interference can be mathematically described as a stochastic Poisson-model process, in which the occurrence of a random impulse in each range resolution cell is modeled as a Poisson event and the power of each random impulse is distributed according to the exponential law with a constant parameter. The other model of impulse noise is used for analysis of CFAR detectors in [8-14] where it is assumed that the impulse interference arrives from two independent impulse-noise sources operating in parallel. Each one of them generates a random impulse sequence with the same power intensity and the same average repetition frequency. In that case the impulse interference can be mathematically described as a stochastic Binomial-model process, in which the occurrence of a random impulse in each range resolution cell is modeled as a binomial event. The power of a random impulse generated by each impulse-noise source is distributed according to the exponential law with the same constant parameter.

The detectability of CFAR detectors can be evaluated in two possible ways. The first one is to estimate the detectability losses in Signal-to-Noise Ratio (SNR) with respect to the situation of no impulse noise. These detectability losses are defined for given values of the probability of detection and false alarm. This conventional approach is used in [6, 7] for evaluating the performance of CFAR pulse train detectors in the presence of Poisson impulse noise.

The other method for estimating the detectability of CFAR detectors is used in this paper. According to this method, the detectability of CFAR detectors is estimated in terms of the detectability losses in the ADT with respect to the situation of no impulse noise. The goal of this paper is to explore the detectability of two types of CFAR detectors in the presence of impulse noise by 1, evaluating the average decision threshold (ADT) of detectors (the usefulness of estimating the ADT for analysis of detectors was firstly demonstrated by Rohling in [3]) and 2) assuming that the impulse noise corresponds to the Binomial model.

Broadly speaking, this paper is an effort to summarize all our theoretical results in the analysis of CFAR pulse and pulse train detectors operating in the presence of Binomial impulse noise. A part of these theoretical results has been published earlier in [6-14].

2. Signal model

2.1. Binomial impulse noise

The Binomial model describes a situation when the impulse noise is derived from two independent and identical impulse-noise sources, each of them generating a random impulse sequence with the same power intensity and the same average repetition frequency [8-14]. The probability of occurrence (e) of a random pulse generated by each impulse-noise source in each range resolution cell can be expressed as $e=F_j t_c$, where F_j is the average pulse repetition frequency and t_c is the transmitted pulse duration. This means that the elements of the reference window are drawn from three classes. The first class represents the receiver noise only with probability $(1-e)^2$. The second class represents a situation when the signal samples are corrupted by a random impulse generated by one of the impulse-noise sources. This situation occurs with a probability $2e(1-e)$. The third class represents a situation when the signal samples are corrupted by a total random pulse that is a sum of pulses generated by the two impulse-noise sources. This situation occurs with a probability e^2. According to the theorem of total probability, the elements of the reference window are independent random variables distributed with the following probability density function (PDF):

$$
\begin{aligned}
f(x_i) = {} & \frac{(1-e)^2}{\lambda_0}\exp\left(\frac{-x_i}{\lambda_0}\right) + \frac{2e(1-e)}{\lambda_0(1+r_j)}\exp\left(\frac{-x_i}{\lambda_0(1+r_j)}\right) + \\
& + \frac{e^2}{\lambda_0(1+2r_j)}\exp\left(\frac{-x_i}{\lambda_0(1+2r_j)}\right), \quad i=1,\dots,N,
\end{aligned} \tag{1}
$$

where λ_0 is the average power of the receiver noise, r_j is the average per pulse interference-to-noise ratio (INR) at the receiver input, and N is the number of samples in the reference window.

In the presence of a wanted signal in the test resolution cell the signal samples are independent random variables distributed with the following PDF:

$$
\begin{aligned}
f(x_0) = {} & \frac{(1-e)^2}{\lambda_0(1+s)}\exp\left(\frac{-x_0}{\lambda_0(1+s)}\right) + \frac{2e(1-e)}{\lambda_0(1+r_j+s)}\exp\left(\frac{-x_0}{\lambda_0(1+r_j+s)}\right) + \\
& + \frac{e^2}{\lambda_0(1+2r_j+s)}\exp\left(\frac{-x_0}{\lambda_0(1+2r_j+s)}\right),
\end{aligned} \tag{2}
$$

where s is the average per pulse signal-to-noise ratio (SNR).

2.2. Poisson impulse noise

The Poisson model describes a real radar situation when the impulse noise arrives from a single impulse-noise source [6-7]. According to this model, in each range resolution cell the signal sample may be corrupted by impulse noise with a constant probability e_0. Therefore, the elements of the reference window are drawn from two classes. One class represents the interference-plus-noise with probability e_0. The other class represents the receiver noise only with probability $1 - e_0$. According to the theorem of total probability, the elements of the reference window are independent random variables distributed with the following PDF:

$$(3) \qquad f(x_i) = \frac{(1-e_0)^2}{\lambda_0}\exp\left(\frac{-x_i}{\lambda_0}\right) + \frac{e_0}{\lambda_0(1+r_j)}\exp\left(\frac{-x_i}{\lambda_0(1+r_j)}\right), \quad i = 1,\ldots, N.$$

In the presence of a desired signal in the test resolution cell the signal samples are independent random variables distributed with the following PDF:

$$(4) \qquad f(x_0) = \frac{(1-e_0)^2}{\lambda_0(1+s)}\exp\left(\frac{-x_0}{\lambda_0(1+s)}\right) + \frac{e_0}{\lambda_0(1+r_j+s)}\exp\left(\frac{-x_0}{\lambda_0(1+r_j+s)}\right).$$

The probability of occurrence of a random pulse in each range resolution cell can be expressed as $e_0=F_j \cdot t_c$, it must be noted that if the probability e_0 is small (e_0<0.1).

The corresponding probabilities of corrupting and not corrupting calculated as a function of the average repetition frequency F_j are shown in Fig. 1. It can be easy seen that when the probability of co-occurrence of random pulses arriving from the two impulse-noise sources is small (e^2 tends to 0), the Binomial model may be approximated with the Poisson models. In that case the probability $2e(1 - e)$ tends to e_0 and the probability $(1 - e)^2$ tends to $1 - e_0$.

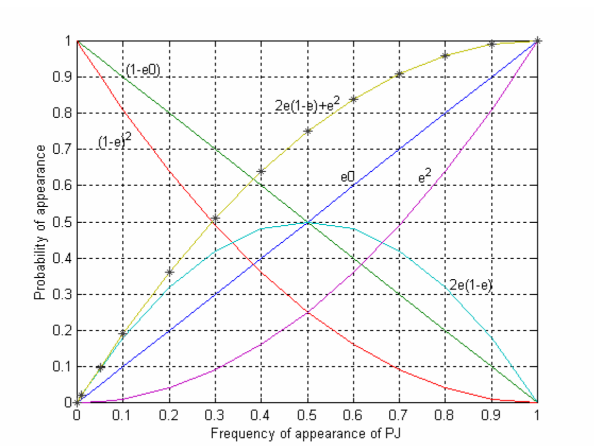

Fig. 1. Probabilities of occurrence of a random impulse and noise only for the Poisson model: e_0 and $1–e_0$. Probabilities of occurrence of a single random impulse, summed impulse and noise only for the Binomial model: e^2, $2e(1–e)$ and $(1–e)^2$

3. Analysis of CFAR pulse detectors

In a conventional CFAR pulse detector the estimate of the noise level V is calculated by using the samples of the reference window $\{x_i\}_N$ surrounding the test cell. The threshold of pulse detection (H_D) is a product of the estimate V and the predetermined detection scale factor T, i.e. $H_D=VT$. The pulse detection is declared, if the sample x_0 from the test resolution cell exceeds the threshold H_D:

$$\begin{cases} H_1: & x_0 \geq H_D \\ H_0: & x_0 < H_D \end{cases}, \tag{5}$$

where H_1 is the hypothesis that the test resolution cell contains a desired signal and H_0 is the hypothesis that the test resolution cell contains the receiver noise only.

According to the decision rule (5), the probability of pulse detection P_D and false alarm P_{FA} are defined as:

$$P_D = \int_0^{\infty} f_V(V)\,dV \int_{VT}^{\infty} f(x_0 / H_1)\,dx_0, \tag{6}$$

$$P_{FA} = \int_0^{\infty} f_V(V)\,dV \int_{VT}^{\infty} f(x_0 / H_0)\,dx_0,$$

where $f_V(V)$ is the PDF of the estimate V, $f(x_0/H_1)$ is the conditional PDF of the test sample under hypothesis H_1, and $f(x_0/H_0)$ is the conditional PDF of the test sample under hypothesis H_0. The detection scale factor T is determined to maintain a required probability of false alarm P_{FA}.

According to [3], the average decision threshold (ADT) of a CFAR pulse detector is defined as a normalized value:

$$\mathrm{ADT}_{\mathrm{CFAR}} = E(TV) / \lambda_0, \tag{7}$$

where E is the mathematical expectation of V calculated as

$$E(V) / \lambda_0 = -\frac{d}{dT} M_V(T / \lambda_0)\Big|_{T=0}. \tag{8}$$

Mostly, the efficiency of detection is evaluated in terms of the detectability losses in SNR with respect to the situation of no impulse noise. The detectability losses are defined for given values of the probabilities of detection and false alarm.

In this paper the detectability of pulse detectors is estimated by the detectability losses in ADT according to [3]. In that case, the detectability losses (Δ) are defined as the ratio of the two ADTs defined for given values of the probability of detection and false alarm. These losses are calculated as (in dB):

$$\Delta = 10\lg\frac{\mathrm{ADT}_1}{\mathrm{ADT}_2} = 10\log\frac{E(T_1V_1)}{E(T_2V_2)} \text{ for } P_{FA1} = P_{FA2},\quad P_{D1} = P_{D2} = 0.5, \tag{9}$$

where P_{FA1} and P_{D1} are the probabilities of detection and false alarm calculated for the case when the impulse noise is present at the receiver input, and P_{FA2} and P_{D2} are the probabilities of detection calculated for the case of no impulse noise.

3.1. Analysis of a CA CFAR pulse detector

In a conventional CA CFAR pulse detector proposed by Finn and Johnson, the noise level is estimated by averaging the outputs of the reference cells surrounding the test cell [1],

$$V = \sum_{i=1}^{N} x_i. \tag{10}$$

CA CFAR detector in Binomial impulse noise. For a conventional CA CFAR pulse detector, where the noise level is estimated by (10), the probability of pulse detection is readily computed using the expressions (4) and (6):

$$\begin{aligned} P_D = {} & (1-e)^2 M_V\left(\frac{T}{\lambda_0(1+s)}\right)+ \\ & +2e(1-e)M_V\left(\frac{T}{\lambda_0(1+r_j+s)}\right)+e^2 M_V\left(\frac{T}{\lambda_0(1+2r_j+s)}\right), \end{aligned} \tag{11}$$

where $M_V(.)$ is the moment generating function (MGF) of the noise level estimate V.

According to (10), the MGF of the noise level estimate V is calculated as a product of the MGF of all samples in the reference window, i.e. $M_V(U)=M_x^N(U)$, where $M_x(U)$ is the MGF of the random variable x_i and defined as

$$M_x(U) = \int_0^{\infty} \exp(-Ux) f(x)\, dx. \tag{12}$$

In the case of Binomial impulse noise, the PDF of each sample in the reference window $f(x)$, is defined by (3), and therefore the corresponding MGF is:

$$M_x(U) = \frac{(1-e)^2}{1+U\lambda_0} + \frac{2e(1-e)}{1+U\lambda_0(1+r_j)} + \frac{e^2}{1+U\lambda_0(1+2r_j)}. \tag{13}$$

Using (13), the MGF of the estimate V is calculated

$$M_V(U) = \sum_{i=0}^{N} \frac{C_N^i e^{2i}}{\left(1+U\lambda_0(1+2r_j)\right)^i} \sum_{j=0}^{N-i} \frac{C_{N-i}^j \left(2e(1-e)\right)^j}{\left(1+U\lambda_0(1+r_j)\right)^j} \frac{(1-e)^{2(N-i-j)}}{(1+U\lambda_0)^{N-i-j}}. \tag{14}$$

Replacing $M_V(U)$ in (11) by (14), the analytical expression for calculating the probability of pulse detection takes the form [8, 14],

$$P_{\mathrm{D}} = \sum_{i=0}^{N} C_N^i e^{2i} \sum_{j=0}^{N-i} C_{N-i}^j \left(2e(1-e)\right)^j (1-e)^{2(N-i-j)} \left\{R_1+R_2+R_3\right\}, \tag{15}$$

where $R_1 = \dfrac{(1-e)^2}{\left(1+\dfrac{T(1+2r_j)}{1+s}\right)^i \left(1+\dfrac{T(1+r_j)}{1+s}\right)^j \left(1+\dfrac{T}{1+s}\right)^{N-i-j}}$,

$$R_2 = \frac{2e(1-e)}{\left(1+\frac{T(1+2r_j)}{1+r_j+s}\right)^i \left(1+\frac{T(1+r_j)}{1+r_j+s}\right)^j \left(1+\frac{T}{1+r_j+s}\right)^{N-i-j}},$$

$$R_3 = \frac{e^2}{\left(1+\frac{T(1+2r_j)}{1+2r_j+s}\right)^i \left(1+\frac{T(1+r_j)}{1+2r_j+s}\right)^j \left(1+\frac{T}{1+2r_j+s}\right)^{N-i-j}}.$$

The probability of false alarm is evaluated by (15), where R_1, R_2 and R_3 are calculated for $s = 0$.

3.2. CA CFAR detector in Poisson impulse noise

In case of Poisson impulse noise, the analytical expression for calculating the probability of pulse detection is obtained in [6, 14],

$$P_\mathrm{D} = \sum_{i=0}^{N} C_N^i e_0^i \left(1-e_0\right)^{N-i} \left\{ \frac{e_0}{\left(1+\frac{(1+r_j)T}{1+r_j+s}\right)^i \left(1+\frac{T}{1+r_j+s}\right)^{N-i}} + \frac{1-e_0}{\left(1+\frac{(1+r_j)T}{1+s}\right)^i \left(1+\frac{T}{1+s}\right)^{N-i}} \right\}. \tag{16}$$

It can be easy seen that the same expression can be obtained from (15) under the assumption that $e^2 \to 0$, $2e(1-e) \to e_0$ and $(1-e)^2 \to 1-e_0$. It is the case when the probability of co-occurrence of the two random pulses in each range resolution cell becomes negligible, i.e. e^2 tends to 0, but the probability of occurrence of a random pulse derived from one of two impulse-noise sources is non-zero, i.e. $2e(1-e)>0$. The probability of false alarm is calculated by (16), setting $s = 0$.

3.3. Average decision threshold of CA CFAR detectors

In the case of Binomial impulse noise, the average decision threshold (ADT) of a CA CFAR pulse detector is calculated using equations (8), (14). After substituting $U=T/\lambda_0$ into (14), differentiating it with respect to T and substituting $T = 0$, the expression for calculating the ADT takes the form [8, 14]

$$\mathrm{ADT} = T\sum_{i=0}^{N} C_N^i e^{2i} \sum_{j=0}^{N-i} C_{N-i}^j \left(2e(1-e)\right)^j \left(1-e\right)^{2(N-i-j)} \left(N + r_j\left(2i+j\right)\right), \tag{17}$$

where the detection scale factor T is found as a solution of the equation (15), for a required value of the probability of false alarm P_FA and s=0.

In case of Poisson impulse noise, the expression for calculating the ADT can be easy obtained from (17) assuming that $e^2 \to 0$, $2e(1-e) \to e_0$ and $(1-e)^2 \to 1-e_0$ [11, 14]:

(18) $$\mathrm{ADT} = T\sum_{i=0}^{N} C_N^i e_0^i \left(1-e_0\right)^{N-i}\left(N+ir_j\right),$$

where the detection scale factor T is found as a solution of the equation (16), for a required value of the probability of false alarm P_{FA} and s=0.

For comparison, in case of no impulse noise, the expression for calculating the ADT is obtained in [2]:

(19) $$\mathrm{ADT}=TN\text{, where } T=(P_{\mathrm{FA}})^{-1/N}-1.$$

3.4 Analysis of an excision CFAR pulse detector

In an excision CFAR pulse detector (EXC CFAR), the noise level estimate V is formed as an average mean of nonzero samples at the output of the limiter, which nulls all the input samples that exceed an excision threshold [5, 7, 14]:

(20) $$V=(1/k)\sum_{i=1}^{k} y_i, \quad \text{where} \quad y_i=\begin{cases} x_i & \text{if } x_i \le B_E \\ 0 & \text{otherwise} \end{cases},$$

where k is the number of nonzero samples at the limiter output, and B_E is the excision threshold.

Excision CFAR detector in Binomial impulse noise. In case of Binomial impulse noise, the PDF of nonzero samples y_i at the limiter output may be expressed as

(21) $$f\left(y_i\right)=\frac{\left(1-e\right)^2\exp\left(\frac{-y_i}{\lambda_0}\right)}{\lambda_0\left(1-\exp\left(\frac{-B_E}{\lambda_0}\right)\right)}+\frac{2e\left(1-e\right)\exp\left(\frac{-y_i}{\lambda_0\left(1+r_j\right)}\right)}{\lambda_0\left(1+r_j\right)\left(1-\exp\left(\frac{-B_E}{\lambda_0(1+r_j)}\right)\right)}+ \\ +\frac{e^2\exp\left(\frac{-y_i}{\lambda_0\left(1+2r_j\right)}\right)}{\lambda_0\left(1+2r_j\right)\left(1-\exp\left(\frac{-B_E}{\lambda_0(1+2r_j)}\right)\right)}.$$

The probability that an input sample x_i survives at the limiter output is

(22) $$P_E=1-\left(1-e\right)^2\exp\left(\frac{-B_E}{\lambda_0}\right)-2e\left(1-e\right)\exp\left(\frac{-B_E}{\lambda_0(1+r_j)}\right)-e^2\exp\left(\frac{-B_E}{\lambda_0(1+2r_j)}\right).$$

The probability that k out of N samples of the reference window survive at the limiter output is

(23) $$q(k)=C_N^k P_E^k(1-P_E)^{N-k}.$$

The MGF of the random non-zero variable y_i at the limiter output is given by

(24) $$M_y(U)=\frac{\left(1-e\right)^2\left(1-\exp\left(R_1-B_EU\right)\right)}{\left(1-\exp\left(R_1\right)\right)\left(1+U\lambda_0\right)}+ \\ +\frac{2e\left(1-e\right)\left(1-\exp\left(R_2-B_EU\right)\right)}{\left(1-\exp\left(R_2\right)\right)\left(1+U\lambda_0\left(1+r_j\right)\right)}+\frac{e^2\left(1-\exp\left(R_3-B_EU\right)\right)}{\left(1-\exp\left(R_3\right)\right)\left(1+U\lambda_0\left(1+2r_j\right)\right)},$$

where

(25) $$R_1 = \frac{-B_E}{\lambda_0}, \quad R_2 = \frac{-B_E}{\lambda_0(1+r_j)}, \quad R_3 = \frac{-B_E}{\lambda_0(1+2r_j)}.$$

Since the random variables y_i, $1 \le i \le k$, are independent, the MGF of the estimate V takes the form:

(26) $$M_V(U,k) = M_y^k(U/k).$$

Using equations (23)-(26) we obtain the following expression for calculating the MGF of the noise level estimate V:

(27) $$M_V(U,k) = \sum_{i=0}^{k} C_k^i \left\{ \frac{e^2 (1-\exp(R_3 - B_E U/k))}{(1-\exp(R_3))(1+U\lambda_0(1+2r_j)/k)} \right\} \times$$
$$\times \sum_{j=0}^{k-i} C_{k-i}^j \left\{ \frac{2e(1-e)(1-\exp(R_2 - B_E U/k))}{(1-\exp(R_2))(1+U\lambda_0(1+r_j)/k)} \right\}^j \left\{ \frac{(1-e)^2 (1-\exp(R_1 - B_E U/k))}{(1-\exp(R_1))(1+U\lambda_0/k)} \right\}^{k-i-j}.$$

The conditioning on k is removed by averaging the last result with coefficients $q(k)$ defined by (23):

(28) $$M_V(U) = \sum_{k=1}^{N} C_N^k P_E^k (1-P_E)^{N-k} M_V(U,k).$$

The analytical expression for calculating the probability of pulse detection is obtained in [9, 14],

(29) $$P_D = \sum_{k=1}^{N} C_N^k P_E^k (1-P_E)^{N-k} \left\{ (1-e)^2 M_V\left(\frac{T}{\lambda_0(1+s)}, k \right) + \right.$$
$$\left. + 2e(1-e) M_V\left(\frac{T}{\lambda_0(1+r_j+s)}, k \right) + e^2 M_V\left(\frac{T}{\lambda_0(1+2r_j+s)}, k \right) \right\}.$$

The probability of false alarm is calculated by (29) setting $s = 0$.

Excision CFAR detector in Poisson impulse noise. In case of Poisson impulse noise, the analytical expression for calculating the probability of detection of an EXC CFAR pulse detector is obtained in [7, 14],

(30) $$P_D = \sum_{k=1}^{N} C_N^k P_E^k (1-P_E)^{N-k} \left\{ (1-e_0) M_V\left(\frac{T}{\lambda_0(1+s)}, k \right) + \right.$$
$$\left. + e_0 M_V\left(\frac{T}{\lambda_0(1+r_j+s)}, k \right) \right\}.$$

It can be easy seen that the same expression can be obtained from (29) assuming that $e^2 \to 0$, $2e(1-e) \to e_0$ and $(1-e)^2 \to 1-e_0$. It is the case when the probability of co-occurrence of two random pulses in each range resolution cell becomes negligible, i.e. e^2 tends to 0, but the probability of occurrence of a random pulse from one of two impulse-noise sources is non-zero , i.e. $2e(1-e)>0$. The probability of false alarm is calculated by (30), setting $s = 0$.

Average decision threshold of excision CFAR detectors. In case of Binomial impulse noise, the ADT of an excision CFAR pulse detector is calculated by the following expression:

(31) $$\mathrm{ADT} = -T\sum_{k=1}^{N} C_N^k P_E^k \left(1-P_E\right)^{N-k} \left(\frac{d}{dT}\left(M_V\left(\frac{T}{\lambda_0}, k \right)\right)\right)\Bigg|_{T=0}.$$

After substituting $U=T/\lambda_0$ into (27), differentiating it with respect to T and substituting $T = 0$ into (31), the expression for calculating the ADT takes the form

(32) $$\begin{aligned}\mathrm{ADT} = T\sum_{k=1}^{N} C_N^k P_E^k \left(1-P_E\right)^{N-k} \sum_{i=0}^{k} C_k^i \left(\frac{e^2}{\left(1-\exp\left(R_3\right)\right)} \right)^i \times \\ \times \sum_{j=0}^{k-i} C_{k-i}^j \left(\frac{2e(1-e)}{\left(1-\exp\left(R_2\right)\right)} \right)^j \left(\frac{(1-e)^2}{\left(1-\exp\left(R_1\right)\right)} \right)^{k-i-j} \times \\ \times \frac{1}{k}\left(1-\exp\left(R_3\right)\right)^i \left(1-\exp\left(R_2\right)\right)^j \left(1-\exp\left(R_1\right)\right)^{k-i-j} \times \\ \times \left(i\left(\frac{B_E \exp\left(R_3\right)}{1-\exp\left(R_3\right)} + \left(1+2r_j\right) \right) + j\left(\frac{B_E \exp\left(R_2\right)}{1-\exp\left(R_2\right)} + \left(1+r_j\right) \right) + \right. \\ \left. + \left(k-i-j\right)\left(\frac{B_E \exp\left(R_1\right)}{1-\exp\left(R_1\right)} + 1 \right) \right),\end{aligned}$$

where the detection scale factor T is found as a solution of the equation (29) for a required value of the probability of false alarm P_{FA} and s=0.

In case of Poisson impulse noise, the expression for calculating the ADT can be easy obtained from (32) assuming that $e^2 \to 0$, $2e(1 - e) \to e_0$ and $(1- e)^2 \to (1- e_0)$ [14]:

(33) $$\begin{aligned}\mathrm{ADT} = T\sum_{k=1}^{N} C_N^k P_E^k \left(1-P_E\right)^{N-k} \sum_{j=0}^{k} C_k^j \left(\frac{e_0}{\left(1-\exp\left(R_2\right)\right)} \right)^j \left(\frac{1-e_0}{\left(1-\exp\left(R_1\right)\right)} \right)^{k-j} \times \\ \frac{1}{k}\left(1-\exp\left(R_2\right)\right)^j \left(1-\exp\left(R_1\right)\right)^{k-i-j} \left(j\left(\frac{B_E \exp\left(R_2\right)}{1-\exp\left(R_2\right)} + \left(1+r_j\right) \right) + \right. \\ \left. + \left(k-j\right)\left(\frac{B_E \exp\left(R_1\right)}{1-\exp\left(R_1\right)} + 1 \right) \right),\end{aligned}$$

where the detection scale factor T is found as a solution of the equation (30) for a required value of the probability of false alarm P_{FA} and $s = 0$.

4. Numerical results

4.1. CFAR pulse processors in the presence of impulse interference with unknown parameters

The presence of impulse interference with unknown parameters in both, the test resolution cells and the reference cells, is a situation when CFAR processors do not keep the constant probability of false alarm. In calculations of the false alarm probability for the case of strong impulse interference with varying parameters, we used the value of a scale factor obtained for the homogeneous background. The numerical results for the probability of false alarm are obtained for the following parameters: the average receiver noise power (λ_0) equals 1, the average interference-to-noise ratio (r_j) equals 5 dB and 30 dB, the impulse interference probability varies from 0 to 1, the number of reference cells (N) equals 16, the probability of false alarm is $P_{\text{fa}} = 10^{-6}$ and, finally, the excision threshold is B_E=2.

The numerical results, depicted in Figs. 2 and 3, shows the influence of a scale factor over the probability of false alarm in CA CFAR and EXC CFAR processors, which operate with the fixed scale factor in the presence of strong impulse interference. It can be easy seen, that the performance of EXC CFAR processors is more stable under conditions of impulse noise than the performance of CA CFAR processors.

It can be seen that the CFAR processors under study fail to maintain the constant level of false alarm, using the fixed scale factor for threshold formation. This problem can be overcome, if the scale factor is adapted to varying parameters of impulse interference. We propose to choose the value of a scale factor from a matrix, which contains the values of a scale factor, preliminary calculated for different impulse interference parameters.

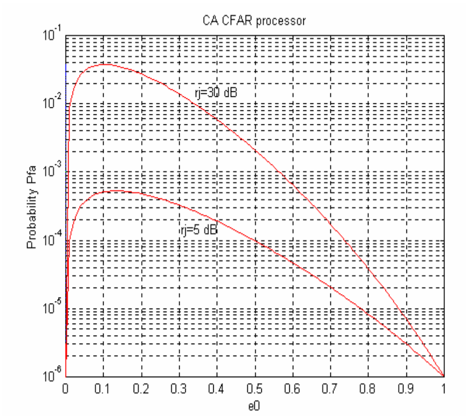

Fig. 2. False alarm probability is not maintained to be constant (P_{fa}=10^{-6}) – CA CFAR

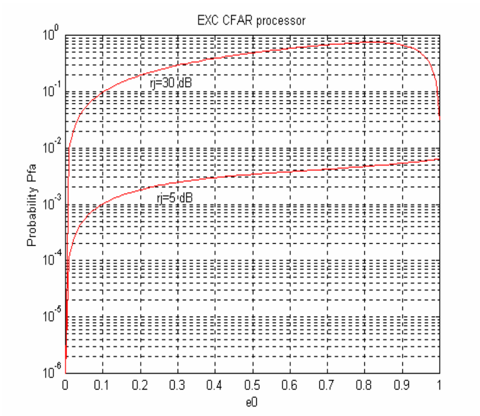

Fig. 3. False alarm probability is not maintained to be constant (P_{fa}=10^{-6}) – EXC CFAR

4.2. CFAR pulse processors in the presence of impulse interference with known parameters

For both pulse detectors, CA CFAR and EXC CFAR, the ADT can be evaluated analytically using equations (18, 33). The same results can be also obtained using the SNR corresponding to the detection probability of 0.5 evaluated by equations (16, 30). The following input parameters are used in calculations of the ADT: λ_0=1 is the average power of the receiver noise; r_j = 30 dB is the INR; the probability of occurrence of a random pulse varying in the range from 0 to 0.1; N = 16 is the size of a reference window; $P_{\text{fa}} = 10^{-6}$ is the probability of false alarm, and B_E = 2 is the excision threshold.

The ADT values calculated for the two pulse detectors are plotted in Fig. 4 as a function of the probability of occurrence of a random pulse in each range resolution cell.

The losses of the CFAR processor decrease when the excision threshold for censoring of the impulse interferences is used.

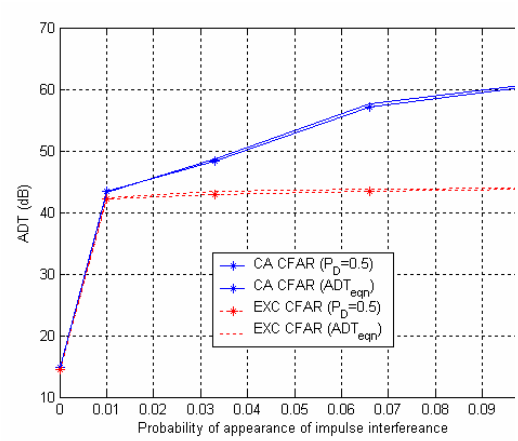

Fig. 4. Average decision threshold of CA and EXC CFAR processors in the presence of Poisson distribution of randomly arriving impulse interference. The ADT is obtained as the SNR required for the adjustment of P_D=0.5, and the ADT equation

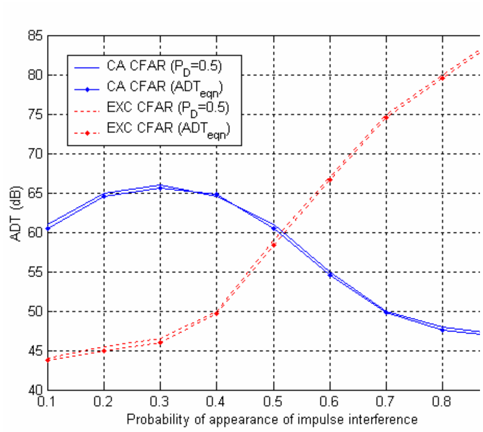

Fig. 5. Average decision threshold of CA and EXC CFAR processors in the presence of Binominal distribution flow from impulse interference, received as the SNR required for the adjustment of P_D=0.5, and the ADT equation

The ADT of the CA CFAR and EXC CFAR detectors in the presence of Binominal distribution flow from impulse interference are evaluated analytically using equations (17), (32), and are plotted in Fig. 5. The same results can be also obtained using the SNR corresponding to the detection probability of 0.5 evaluated by equations (15), (29). The following input parameters are used in calculations of the ADT: $\lambda_0 = 1$ is the average power of the receiver noise; $r_j = 30$ dB is the INR; $e \in [0.1, 0.9]$ is the probability of occurrence of a random pulse varying in the range from 0.1 to 0.9; $N = 16$ is the size of a reference window; $P_{fa} = 10^{-6}$ is the probability of false alarm, and $B_E = 2$ is the excision threshold.

On the one hand, the noise level estimate in a CA CFAR pulse detector increases with the probability of appearance of impulse noise in the range cell. On the other hand, in order to maintain the constant false alarm rate, the scale factor T should be decreased when the pulse interference frequency increases. It can be seen from Fig. 5 that the ADT value of a CA CFAR pulse detector increases when the probability of occurrence of impulse noise is relatively low and is within the interval [0.1, 0.3], and decreases when the probability of occurrence of a random pulse exceeds the value of 0.3.

In an excision CFAR pulse detector, the impulse noise is censored before noise level estimation, and because of this the noise level estimate in the reference window is maintained to be constant (Fig. 5). In order to maintain the constant false alarm rate, the scale factor T should be increased with the impulse interference frequency. It can be seen from Fig. 5 that the ADT value of an excision CFAR pulse detector increases very slowly when the probability of occurrence of a random pulse is relatively low and is within the interval [0.1, 0.3], and increases very quickly when this probability exceeds the value of 0.3.

The numerical results obtained show that excision CFAR pulse detectors are most suitable for practical application when the probability of occurrence of a random pulse is relatively small and does not exceed the value of 0.5. In cases when this probability is greater than 0.5, CA CFAR pulse detectors are more appropriate. As shown in Fig. 5, the numerical results, obtained analytically by equations (19), (32) and the ones calculated by equations (15), (29) using the approach of the SNR corresponding to the probability of detection of 0.5, are approximately identical. They differ from each other by about 1 dB. All numerical results are obtained in MATLAB computational environment.

5. Conclusions

In this paper the detectability of CFAR pulse detectors is numerically analyzed in the presence of Poisson and Binomial impulse noise. The numerical results presented in the paper show that in Poisson and Binomial impulse noise the excision CFAR pulse detectors are more appropriate in cases when the probability of occurrence of a random pulse is within the interval (0, 0.5). In the case when the probability of occurrence of a random impulse is high and exceeds 0.5 (Binomial impulse noise), the usage of CA CFAR pulse detectors is more appropriate.

For comparison, the analytical expressions for calculating the quality characteristics of detectors are presented not only for the Binomial model, but for the Poisson model as well. It is shown that the expressions for the Poisson model can be easy obtained from the corresponding expressions for the Binomial model. Therefore, the proposed detectors can be used in different radar systems and secondary applications of communication technologies.

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