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Distribution of Active and Reactive Energy in a Power Line

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Abstract: *The present paper discusses the main electrical parameters, used to evaluate the operation and control of a power line in a stationary mode. A large part of these parameters has been heuristically introduced, which requires their precise definition, aimed to obtain a conformable assessment, done in details in IEEE Standard* ***1459-2000****. Their definitions are presented in the paper without any comment, but attention is stressed on the digital approach, used nowadays to compute them. Some conclusions are made concerning Reactive power and Reactive energy, related with the existence of different definitions.*

Some approaches have been presented for the optimal compensation of the Reactive energy, particularly important for parametric and non-linear loads.

An approach is offered for the simultaneous assessment in a united computational procedure of the Active, Reactive, and Apparent energy. The method is based on actually measurable electrical features, and may be considered as a new definition of these Energies and Powers, functionally related with the terms already known.

Keywords: *Powers – active, reactive, complete, non-active; distortions; power factor; spectral analysis; time domain analysis; Hilbert transform.*

1. Introduction

The power line is a universal source of energy. It is easily accessed, the methods and tools developed enable its practical transformation into any other type, necessary for production purposes and everyday life – mechanical, heat, magnetic, chemical. It has gained some new, unusual functions in the recent years – to transmit information. Nowadays it mainly transfer bidirectional information for its own needs – quality and quantity of the power consumed, energy consumed, actual tariffs. Intensive expansion of its information functions up to its turning into universal energy and information environment is expected in the future. The network can accomplish these functions, supporting almost sinusoidal voltage and current waveforms, independently on the type and number of the users and the power consumed. This feature, generally defined as ***Quality of the electric service***, depends on the electrical energy source, the transmitting network, the distribution network and the users. Each one of these parts of the network puts in its contribution to the quality of service and must accept its responsibility.

A number of features are used to evaluate the amount and quality of the energy which is supplied and used. The most significant ones are those, accepted in IEEE 1459-2000 Standard [1]. Some new definitions of the different types of powers, of the Power factor and distortions are given in it, based on the contemporary power theory, developed in the last years. This theory accounts also the new types of users, causing nonsinusoidal currents and voltages and asymmetric loading of the three-phase network.

The evaluation of the status and the efficient control of the power line and the loads connected to it require the determination of many features, which is done with the help of the modern methods for digital signals processing.

The purpose of the present paper is to discuss the main parameters that must be calculated in a more or less developed system for control of the network and the loads in a stationary mode. The definitions of the different features introduced must emphasize the role, and hence – the responsibilities of the main network parts, being the following:

– The loads, including the circuits, connected for improving the efficiency and the quality of the energy consumed;

– The distributing and transmitting system;

– The power supplies.

The main approaches towards the determination of some parameters, which are not directly measurable and the obtaining of which is related with more complicated procedures, are presented. The methods for determination and compensation of the Reactive power are thoroughly discussed.

New energy quantities, that allows direct definition, are introduced, which enables a simultaneous computation procedure for Active, Reactive and Apparent energy. This approach may be accepted as a new definition of the Reactive power as well.

For some of the values only their definition and interpretation of the meaning is given, without any consideration of the computing procedure, if it follows directly from the definition. For example, the determination of the harmonic content and of the effective values of the voltages and currents is not discussed.

2. Main definitions, related with the electrical power. Reactive power

The classical theory of power and energy in an electrical network is built on the basis of sinusoidal voltages and currents, and linear loads. It is physics based, comprehensible and efficient to use. This theory operates with three main notions of powers and energies: Active, Reactive and Apparent. At that the responsibilities of the manufacturers, suppliers and users of electrical energy are distinguished very correctly. The theory gives a simple solution to the problem of compensating the Reactive energy.

According to Cauchy-Schwarz' inequality, the following could be written:

(1) $$\frac{1}{T}\int_0^T v(t)\, i(t)\, dt \le \|v(t)\|\,\|i(t)\|,$$

where $v(t)$ and $i(t)$ are the momentary values of the voltage and currents respectively, T is their period, $T = 1/f$, f is the frequency of the voltage, and $\|x\|$ denotes the Euclidean norm of the variable x, which is determined as:

$$\| x(t) \| = X = \sqrt{\frac{1}{T}\int x^2(t)dt}\ .$$

The Euclidean norm coincides with the widely used in electrical circuits theory Effective value of the variables: $V_{\text{eff}} \equiv V$ and $I_{\text{eff}} \equiv I$. Herein and further on the indexing of the effective value will be omitted for simplicity of the denotations.

Equality (1) is achieved only at proportionality between the variables

(2) $$i(t) = k\ v(t) \quad \forall k \in \Re\ .$$

This is the reason to decompose the current consumed into two components – a in-phase one and an orthogonal one to the voltage. The first one defines the Active Power (AP), while the second one – the Reactive Power (RP).

At sinusoidal voltage supply and linear load, the current is sinusoidal as well, but phase-shifted at φ rad in respect to the voltage. Then the instantaneous value of the power $p(t)$ is

(3) $$p(t) = V_m \sin(\theta) I_m \sin(\theta - \varphi)\ , \quad \theta = \omega\, t = 2\pi\, ft.$$

Using the Euclidean norm above mentioned, it could be written:

(4) $$p(t) = VI\cos\varphi(1 - \cos 2\theta) - VI\sin\varphi \sin 2\theta$$

or

(5) $$p(t) = P\,(1 - \cos 2\theta) - Q\sin 2\theta\ ,$$

where

(6) $$P = VI\ \cos\varphi\ ,$$

(7) $$Q = VI\ \sin\varphi\ .$$

The actually consumed power P, averaged over one period or over an integer number of voltage periods, is considered to be the AP.

In the expression (4) $\cos\varphi$ is a measure of the usage of the current and the voltage and it is denoted as a Power factor P_F. On its basis the users may be imposed certain penalties according to the degree of declination of P_F compared to 1.

The Active power P, as well as the Active energy, related with it, is actually measurable. It requires efficient averaging:

– In the late electro-mechanic meters this is realized by inertial mass in a rotating mechanical system;

– In contemporary computer-aided methods, integration after *Riemann* is generally used, with a quantization step, sampling of $v(t)$ and $i(t)$ is selected as a function of the signal waveform and the wished accuracy of computation.

The magnitude Q, shown by (7), presenting the second component in (4), is accepted as Reactive power. This component of the total power is altered with the doubled frequency of the voltage alteration and its average value is zero for every period of the network voltage.

The RP is given a sign, coinciding with the sign of the phase difference:

– Positive, when the current gets ahead of the voltage;

– Negative, when it is behind.

The equivalent RP from several consumers is equal to the algebraic sum of the Reactive powers of the particular consumers. Thus some closely located loads with opposite Reactive powers compensate each others mutually to a given extent and the network gives out only the difference. In a similar way every user may use reactive loads mainly – coils or capacitors, most often capacitors, which partially or completely compensate the reactive power of the main load to some extent

The RP thus defined is related with energy storing in the magnetic or the electric field of the consumer. There exists proportionality between the maxima of the accumulated energy and the energy, supplied by a source with power Q for one period of the network voltage. At sinusoidal voltages and currents every load, at first approximation, can be equivalently presented by a parallel connected resistor and capacitor C, or by a resistor and coil L, connected in series. The following equalities are obtained then:

(8) $$\frac{1}{2}CV_m^2 = \frac{T}{2\pi}Q \quad \text{and} \quad \frac{1}{2}LI_m^2 = \frac{T}{2\pi}Q.$$

Another important power feature, inherent to every load, is the Apparent Power (ApP) S, determined by the expression:

(9) $$S = VI.$$

This power is not physically definable, but it is an important quantitative parameter. It defines the power limit that the consumer can get without alteration of the losses in the network. It is related with the Active and Reactive power according to the relation:

(10) $$S^2 = P^2 + Q^2.$$

To a considerable extent S is proportional to the losses in the network.

In order to determine the Apparent energy, the norms of $i(t)$ and $v(t)$ must be defined in the boundary case for each period of the network cycle and their product has to be summed for the periods observed.

The Reactive power and its integral along the axis of time, the Reactive energy, introduced by F r y z l e [2], is related with the power that is not used by the load at one and the same losses in the feeders system and in the current source. Potentially it could be fully compensated near to the load, without additional loading of the network. This power is directly related with the efficiency of the energy consumption.

As it will be further exposed, the RP is an abstract, but very efficient value for estimation of the power, unutilized by the load. In its larger part it is connected with the energy, traveling in both directions between the load and the network, with accumulating and giving out of energy into and by the electric field of the capacitors and the magnetic field of the coils.

According to the definition, given in IEEE 1459-2000, the RP contains also power, connected with the different spectral content of the current and the voltage through and over the non-linear loads.

In order to define the RP, Fryzle [2] uses the time-domain approach and splits the current *i*(*t*) in two components – active $i_{\mathrm{a}}(t)$ and reactive $i_{\mathrm{r}}(t)$. The first one is considered coinciding in phase with the voltage *v*(*t*), while the second one – orthogonal to it:

(11)
$$i_{\mathrm{a}}(t) = \frac{P}{V^2} v(t), \qquad i_{\mathrm{r}}(t) = i(t) - i_{\mathrm{a}}(t)$$

Here *P* is the Averaged active power and *V* – the effective value of *v*(*t*).

The active component thus defined does not impose any constraints on the voltage waveform.

As Filipski's work [3] has already shown, the measuring of these components as analog values is relatively not very sophisticated, which is true also for the calculation of the RP in this way. Unfortunately, the reactive current, determined in this way, and hence – the Reactive energy, does not show how and to what extent it could be compensated at nonsinusoidal voltages.

In order to decrease this uncertainty, in [4] and [5] the Reactive component obtained by Fryzle, is additionally splitted into two orthogonal components. The first component is the one that could be compensated by reactivity – a coil or a capacitor, parallel connected to the load, while the second one is without a possibility for such compensation. In the analysis of the first component the authors use applied voltage that could differ from the sinusoidal shape. The current, consumed by the load *i*(*t*) (the Reactive, defined by Fryzle, can also be used without any change in the result), is projected upon the normalized forms of $\dot{v}'(t) \equiv \frac{dv(t)}{dt}$ and $w(t) = \int v(t)dt$ respectively:

(12)
$$\tilde{v} = \frac{v'(t)}{\| v'(t) \|} \quad \text{and} \quad \tilde{w}(t) = \frac{w(t)}{\| w(t) \|}.$$

It is assumed that the character of the load – of inductive or capacitive type, is known, so that two projections are made. The first projection – $i_{\mathrm{r}C}$, defined as capacitive, gives the current, that would flow through the capacitor in an equivalent parallel circuit of the load, while the second one, the inductive – $i_{\mathrm{r}L}$, determines the current through the parallel connected coil in the equivalent circuit.

(13)
$$i_{\mathrm{r}C} = \frac{v'(t)}{\| v'(t) \|^2} \frac{1}{T} \int i(t) \frac{dv}{dt} dt \; ;$$

(14)
$$i_{\mathrm{r}L} = \frac{w(t)}{\| w(t) \|^2} \frac{1}{T} \int i(t) w(t) dt.$$

The reactive energy, related with these components, is respectively

(15)
$$Q_C = \frac{\| v(t) \|}{\| v'(t) \|} \frac{1}{T} \int_0^T i(t) \frac{dv}{dt} dt$$

or

(16)
$$Q_L = \frac{\| v(t) \|}{\| w(t) \|} \frac{1}{T} \int_0^T i(t)\, w(t) dt$$

and for its compensation it is necessary to connect reactance, obtaining the same energy, but with an opposite sign.

In most of the cases at sofisticated, nonsinusoidal voltage waveform, the load cannot be equivalently presented by parallel connected resistor and reactance. Then partial compensation is possible using a complex compensating circuit of coils and capacitors or solving the optimization problem for minimization of the total reactive current.

It has been shown in [6] that in the presence of harmonics of high order, even at strongly expressed inductive load, the compensation with a simple circuit may prove pointless and could even increase the total reactive current.

This approach towards compensation is acceptable due to its simplicity. At powerful loads however, when the network impedance is not negligible, the voltage waveform is changed by the compensation circuit and the problem becomes too complicated.

The presentation of the actual waveform of the voltages and currents in the frequency domain could be done by their harmonics. This is possible because of their periodical character and it directly gives a feeling for their distortion and distinction from the ideal sinusoidal waveform.

(17)
$$v(t) = \sqrt{2} \sum_k V_k \sin(k\omega_0 t + \alpha_k) \; ,$$

(18)
$$i(t) = \sqrt{2} \sum_k I_k \sin(k\omega_0 t + \beta_k) \; .$$

At non-linear, non-harmonic voltages and currents, the presentation of which through Fourier series contains harmonics, the Reactive power can be determined in relation with the definition, proposed by B u d e a n u [7]:

(19)
$$Q_{\mathrm{B}} = \sum U_k I_k \sin \varphi_k \; , \qquad k = 1, 2, \ldots,$$

where $\varphi_k = \alpha_k - \beta_k$.

This is in fact the first definition of RP Q_B, denoted according to Budeanu, concerning nonharmonic in shape, nonsinusoidal voltage.

The determination of the spectral components gives complete information about the synthesis of a two-pole circuit of reactive components for complete compensation. It has been shown in [8] that in the presence of N considerable harmonics, complete compensation of the reactive power can be achieved with the help of a two-pole circuit of $N\,(2N-1)$ reactive components.

A study is discussed in [9] which has shown that satisfactory, though not complete compensation can also be achieved with the help of a two-poles circuit with much smaller number of components.

Definition (19), though theoretically well sustained and with certain practical significance, contradicts to definition (10) of the Apparent power S. According to it, the Apparent power S must be a geometric sum (Descartes' length) of the Active and Reactive powers and besides this it is determined as a product of the effective values of the current and the voltage, in which all spectral components participate. In Budeanu's definition the products of the homonymous components only are included in the Reactive power, the greater part of them being zeroes.

In order to preserve the requirement for a geometric sum when using Q_B, additional Budeanu's power can be added on the basis of the available harmonics D_B, thus obtaining for the Apparent power S the expression

$$S^2 = P^2 + Q_B^2 + D_B^2 \,. \tag{20}$$

3. Qualitative features of the power line at nonlinear loads

It is usually assumed that the electrical power supplies generate sinusoidal voltages. This is true for powerful supplies. The contemporary energy systems include a large number of distributed supplies with renovated primary energy – from wind, or from biological products, which, due to its nature cannot produce constant power and pure sinusoidal voltage. However the prime reason for nonsinusoidal currents and voltages are the non-linear loads, intensively coming into use – different types of invertors, motors with electronic-controlled speed, office and laboratory devices, high efficient lighting systems. Thus many harmonics, as well as quick spicks of the voltage (known as Flicker) are injected into the network.

Unfortunately the idealized model, which depicts the three components power, supplied to the loads, is not valid at nonsinusoidal voltages and currents. The nonsinusoidal in shape voltages and currents are in contradictions with the basic property of the network – the invariability of the shape, magnitude and frequency of the voltage supplied.

In 1995 an IEEE Working Group has issued some recommendations, a part of them being included in IEEE1459-2000 Standard for parameters, which could present the networks and loads at nonsinusoidal currents and voltages [9].

The main idea of the Group is to define the electrical parameters of the network – voltages, currents, powers in a stationary mode. Under these conditions, setting stationary in the periodicity of the processes in the network, the voltages and the currents are decomposed into dc, fundamental and harmonic terms. The evaluation parameters are formulated in the frequency domain, using these components namely. All harmonic components, without the fundamental ones, are treated as “contaminations” and must be rejected. The Group has not followed the purpose to make the features offered measurable in practice. But they must be able to present adequately the significant qualitative parameters of the network, and to be obtained as a result of computing procedures.

It should be noted, that this approach cannot evaluate directly the network transient response, sub-harmonics, integer harmonics, Flicker. Some other parameters must be defined in the Time-domain for such non-stationary processes and other approaches used – purely Time-domain, Frequency-Time, linear, quadratic.

According to IEEE1459-200 Standard, the voltages and currents are presented by their harmonic components:

(21) $$v(t) = V_0 + \sqrt{2}\sum_{h_v} V_h \sin(h\omega t + \alpha_h) ,$$

(22) $$i(t) = I_0 + \sqrt{2}\sum_{h_i} I_h \sin(h\omega t + \beta_h) ,$$

where V_0 and I_0 are dc components of the voltage and current.

The effective values (Euclidean norms) are determined by the geometric sum of the separate components:

(23) $$V = \sqrt{\sum_{h_v} V_h^2} , \qquad I = \sqrt{\sum_{h_i} I_h^2} .$$

The fundamental components V_1 and I_1 are separately defined, as well as the effective values of all the harmonics, indexed by H:

(24) $$V^2 = V_1^2 + I_H^2 ,$$

where $V_H = \sqrt{\sum_{h \neq 1} V_h^2}$ and

(25) $$I^2 = I_1^2 + I_H^2 ,$$

where $I_H = \sqrt{\sum_{h \neq 1} I_h^2}$.

Apparent powers S are introduced for the complex load, for the fundamental components, the harmonics and the interaction of the fundamental components with the harmonics

(26) $$S^2 = V^2 I^2 = V_1^2 I_1^2 + S_N^2 = (V_1 I_1)^2 + (V_1 I_H)^2 + (V_H I_1)^2 + (V_H I_H)^2 = S_1^2 + S_N^2 ,$$

where

(27) $$S_N = \sqrt{S^2 - S_I^2}$$

is the nonfundamental apparent power.

The nonfundamental apparent power S_{N} includes the Apparent power without the Fundamental apparent and it contains three components.

– The first component $V_1 I_H$ (power of current distortion), is the biggest one. It is to the greatest extent connected with load’s non-linearities and interpreted as a current part. This power is non-active, its average value is zero and it is not directly measurable.

– The second component $V_H I_1$ (power of voltage distortion), is related with the harmonics content in the voltage. Usually it is not large and as far as available, it is caused by distortion in the currents, passing through the loads in the circuit.

– The third component, Apparent power from the harmonics, S_H:

$$S_H^2 = U_H^2 I_H^2 = P_H^2 + Q_H^2 + Q_{HN}^2 , \tag{28}$$

shows the contribution of the harmonics to the Total apparent power. It contains active P_H and reactive Q_H power of the different spectral components and additional Nonfundamental Reactive power Q_{HN}:

$$P_H = \sum_{h \neq 1} I_h V_h \cos\varphi_h , \tag{29}$$

$$Q_H = \sum_{h \neq 1} V_h I_h \sin\varphi_h , \tag{30}$$

$$Q_{HN} = \sqrt{S_H^2 - P_H^2 - Q_H^2} . \tag{31}$$

The harmonic reactive power is defined after Budeanu [7]. At complete spectral presentation of $v(t)$ and $i(t)$, P_H and Q_H can be determined, while the Nonfundamental Reactive power Q_{HN} can be obtained through a computational procedure only.

The apparent power of every harmonic, including the fundamental one, can be presented as a geometric sum of the Active and Reactive power:

$$S_1^2 = P_1^2 + Q_1^2 , \tag{32}$$

$$S_h^2 = P_h^2 + Q_h^2 . \tag{33}$$

The Active power of all harmonics participates in the formation of the Total useful power and they must be taken into account. In the practical cases the Active power from the harmonics is useless and harmful for the consumer. It is usually related with undesirable losses of energy and hence this must impose some penalties on the energy supplier. Up to now it is indirectly accounted by the voltage distortion.

The Reactive power from the different harmonics is usually added to the remaining Nonfundamental Reactive powers, because the average value of the energy, supplied by them, is zero.

The degree of the contamination, of the load currents and voltages deviations from the sinusoidal shape is expressed by the Total Harmonic Distortions THD_T:

$$\mathrm{THD}_T^2 = \frac{S_N^2}{S_1^2} = \left(\frac{I_H}{I_1}\right)^2 + \left(\frac{V_H}{V_1}\right)^2 + \left(\frac{I_H}{I_1}\right)^2 \left(\frac{V_H}{V_1}\right)^2 . \tag{34}$$

The total harmonic distortions THD_T is presented as a geometric sum (Descartes' length) of three components, due to the presence of harmonics in the current, in the voltage, and in the product of them.

The Current distortions have got a determining significance in the Total distortions. In many cases they can get values greater than 1. For a network with small internal source resistance, the voltage shape is influenced to a minor extent by the Current distortions and hence THD_T remains low.

The part of the remaining components in the Total harmonic distortions is usually small.

The Total Power Factor P_F is accepted as a generalized index of the efficiency of the energy supplied, which involves the presence of harmonics, as well as the presence of Reactive powers. It is determined by the relation:

$$P_F = \frac{P}{S} = \frac{P_1 + P_H}{S} . \tag{35}$$

The Report of the above mentioned Group has defined the Reactive energy of the fundamental frequency only and it is added to the Total Nonactive energy. The determination of the Reactive energy for the harmonics is not foreseen. This might be considered a lapse.

Taking into account the Reactive powers due to the harmonics enables the evaluation of the necessity for their compensating. Some cogent cases are shown in [6], where the compensation of the Reactive power for the Fundamental frequency leads to considerable increment of the Reactive power for some of the harmonics up to an extent that makes the compensation itself pointless.

The recommendations of the Group offer indeed a many-sided evaluation of the quality of service and consumed energy.

It should be noted, that most of the features are not directly measurable, but defined with the *apriori* assumption that full spectral analysis of the currents and voltages is being accomplished.

4. Determination of reactive power

The definition of Reactive power, mainly pointed as a practical useful parameter, though not directly measurable, continues to split the specialists. The working Group recommends separate determination of the Reactive power at the fundamental frequency and for the harmonics this power is added to the Nonactive power. Such an approach supposes the accomplishment of spectral analysis and does not regard the possibility to compensate the Reactive power, caused by the high order harmonics. This could be acceptable, but at sufficiently “pure” network voltage.

Independently on the recommendation, most of the researchers continue to use the complete Reactive energy, because of its more informative character and also because of the more direct way for its determination, which does not require a full spectral analysis.

The most direct approach in solving this task is to implement Hilbert transformation on one of the variables – the current or the voltage and after that to multiply the result by the other one. The average value of the product obtained gives the Reactive power, defined according to Budeanu. The instantaneous value of the Reactive power for k -th harmonic is

(36) $$q_k(t) = 2V_m I_m \sin\theta \sin\left(\theta - \varphi + \frac{\pi}{2}\right) =$$

$$= V_m I_m \sin\varphi\,(1 - \cos 2\theta) + V_m I_m \cos\varphi \sin 2\theta,$$

and the total Reactive power will be

(37) $$Q = \frac{1}{2}\sum_{k=1}^{N} V_k I_k \sin\varphi_k .$$

The Nonactive power thus defined is obtained as a sum of the Reactive powers for all harmonics. This expression has its physical meaning. The power of every harmonic causes bidirectional traveling of energy between the load and the source. The summed power obtained gives the power that can be compensated.

It must be noted, that this Nonactive power is discussed [1] by the IEEE Standard above mentioned, where it is defined as combining all the nonactive components – with the fundamental frequency, and with the harmonics also.

Such a definition of the Reactive power has a considerable disadvantage, that for different frequencies the Power may have different polarity, as a result of which the Total power will be diminished. So the compensation of P_F at that is possible separately for the frequency components only.

The Reactive power Q_{hN} above obtained is not related with any reactivities in the load and is a result of non-linearities only. It cannot be compensated in a simple way.

Hilbert's transformation is given by the expression

(38) $$H\left[x(t)\right] = \hat{x}(t) = \frac{1}{\pi}\int \frac{x(s)}{t-s}dt .$$

The integral is considered in the basic meaning of *Cauchy*. As a result of this transformation each frequency component of $x(t)$ is shifted at $\frac{\pi}{2}$, preserving its magnitude.

The transformation of Hilbert of one function can be implemented as a linear operation by a linear filter with a transfer function

(39) $$H(e^{jw}) = \left| \begin{array}{ll} -j & 0 < w < \pi \\ j & -\pi < w < 0 \end{array} \right. .$$

The implementation of an ideal Hilbert's filter with one input and one output is very difficult. Usually a couple of filters are used and the transformed signals are obtained at the output. A similar approach is realized and the model is investigated in [11]. The author uses filters with IIR of 7th order. The influence of the harmonics up to 19th order is accounted and in conformance with this a sampling frequency of 2 kHz is selected. A Low-frequency filter is used at the output after the multiplier in order to average the instantaneous Reactive power. The filter study shows that:

– the results do not depend on the alteration of the frequency;

– the Impulse response of the filter to instant changes in the frequency is of the order of three periods of the fundamental frequency;

– the phase-shifting deviation with respect to 90 deg for the whole frequency range within (40-950) Hz is less than 2.10^{-2} deg.

Another more simplified approach to phase-shifting can be implemented, shifting the samples of one of the variables at 1/4 of the network cycle *T.* But for the harmonic components the phase-shifting is already with $h\frac{\pi}{2}$, where *h* is the harmonic's number.

A similar approach has been offered in [12]. The phase-shifting is accomplished through an integrator. After a Multiplier an integrator is connected, that measures the reactive energy in *var*. The electric Power is determined by a Differentiator of the Reactive energy.

In [13] a Time-domain approach is proposed to determine the Reactive energy. The expression for the instant power (3) is integrated separately for the intervals $\left(\frac{\pi}{2}, \pi\right)$ and after that in the interval $\left(\frac{3}{2}\pi, 2\pi\right)$, and $\left(0, \frac{\pi}{2}\right)$ and $\left(\pi, \frac{3}{2}\pi\right)$.

The difference from the results obtained gives a value, proportional to the Reactive energy for one period of the network cycle. Though not complicated, this approach requires the fundamental frequency's "zero"-detection.

The term, used for Reactive energy, is related with the energy, which is bidirectionally exchanged between the load and the network, during the time its average value remaining zero.

For non-linear loads and nonsinusoidal network voltage, Nonactive power and energy is introduced, which is with a zero-average value and is not related with the bidirectional energy traveling.

5. Determination of the powers, based on the energy merges between the network and the load

The energy, circulating between the load and the network, which is observed in the physical processes, is defined as Reactive energy, and its averaged value is the Reactive power. With this definition the values considered not only become actually measurable values, but they are also directly related with the real process of *Riemann*'s integration during the digital measurement of the Active power and energy.

At negative phase difference φ, at which the current goes behind the voltage, the energy is returned to the network within the interval $(0, \varphi)$, while at positive, within the interval $(\pi - \varphi, \pi)$.

At sinusoidal currents and voltages and negative phase difference, a conditional Negative energy E_{-} may be defined for one semi-period of the network waveform, given back to the network –

(40)
$$E_{-} = \int_{0}^{\frac{\varphi}{2\pi}T} p(t)dt = \frac{T}{2\pi} VI\left[\varphi\cos\varphi - \sin\varphi\right]$$

and Positive energy E_{+}, used by the load, connected to the network –

(41) $$E_+ = \int_{\frac{\varphi}{2\pi}T}^{\frac{T}{2}} p(t)dt = \frac{T}{2\pi}VI\left[\sin\varphi + \left(\pi - \varphi\right)\cos\varphi\right].$$

The active energy for the same semi-period is evidently the algebraic sum of the two energies:

(42) $$E_{\mathrm{a}} = E_+ + E_- = \frac{T}{2}VI\cos\varphi \,.$$

By definition the Reactive energy E_{r} for the same semi-period is

(43) $$E_{\mathrm{r}} = \frac{T}{2}VI\sin\varphi$$

and it is defined by the values, obtained for E_+ and E_-

(44) $$E_{\mathrm{r}} = \phi\, E_{\mathrm{a}} - \pi\, E_-$$

that leads to:

(45) $$E_{\mathrm{r}} = \varphi\, E_+ + (1-\pi)\, E_- \,.$$

The Apparent power S can be defined as a geometric sum of the Active and Reactive power:

(46) $$S = \sqrt{\frac{4}{T^2}\left(E_{\mathrm{a}}^2 + E_{\mathrm{r}}^2\right)}\,,$$

therefrom obtaining

(47) $$S = \frac{2}{T}\sqrt{(E_+ + E_-)^2 + [\varphi\, E_+ + (1-\pi)\, E_-]^2}\,.$$

The phase difference φ can be determined by the relation

(48) $$\frac{E_-}{E_+} = \frac{1}{1 + \dfrac{\pi}{\mathrm{tg}\,\varphi - \varphi}}\,.$$

So one can receive:

(49) $$\phi = \psi(E_+, E_-)\,.$$

The suggested definitions for Active (42), Reactive (45) and Apparent (47) energy are based on the real measurements results. They allow the simultaneous calculation of the Active, as well as Reactive and Apparent powers with the aid of a combined, relatively simple computing procedure. In addition, the Reactive power (45) is analytic-synonymously related with the well known definition of Reactive power (43) and the so-calculated values can be reduced to it.

6. Conclusion

The requirements towards the quality of the energy produced and the efficiency of the electric energy consumed increase, to a great extent as a result of the expanding use of the electronic controlled consumers, utilizing mainly nonsinusoidal current.

In conformance with the definitions, introduced in IEEE1459-2000 Standard, some basic values are discussed, concerning the parameters of the power line and the electric energy consumers. Their monitoring allows efficient observation and control of the different network's users.

Different methods are presented, which measure and compensate the Reactive power.

A new approach is suggested for the definition of the Active and Reactive powers and energies, enabling a single computing process.

New, energy-related definitions are proposed (40) and (41), based on the use of two data-arrays with the digitized versions of the network voltage and load's current. The so received results may be used to compute the Active (42), Reactive (45) and Apparent (47) energy as well as Phase angle (49) between the network voltage and the load current in a consistent computational procedure.

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