BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **8**, No 1

Sofia • 2008

Adaptive Dynamical Polling in Wireless Networks

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Abstract: *We consider a polling system with adaptive polling mechanism describing the performance of broadband wireless Wi-Fi and WiMax networks. A server visits queues in cyclic order depending on the state of queues in the previous cycle in the following way. A queue is skipped (not visited) by a server if it was empty in the previous cycle. The skipped queues are polled in the next cycle only. Such a polling mechanism is referred to as adaptive one. We propose to reduce an adaptive polling mechanism to a Bernoulli polling scheme allowing investigation of the model with the mean value analysis.*

Keywords: *polling system, adaptive mechanism, mean value analysis.*

1. Introduction

The models of polling systems whose study dates from the late 1950’s found wide use in the public health systems, air and railway transportation, and communication systems. The number of works on the polling systems is quite large. Classification of polling systems, methods and results of their investigation are reviewed in [4, 5, 6, 11, 13, 14].

The rapid development of the telecommunication networks and cellular communications in particular which is often called “the wireless revolution” has made it necessary to create and investigate the models describing the features of such systems and networks [10]. The polling models for investigating the characteristics of personal and local wireless networks are analyzed in [3, 7]; those intended for the regional wireless broadband regional networks with centralized control, in [8, 9].

The adaptive polling mechanism with queue skips in a cycle adequately describes the performance of broadband wireless Wi-Fi and WiMax networks where the number of abonent stations is large. When base station polls the abonent ones cyclically it can be impossible to poll all stations in a cycle thus some of them have to be skipped. One of the criteria to skip (not to poll) a queue (abonent station) in a cycle is its emptiness at the previous polling moment. The polling moment of a queue is referred to as a moment when the server (base station) checks if there are packets in a queue to be transmitted. Unfortunately the adaptive mechanism is hard enough to be analyzed so we use approximation methods and polling schemes, e.g. a threshold polling scheme [12].

In the present paper we show how the adaptive polling mechanism can be reduced to a Bernoulli scheme and develop an approximate algorithm for calculation of the mean waiting time in a queue on the base of mean time analysis [15].

2. Model

We consider a polling system with a single server and N queues, $N \geq 2$. Each queue has infinite buffer capacity. The server visits and serves the queues in a cyclic adaptive order. Such an order is not fixed but changes at the beginning of a cycle depending on the states of queues in the previous cycle.

The i-th queue has its own Poisson input of customers with rate λ_i. The service times in the queue i are independent, identically distributed random variables with a mean b_i and second moment $b_i^{(2)}$. Service at each queue is a gated one: when the server visits a queue it serves all, and only, customers present in the queue at the polling instant.

When the server visits the queue the setup time is incurred of which the first and second moment are denoted by g_i and $g_i^{(2)}$, $i = \overline{1, N}$.

We refer to a queue polling instant as a moment when the server has completed the setup time and ready to serve the queue. It is supposed that the server does not know the queue length until the setup time is expired. If a queue is empty at its polling moment the server will skip (not visit) this queue in the next cycle. If all queues are to be skipped the server initiates an empty cycle, i.e. takes a vacation having an exponential distribution with mean τ and then polls all queues starting from queue 1.

The occupation rate ρ_i at queue i is defined as $\rho_i = \lambda_i b_i$, $i = \overline{1, N}$. The total occupation rate is $\rho = \sum_{i=1}^{N} \rho_i$. The necessary and sufficient condition for the stability of the polling system under consideration is $\rho < 1$ [2].

In the next section we develop the approximate approach and reduce the adaptive mechanism to Bernoulli polling, [1]. The system under consideration differs from the one in [1] by the fact that the setup time is incurred only if a queue is visited for service.

3. Mean cycle length and probability of a queue visit

Suppose that the queue i is visited in the current cycle with a probability u_i. Suppose that the probability does not depend on the number of the cycle. In that case the adaptive polling mechanism can be approximated by a Bernoulli scheme which is described as follows. The set of probabilities (u_1, …, u_N), is fixed, $0< u_i \le 1$, $i=\overline{1,N}$. The queue is served in the cycle with a probability u_i and with additional probability the server moves to the next queue.

For the adaptive mechanism the probabilities u_1, …, u_N depend on the mean cycle length C and can be calculated as

$$u_i = 1 - u_i + u_i(1 - e^{-\lambda_i C}),$$

where C is the mean cycle length. The cycle length means the time for the server to visit queues from 1 to N excluding queues to be skipped. Let us give a short explanation for the formula above. A queue is visited in a cycle when it was skipped in the previous cycle (with a probability $1 - u_i$) or it was visited in the previous cycle (with a probability u_i) and customers arrived to the queue during the intervisit time (the time between two successive visits to the queue).

It follows from the equation above that

(1) $$u_i = \frac{1}{1+e^{-\lambda_i C}}, \quad i=\overline{1,N}.$$

The mean queue length is determined by the formula

(2) $$C = \frac{\sum_{i=1}^{N} g_i u_i + \tau \prod_{i=1}^{N} (1-u_i)}{1-\rho}.$$

The relations (1) and (2) give the system of equations for calculation of the unknown values C and u_1, …, u_N.

The second way to determine the probabilities u_i, $i=\overline{1,N}$, can be applied when the probability that a queue is empty at a polling instant can be calculated or estimated. This way is described as follows.

Consider the stochastic process $c_{i,j}$, $j\ge 1$, where $c_{i,j}$ is the status of the queue i in j-th cycle, that is $c_{i,j}=0$ if the queue is skipped and $c_{i,j}=1$ otherwise.

The state of the process $c_j^{(i)}$, $j \ge 1$, depends on its previous state and the queue i state in the j-th cycle. If $c_{j-1}^{(i)} = 1$ and the queue i is empty at the polling instant in the (j – 1)-th cycle, we have $c_j^{(i)} = 0$. Otherwise, $c_j^{(i)} = 1$. The probability u_i that the queue i is visited by the server in an arbitrary cycle is the stationary state probability that $c_j^{(i)} = 1$,

$$u_i = \lim_{j\to\infty} P\{c_j^{(i)} = 1\}, \quad i=\overline{1,N}.$$

Let $\pi_0^{(i)}$ be the stationary state probability that the queue i is empty at a polling instant and $x_{lk}^{(i)}$, $l,k=\overline{0,1}$, be one step transition probabilities of the process $\{c_j^{(i)}, j\ge 1\}$,

(3) $$x_{00}^{(i)} = 0, \quad x_{01}^{(i)} = 1,$$

(4) $$x_{10}^{(i)} = \pi_0^{(i)}, \quad x_{11}^{(i)} = 1 - \pi_0^{(i)}.$$

The probability u_i can be calculated from the balance equation

$$u_i = P\{c_j^{(i)} = 0\} x_{01}^{(i)} + P\{c_j^{(i)} = 1\} x_{11}^{(i)}.$$

Hence,

$$u_i = (1 - u_i) \cdot 1 + u_i (1 - \pi_0^{(i)})$$

and from (3) we have

(5) $$u_i = \frac{1}{1 + \pi_0^{(i)}}.$$

Note that probabilities $\pi_0^{(i)}$, $i = \overline{1, N}$, are unknown and the formula (5) can only be exploited when these probabilities are calculated or estimated.

4. Mean queue length

In this section we derive the approximation for the mean queue length at an arbitrary time on the base of mean value analysis [15].

Let θ_i be the average time the server spends in the queue i plus the average setup time to queue i +1 under condition that the queue i +1 is visited by the server, $i = \overline{1, N}$. We suppose that in the empty cycle the server is cyclically visiting all the queues and it spends the mean time τ / N in each queue without customer service. The value θ_i is defined as

$$\theta_i = \rho_i C + g_{i+1} u_{i+1} + v\tau / N, \quad i = \overline{1, N},$$

where $v = \prod_{i=1}^{N} (1 - u_i)$ is the probability that a cycle is empty, $I_{\{i=N\}}$ equals 1 if $i = N$ and equals 0 otherwise. As in [15] we define the (i, j)-period as the sum of j consecutive visit times starting from queue i, the mean of the period is defined as

$$\theta_{i,j} = \sum_{n=i}^{i+j-1} \theta_n, \quad i, j = \overline{1, N}.$$

The fraction of time the system spends in the (i, j)-period is given by

$$q_{i,j} = \frac{\theta_{i,j}}{C}, \quad i, j = \overline{1, N}.$$

The mean of a residual (i, j)-period is given by

$$R_{\theta_{i,j}} = \frac{\theta_{i,j}^{(2)}}{2\theta_{i,j}}, \quad i, j = \overline{1, N},$$

where $\theta_{i,j}^{(2)}$ is the second moment of (i, j)-period length.

Denote by $L_{i,j}$ the mean queue i length at an arbitrary epoch of visiting the queue j, $i, j = \overline{1, N}$. The corresponding unconditional queue length is defined as

$$L_i = \sum_{n=1}^{N} q_{n,1} L_{i,n}, \quad i = \overline{1,N}.$$

The value $L_{i,j}$ in the case $i = j$ is the sum of two variables $\overline{L}_i$ and $\tilde{L}_{i,j}$. The value $\overline{L}_i$ is the number of customers to be served at an arbitrary epoch of visit to queue i. The value $\tilde{L}_{i,i}$ is the number of customers that arrived during the service time of the queue and will be served in the next cycle. In case $i \neq j$ $L_{i,j} = \tilde{L}_{i,j}$. That is

$$L_{i,j} = \overline{L}_i \frac{\rho_i}{u_i} I_{\{i=j\}} + \tilde{L}_{i,j}, \quad i, j = \overline{1,N}.$$

The corresponding unconditional mean queue length L_i is calculated as

(6) $$L_i = \tilde{L}_i + \overline{L}_i \frac{\rho_i}{u_i} = \sum_{n=1}^{N} q_{n,1} \tilde{L}_{i,n} + \overline{L}_i \frac{\rho_i}{u_i}, \quad i = \overline{1,N}.$$

One more equation for the value L_i can be derived by Little's law

(7) $$L_i = \lambda_i W_i,$$

where W_i is the mean waiting time in the queue i (the time from a customer's arrival at queue i until its service starts).

The customer arriving to queue i has to wait for the service of all customers $\tilde{L}_i$ waiting before the gate on its arrival. Further, it has to wait until the first polling instant of queue i equalling a residual (i, N)-period, i.e., a residual cycle. And with probability $1 - u_i$ the queue i was not visited in the previous cycle, so the customer has to wait one more cycle. Thus, the mean waiting time W_i is given by

(8) $$W_i = \tilde{L}_i b_i + R_{\theta_{i,j}} + (1 - u_i)C, \quad i = \overline{1,N},$$

which, in combination with Little's Law (7), gives us the following relation

(9) $$L_i = \tilde{L}_i + \lambda_i R_{\theta_{i,j}} + \lambda_i (1 - u_i)C, \quad i = \overline{1,N}.$$

The number of customers at an arbitrary moment of the period (i, j) is the number of Poisson arrivals during the age of the period plus the arrivals during the cycle if the queue was not visited in the previous cycle,

(10) $$\sum_{n=i}^{i+j-1} \frac{q_{n,1}}{q_{i,j}} \tilde{L}_{i,n} = \lambda_i R_{\theta_{i,j}} + \lambda_i (1 - u_i)C, \quad i, j = \overline{1,N}.$$

Substituting (6) in (9) we get

(11) $$(1 - \rho_i) \sum_{n=1}^{N} q_{n,1} \tilde{L}_{i,n} + \frac{\rho_i}{u_i} \overline{L}_i = \lambda_i R_{\theta_{i,j}} + \lambda_i (1 - u_i)C, \quad i = \overline{1,N}.$$

Note that equations (10) and (11) form the system of $N(N+1)$ linear equations for $\overline{L}_i$, $\tilde{L}_{i,j}$ and $R_{\theta_{i,j}}$. To calculate the unknown mean residual (i, j)-periods from this system, below we obtain dependence of $R_{\theta_{i,j}}$ on $\overline{L}_i$ and $\tilde{L}_{i,j}$.

The mean residual (i, 1)-period lasts at least the sum of the service times of the customers behind the gate if the queue i is visited for service. With probability $\frac{\rho_i C}{\theta_{i,1}}$ the mean residual service time

$$R_{b_i} = \frac{b_i^{(2)}}{2b_i}$$

and the mean setup time for queue i + 1 is added given that the queue i + 1 is not skipped. Further, with a probability $\frac{u_{i+1}s_{i+1}}{\theta_{i,1}}$ the mean residual setup time for the queue i + 1

$$R_{g_{i+1}} = \frac{g_{i+1}^{(2)}}{2g_{i+1}}$$

is generated. Finally, the mean residual (i, 1)-period equals to the mean residual time server spends at queue i, that is, τ / N if the cycle is empty (with probability v). Thus, we have

(12) $$R_{\theta_{i,1}} = u_i b_i \bar{L}_i + \frac{\rho_i C}{\theta_{i,1}}\left(R_{b_i} + u_{i+1} g_{i+1}\right) + \frac{u_{i+1} g_{i+1}}{\theta_{i,1}} R_{g_{i+1}} + \frac{v\tau}{N}.$$

Consider the case of the mean residual (i, 2)-period. With probability $\frac{q_{i,1}}{q_{i,2}}$, the value of $R_{\theta_{i,2}}$ equals $R_{\theta_{i,1}} + g_{i+2}u_{i+2}$ plus the service times of customers present in the queue i + 1 at an arbitrary moment when the server visits the queue i and of customers arriving to the queue i + 1 during the mean time $R_{\theta_{i,1}}$ given that the queue i + 1 is visited. With additional probability $\left(1 - \frac{q_{i,1}}{q_{i,2}}\right)$, it equals $R_{\theta_{i+1,1}}$. Thus,

(13) $$R_{\theta_{i,2}} = \frac{q_{i,1}}{q_{i,2}}\left(R_{\theta_{i,1}} + u_{i+2}s_{i+2} + (\lambda_{i+1}R_{\theta_{i,1}} + \tilde{L}_{i+1,i})b_{i+1}u_{i+1}\right) + \\ + \left(1 - \frac{q_{i,1}}{q_{i,2}}\right)R_{\theta_{i+1,1}} = \frac{q_{i,1}}{q_{i,2}}\Big(R_{\theta_{i,1}}(1 + \rho_{i+1}u_{i+1}) + \\ + u_{i+2}s_{i+2} + \tilde{L}_{i+1,i}b_{i+1}u_{i+1}\Big) + \left(1 - \frac{q_{i,1}}{q_{i,2}}\right)R_{\theta_{i+1,1}}, \quad i = \overline{1,N}.$$

The values $R_{\theta_{i,j}}$ for $j = \overline{2,N}$ can be obtained in a similar way,

(14) $$R_{\theta_{i,j}} = \frac{q_{i,1}}{q_{i,j}}\left(R_{\theta_{i,1}}\prod_{n=1}^{j-1}(1 + \rho_{i+n}u_{i+n}) + \right. \\ \left. + \sum_{n=1}^{j-1}\left(u_{i+n+1}s_{i+n+1} + \tilde{L}_{i+n,i}b_{i+n}u_{i+n}\right)\prod_{m=n+1}^{j-1}(1 + \rho_{i+m}u_{i+m})\right) + \\ + \left(1 - \frac{q_{i,1}}{q_{i,j}}\right)R_{\theta_{i+1,j-1}}, \quad i = \overline{1,N},\ j = \overline{2,N}.$$

Finally, the equations (12)-(14) form a set of N^2 linear equations. Solving the equations (10)-(11) and (12)-(14), we get the unknowns $\overline{L}_i$, $\tilde{L}_{i,j}$ and $R_{\theta_{i,j}}$. Then, the unconditional mean queue lengths and mean delays are easily calculated from (6) and (8).

5. Numerical example

To illustrate the obtained results we present numerical examples. We compare the approximate results presented above with simulation results.

Let us consider a symmetric polling system with two queues and exponentially distributed service times. In this case we omit the subscript i for the queue characteristics. The mean service time b = 0.311, mean setup time g = 0.091. The approximate results (column "T"), simulation results (column "E") and relative error of comparison (column "Δ") are shown in Table 1. We compare the mean cycle length C, probability u that queue is polled in the cycle and mean queue length L.

Table 1. A symmetric system with two queues

λ, ρ	C, u, L	T	E	Δ, %	T	E	Δ, %
λ = 0.5, ρ = 0.311	C	τ = 0.05			τ = 0.1		
		0.154	0.157	2.06	0.171	0.168	1.77
	u	0.526	0.534	1.5	0.528	0.537	1.7
	L	0.240	0.233	2.9	0.247	0.235	4.9
λ = 1, ρ = 0.622	C	τ = 0.05			τ = 0.1		
		0.300	0.310	3.28	0.326	0.328	0.61
	u	0.595	0.596	0.16	0.600	0.602	0.33
	L	0.697	0.740	5.98	0.723	0.744	2.86

Now let the number of queues in the system be 5. The input intensities are λ_1 = 1, λ_2 = 2, λ_3 = 0.5, λ_4 = 6, λ_5 = 0.5. The mean setup time is the same for all queues and equals 0.05, the mean time of an empty cycle τ = 0.05. Table 2 shows results for two values of the mean service time 0.05 and 0.07.

Table 2. A system with five queues

C, u_i, L_i	T	E	Δ, %	T	E	Δ, %
	b = 0.05, ρ = 0.5			b = 0.07, ρ = 0.7		
C	0.321	0.331	3.0	0.587	0.593	1.05
u_1	0.579	0.620	6.74	0.642	0.678	5.33
u_2	0.654	0.696	6.17	0.764	0.769	0.7
u_3	0.539	0.568	5.25	0.572	0.608	5.98
u_4	0.871	0.851	2.25	0.974	0.910	6.51
u_5	0.539	0.568	5.32	0.572	0.607	5.91
L_1	0.434	0.429	1.15	0.763	0.738	3.33
L_2	0.843	0.793	5.76	1.47	1.400	4.87
L_3	0.222	0.227	2.30	0.393	0.400	1.76
L_4	2.50	2.351	6.18	5.013	4.772	4.96
L_5	0.222	0.229	3.10	0.393	0.405	2.00

The results obtained for the mean service times $b_1 = 0.07$, $b_2 = 0.015$, $b_3 = 0.1$, $b_4 = 0.025$, $b_5 = 0.4$ are shown in Table 3.

Table 3. Nonsymmetric service in queues

C, u_i, L_i	T	E	Δ , %
C	0.321	0.324	0.89
u_1	0.579	0.612	5.44
u_2	0.654	0.676	3.20
u_3	0.539	0.564	4.38
u_4	0.871	0.819	6.19
u_5	0.539	0.566	4.74
L_1	0.473	0.508	6.88
L_2	0.846	0.887	4.73
L_3	0.287	0.285	0.70
L_4	2.416	2.376	1.66
L_5	0.287	0.286	0.35

6. Conclusion

A polling system with adaptive polling mechanism is considered. The adaptive mechanism means that the order in which the server visits queues depends on the states of queues in the previous cycle, i.e. the server does not visit queues that were empty at their polling moments in the previous cycle. The adaptive mechanism is reduced to a Bernoulli one, that is a queue is polled in a cycle with some probability. The mean waiting time in each queue is obtained on the base of mean value analysis.

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