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Robust Root Locus Application in Design and Analysis of Typical Industrial Control System Model*

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Abstract: *This paper presents an approach to controller design in the complex plain, satisfying some performance specifications and an analysis of the obtained system in the presence of parametric uncertainty. The relative location of regions in the complex plane, specifying performance and uncertainty, is analyzed. Transient responses and the notion of robustness are commented. The design example, shown in the paper is based on a triparametric plant model and a PID controller. Simulations are run in Matlab.*

Key words: *Complex plane, Performance, PID controller, Robust Root Locus, Uncertainty.*

1. Introduction

It is well-known that the complex plane analysis and design methods form a powerful tool available to the control engineer. Nowadays, more and more attention is given to the adaptation and the extension of these methods when dealing with modern control systems analysis and design problems in the presence of plant uncertainties [1-12]. Time-domain performance specifications are graphically "mapped" in pole location regions in the complex plane, when all analysis and design considerations are based on the well-accepted "dominating poles" concept. Designing a controller, meeting certain performance specifications, by using the tools and the properties of the complex plane requires an engineering intuition, since there are many choices available to the designer. The construction of the so-called gamma-regions [1] widens the set of tools in the plane, by giving new

* In memoriam of our mentor, colleague and friend Kostadin Gueorguiev Kostov.

alternatives in studying plant parametric uncertainties [2, 3, 7, 8], and in designing controllers, classified in a category tagged by the notion of "robustness".

2. Goal

The overall goal of this paper is to propose a solution to the controller design problem for a typical triparametric industrial plant model, in order to meet certain closed-loop system time-domain performance specifications and to analyze the behavior of the obtained system in the presence of uncertainty in all three dynamic parameters of the plant model.

3. Problems

In order to meet our goals, the following problems are addressed:

a) design of a controller, guaranteeing specified closed-loop system performance – overshoot and settling-time, by using the argument equation, describing the root loci formation;

b) verification of the obtained performance by analyzing transient responses and the relative time-domain specifications and their graphically determined counterparts in the complex plane;

c) specification of parameter uncertainty regions, by plotting root loci for each of the three uncertain real dynamic parameters entering the plant model;

d) analysis of the relative location of the performance specifying "gamma" and uncertainty regions, and interpreting the time-domain performance.

4. Third-order system

It is well-known, that in [4-6], [9-11] a large number of typical industrial processes and plants are modeled by transfer functions including an integrator and two simple lags,

(1) $$G(s) = k_G \left[s(\tau_1 s + 1)(\tau_2 s + 1) \right]^{-1}.$$

Some of the candidate plants that "successfully apply" for description with the transfer function given by (1) are: flexible arm, gripper tool, robot submarine, arm rotating dynamics, vehicle, actuator, motor and joint as well as laser eye surgery system, two camera control, surface grinding wheel control system, milling machine control etc. [5]. Closed-loop systems are characterized by non-oscillating and oscillating transient responses. Under certain conditions the closed-loop system may become unstable. The settling-time decreases until two of the poles "meet" each other, after that the settling-time increases and the transients become oscillatory. The steady-state error $\varepsilon(\infty)$ to input As^{-q} is equal to zero for ($q = 1$), bounded for ($q = 2$) and tends to infinity for ($q > 2$).

5. Controller design

The example considered in this paper is the triparametric model (1) with the following dynamic parameter values:

(2) $$\tau_1 = 0.3\text{ s},\ \tau_2 = 0.2\text{ s},\ k_G = 1.6\,.$$

The goal is to design a controller giving closed-loop system performance specified by – overshoot $\sigma \approx 20\%$ and settling-time $t_s \pm 2\% \approx 2.2\text{ s}$.

The dynamic parameter combination (2) (it is assumed that controller is P type with $k_R = 1$) determines the following indirect performance specifications in the complex plane – damping ratio $\xi = 0.456$ and absolute stability margin $\xi\omega_n = -0.91$ (Fig. 1). Using overshoot and settling-time relations (3) with the damping ratio and undamped natural frequency

(3) $$\sigma = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}\%\ \text{ and }\ t_s \pm 2\% \approx \frac{4}{\xi\omega_n}\text{ s.}$$

The following time-domain specifications are obtained: $\sigma = 20\%$, $t_s \pm 2\% = 4.4$ s.

6. Desired point coordinates calculation

The new desired settling-time is two-times smaller than that of the uncorrected system. From the analytical relations between direct and indirect specification indices (3), one can determine the new value of the absolute stability margin, $\xi\omega_n = -1.82$, that guarantees settling-time $t_s \pm 2\% \approx 2.2\text{ s}$ (Fig. 1).

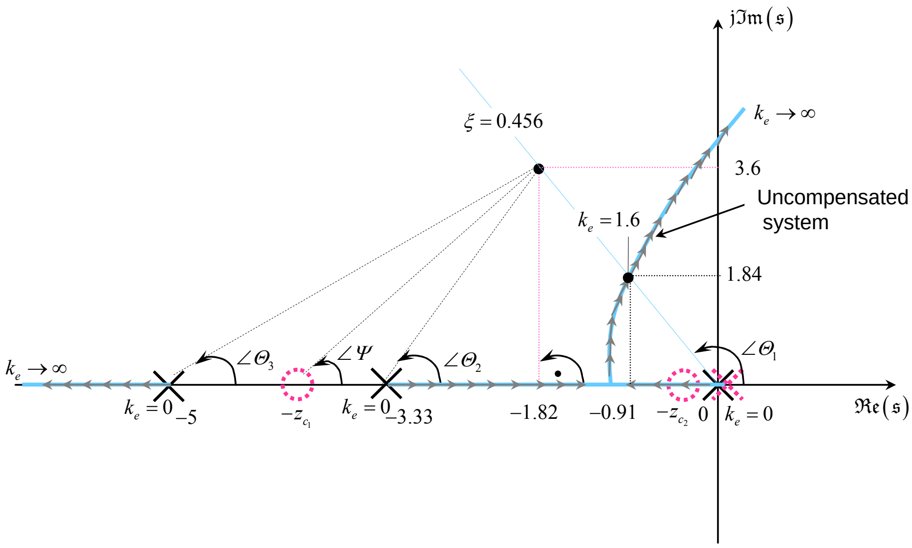

Fig. 1. PID controller design

It is obvious that the desired poles don't lie on the branches of the root locus. Therefore, improving the system speed of response, keeping the overshot at the desired value, is not possible by only altering the Evans-coefficient (the P controller gain) in the right direction. It is necessary to design a controller that alters the root locus, in order to ensure that its branches will contain the point specifying the desired performance.

A possible solution may be found in a PID controller design. The additional integration will ensure zero steady-state error to ramp inputs As^{-q} and the most importantly disturbance rejection. In the case considered in this work, the zeros of the PID controllers will alter the current root locus, improve stability margins and limit the system speed of response.

7. Zero z_{c_1} design

The zero z_{c_1} characterizes the speed of response of the system. Its value is determined by the performance specifying point, which the root locus branches must contain, and is calculated using the argument equation (4) (Fig. 1). It is assumed that $k_e \in [0,+\infty)$:

(4) $$\angle G(s) = \pm(2i+1)\pi = \sum_{i=1}^{m} \psi_i - \sum_{j=1}^{n} \Theta_i \ .$$

Using the geometrical relation given by Fig. 1, the following argument values are calculated:

(5) $$\angle\Theta_1 = 117.3°\ ,\ \angle\Theta_2 = 71.6°\ ,\ \angle\Theta_3 = 48.15°\ .$$

From (4) and (5), the zero argument can be found to be:

(6) $\angle\psi = 56.88°$, and finally $z_{c_1} = -4.14$.

8. Zero z_{c_2} design

Placing the zero z_{c_2} is a delicate stage of the controller design and suggests an iterative procedure. An engineering intuition is required, since complex plane design methods are based on the assumption for a second-order system. In these cases, it is recommended that the zero must be placed in sufficient proximity to the origin of the coordinate system. In our case, the zero is positioned at a distance of 0.01 from the origin of the coordinate system in the complex plane. This helps reducing the influence of the zero on the transient responses and the second-order system approximation is valid.

The design of both zeros z_{c_1} , z_{c_2} completes the design of the PID controller and its transfer function is given by

(7) $$G_{\mathrm{PID}}(s) = \left[(s+4.15)(s+0.01)\right] \mathrm{s}^{-1} ,$$

for the PID controller parameters, one can obtain the following values:

$$k_R = 4.15;\ T_i = 0.71;\ T_d = 0.24.$$

9. Desired performance verification

The structure of the root locus shown in Fig. 2 indicates that the corrected system can indeed be treated as a second-order one.

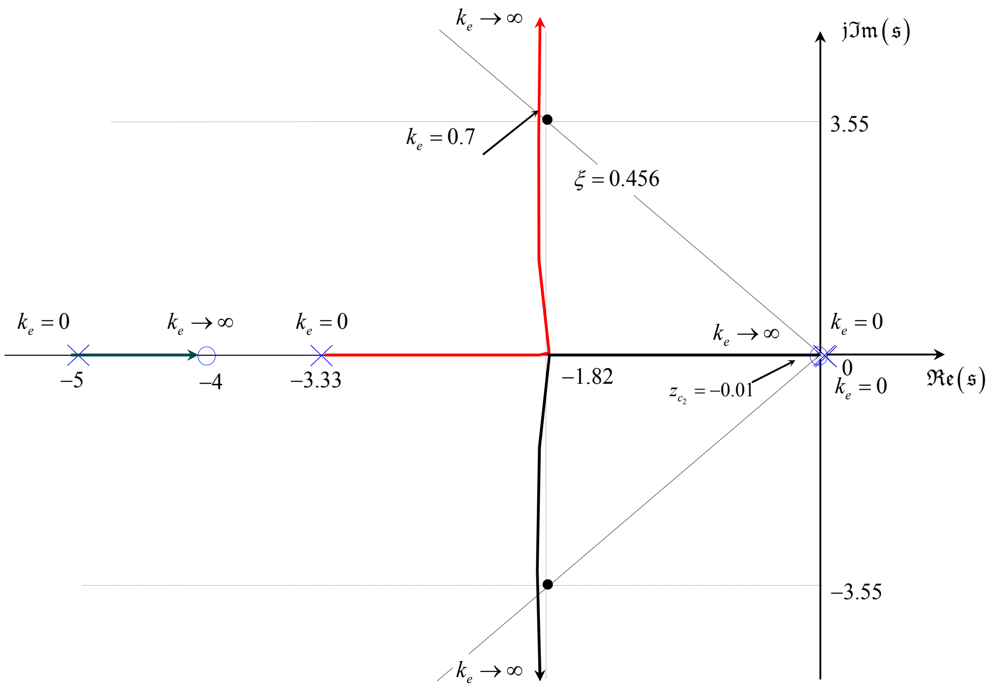

Fig. 2. Root locus of the corrected system

It must also be accounted for the fact that the root locus method is primarily a graphical method, which introduces additional inaccuracy, yielded for example of rounding PID parameter values. This fact, along with the influence of the zeros, and the second-order system assumptions are the reason why the desired point doesn't sit exactly on the corrected root locus branches. In situations like this, the control engineer again needs its intuition in order to choose the correct value of the Evans-gain. In our case, the value $k_e = 0.7$ corresponds to point where the root locus branches cross the constant damping ration loci in the plane at $\xi = 0.456$. Fig. 3 visualizes the transient response of the corrected system.

It is seen that the obtained performance specifications – overshoot of 20% and settling time of 2s, have values that meet the preliminary set performance requirements.

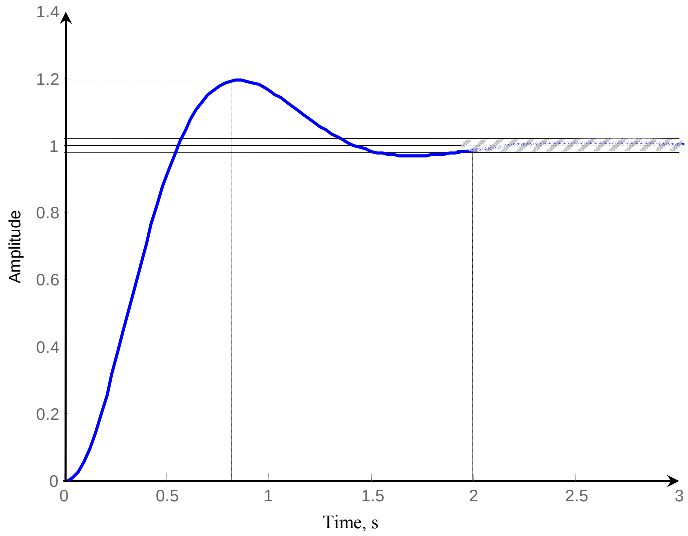

Fig. 3. Transient response of the corrected system

10. Robustness test

The notion of robustness means insensitivity of the system in certain degree to variations in the dynamic parameter values. The specification of a Γ-region, whose boundary is determined by (8), sets a desired dynamic behavior of the control system, and guarantees robustness, in the case when all dominating poles, repositioned in the plane due to plant parameter uncertainties, lie in it:

$$\partial\Gamma = \left\{ s \mid s = -\xi\omega_n(\lambda) \pm j\omega_n\sqrt{1-\xi^2}(\lambda),\ \lambda \in \left[\lambda^- \ \lambda^+\right] \right\}, \tag{8}$$

where λ is a generalized parameter.

Plant parameter uncertainties form an uncertainty region Q_s in the complex plane. The analysis of the relative location and overlapping of those regions enables the interpretation of the robustness properties of control systems.

In our numerical example, the uncertainty in the control system is due to 10% variations of the plant parameters around their nominal values (2),

$$\tilde{k}_G \in \left[k_G^-;\ k_G^+\right];\ \ \tilde{\tau}_1 \in \left[\tau_1^-;\ \tau_1^+\right];\ \ \tilde{\tau}_2 \in \left[\tau_2^-;\ \tau_2^+\right]. \tag{9}$$

The region of desired performance "gamma" is specified by upper and lower bounds on the admissible deviations from the nominal system performance. (nominal parameters (2) and controller (7)) i.e. lower and upper bounds on the damping ratio $\xi \in [0.5;\ 0.3]$ and the speed of response $\xi\omega_n \in [2.5;\ 1.5]$.

The characteristic equation of the closed-loop system in the presence of triparametric uncertainty is given by

$$\tilde{P}(s) = \tilde{\tau}_1\tilde{\tau}_2 s^4 + \left(\tilde{\tau}_1 + \tilde{\tau}_2\right)s^3 + \left(1 + \tilde{k}_G\right)s^2 + \left(z_{c_1} + z_{c_2}\right)s + z_{c_1}z_{c_2}\tilde{k}_G = 0\,. \tag{10}$$

In order to construct the uncertainty region Q_s , a root locus is plotted for each of the three plant parameters as Evans-gain (9). The three needed generalized characteristic polynomials are obtained after the respective modification of (10):

$$\tilde{P}_1(s) = 1 + \tilde{\tau}_1 \frac{\tilde{\tau}_2 s^4 + s^3}{\tilde{\tau}_2 s^3 + \left(1 + \tilde{k}_G\right) s^2 + \left(z_{c_1} + z_{c_2}\right) \tilde{k}_G s + z_{c_1} z_{c_2} \tilde{k}_G}, \tag{11}$$

$$\tilde{P}_2(s) = 1 + \tilde{\tau}_2 \frac{\tilde{\tau}_1 s^4 + s^3}{\tilde{\tau}_1 s^3 + \left(1 + \tilde{k}_G\right) s^2 + \left(z_{c_1} + z_{c_2}\right) \tilde{k}_G s + z_{c_1} z_{c_2} \tilde{k}_G},$$

$$\tilde{P}_3(s) = 1 + \tilde{k}_G \frac{s^2 + \left(z_{c_1} + z_{c_2}\right) s + z_{c_1} z_{c_2}}{\tilde{\tau}_1 \tilde{\tau}_2 s^4 + \left(\tilde{\tau}_1 + \tilde{\tau}_2\right) s^3 + s^2}.$$

Twelve characteristic polynomials "shape" the boundary of Q_s, each one obtained by taking the lower and upper bounds on the variations of the plant parameters. Fig. 4 shows the construction of the uncertainty region, by using the three basic polynomials (11) with the nominal values (2) in which the parameters $\tilde{\tau}_1$, $\tilde{\tau}_2$ and $\tilde{k}_G$ vary from zero to infinity. The uncertain part of each root locus around the nominal values of the parameters can also be seen in Fig. 4.

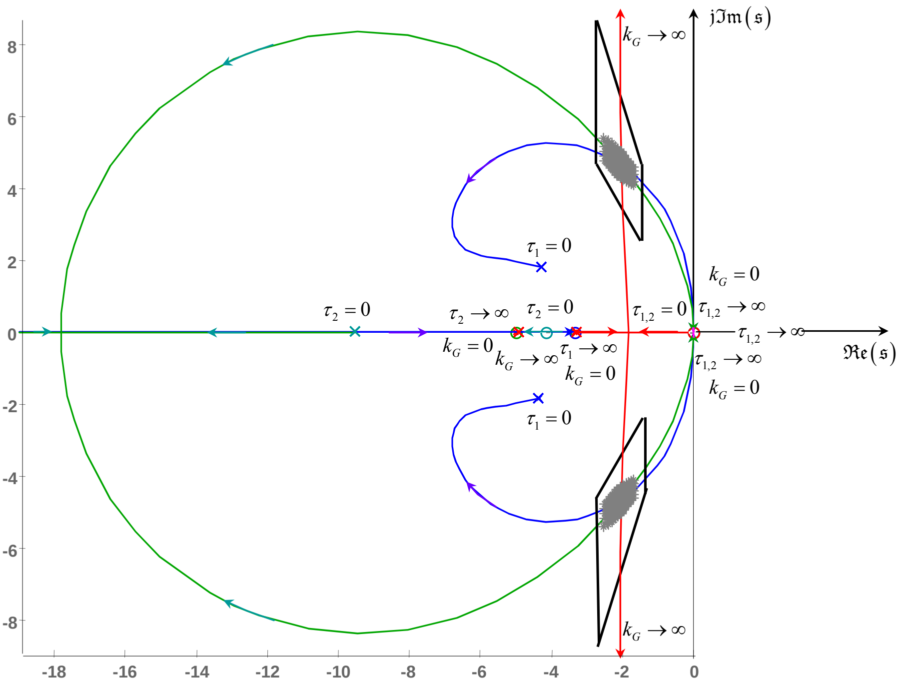

Fig. 4. Q_s construction

Fig. 5 shows only the parts of the root loci resulting from the parameter variations and forming the uncertainty region.

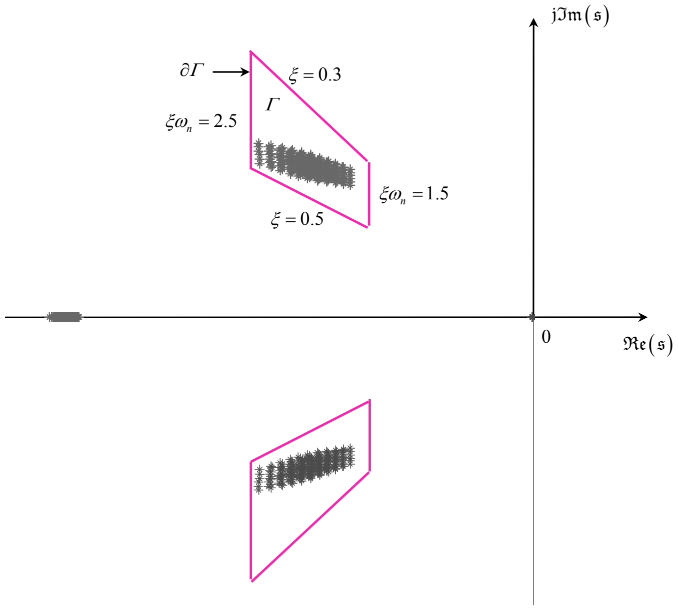

Fig. 5. Robust root locus

In the presence of 10% variation of the plant parameter values, the closed-loop characteristic equation roots dominating the system transient response remain in the performance specifying Γ-region and the overall system possesses robust properties in this sense.

In the time domain (Fig. 6), one can see that the system transient responses are specified by admissible overshoot and settling-time for the entire range of parameter deviations.

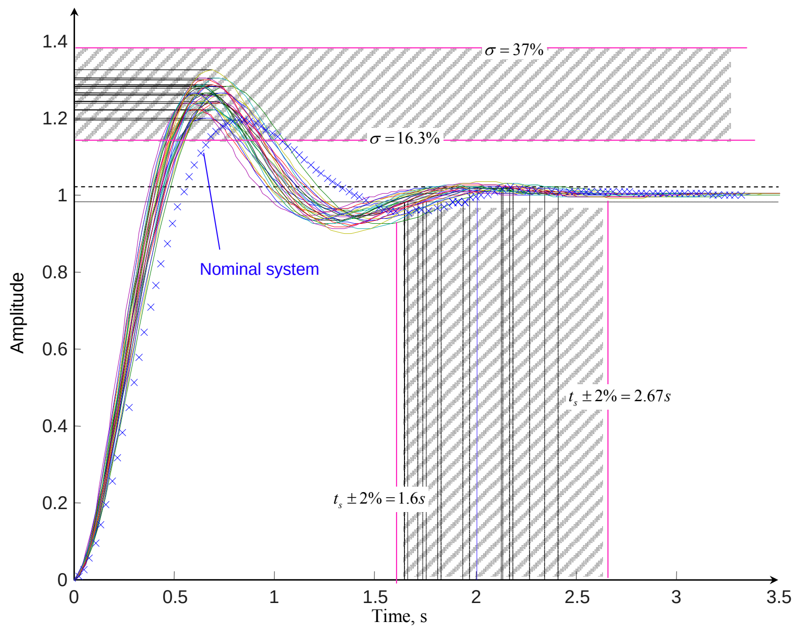

Fig. 6. Transient responses

11. Conclusion

The root locus method, in comparison with other design methods, considerably facilitates the design of a controller guaranteeing desired closed-loop system dynamic behavior, since it offers an adequate interpretation of the time-domain performance. Certain level of engineering skills and intuition is required, due to the existence of more than one solution of a given problem. The complex plane can be used when evaluating the dynamic behavior of control systems in the presence of parametric uncertainties as an alternative to other common approaches. It is relatively easy to form an uncertainty region, showing the combinations of the parameter values which cause changes in system characteristics. Root loci, plotted for different dynamic parameter as Evans-gain show values of the respective parameter, causing loss of stability. The specification of performance regions, different in their properties, can yield more comprehensive analysis of the designed control system, which helps the adequate choice of a controller that will guarantee the desired performance in a wide sense.

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