BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **7**, No 3

Sofia • 2007

Moving Target Hough Detector in Randomly Arriving Impulse Interference[1]

Lyubka Doukovska

Institute of Information Technologies, 1113 Sofia

Abstract: *The Hough detector is investigated with three types of Constant False Alarm Rate (CFAR) processors – an Adaptive censoring Post detection Integration (API CFAR) processor, a Binary Integration (BI CFAR) processor and an Excision with Binary Integration (EXC CFAR BI) processor in the presence of randomly arriving impulse interference. The detection probability and the average detection threshold of a Hough detector are studied with these three types of CFAR processors. The experimental results are obtained by numerical analysis. The target echo signal fluctuates according to a Swerling II case model in the randomly arriving impulse interference with a Poisson distribution of the probability of appearance and a Rayleigh distribution of the amplitudes. The research work is performed in MATLAB computational environment. The analytical results obtained for Hough detector can be used in both radar and communication receiver networks.*

Keywords: *Radar detector, Hough detector, randomly arriving impulse interference, probability of detection, probability of false alarm, detectability profits (losses).*

1. Introduction

Nowadays the algorithms that extract information for target's behavior through mathematical transformation of the signals reflected from a target, find ever-widening practical application. The pioneers in applying the Hough transform to target detection are C a r l s o n, E v a n s and W i l s o n [1], who have proposed

[1] This work is supported by the Bulgarian NSF under Grant MI-1506/2005 and IIT-010089/2007.

the structure of the two-threshold detector of a straight-line trajectory for a highly fluctuating target – Swerling II type target model, and stationary homogeneous interference with known intensity. The paper considers the case when the detection is held under conditions of homogeneous interference with unknown intensity and randomly arriving impulse interference with known parameters. The detector performance presented by C a r l s o n, E v a n s and W i l s o n [1] in this case depends strongly on the parameters of the randomly arriving impulse interference caused by different sources. The occurrence of these disturbances, even with low probability of appearance, worsens detector performance.

It is known that the application of different CFAR processors in homogeneous interference with unknown intensity and randomly arriving impulse interference with known parameters improves detection performance [2, 3, 14-15, 16, 17, 18, 19]. The usage of these processors together with a Hough detector would improve the probability characteristics [4, 5, 6, 10, 11]. It is assumed that the noise amplitude is Rayleigh distributed and therefore the noise power is exponentially distributed. In this paper the limit case is considered, when the increasing of the probability of appearance (e_0) changes the distribution law from Poisson to binomial [4]. A comparative analysis of the performance of different types of CFAR processors – *a Binary Integration* (BI), *an Excision with Binary Integration* (EXC BI), *an Adaptive censoring Post detection Integration* (API) used in the algorithm of Hough detector is carried out.

The signal model is based on the fact that the law of distribution of the impulse interference changes from Poisson to binomial, with the increasing of the probability of appearance of randomly arriving impulse, greater than 0.1 [8, 9]. The binominal model is more general than the Poisson distribution model. Therefore all mathematical formulas for evaluation of both probability measures – the probability of detection and the probability of a false alarm, should be derived for a binominal distribution for a Hough detector.

In this research comparison is accomplisehd between the probability characteristics of a Hough detector with fixed threshold synthesis of homogeneous interference with known power, and three other Hough detector types with two-dimensional CFAR processors, which keep constant false alarm rate under conditions of homogeneous interference with unknown power and randomly arriving impulse interference with known parameters (non-homogeneous background).

The present paper investigates the case for highly fluctuating target – Swerling II type target model detection under conditions of randomly arriving impulse interference is studied. The efficiency of a Hough detector with three types of CFAR processors under conditions of randomly arriving impulse interference for one value of the probability of detection – $P_D = 0.5$ is studied. The efficiency of the Hough detector is estimated with the help of the method described in [12], i.e. the sensibility towards randomly arriving impulse interference. The results presented show that Hough detector is efficient under conditions of decreasing randomly arriving impulse interference.

2. Statistical analysis of a Hough detector

Let us assume that L pulses hit the target, which is modeled according to Swerling case II. The received signal is sampled in a range by using $M + 1$ resolution cells resulting in a matrix with $M + 1$ rows and L columns. Each column of the data matrix consists of the values of the signal obtained for L pulse intervals in one range resolution cell. Let us also assume that the first $M/2$ and the last $M/2$ rows of the data matrix are used as a reference window in order to estimate the "noise-plus-interference" level in the test resolution cell of the radar. In this case the samples of the reference cells result in a matrix X of size $M \times L$. The test cell or the radar target image includes the elements of the $M/2+1$ row of the data matrix and is a vector Z of length L.

In the presence of randomly arriving impulse interference the elements of the reference window are drawn from two classes. One class represents the noise only with probability $1 - e_0$. The other class represents the interference-plus-noise with the probability e_0. The elements of the reference window are independent random variables with the compound exponential distribution law:

(1) $$f_{\mathrm{p}}(x_i) = \frac{1-e_0}{\lambda_0}\exp\left(\frac{-x_i}{\lambda_0}\right) + \frac{e_0}{\lambda_0(1+r_j)}\exp\left(\frac{-x_i}{\lambda_0(1+r_j)}\right), \quad i = 1,\ldots,K,$$

where $K = ML$ and λ_0 is the average power of the receiver noise, r_j / λ_0 is the average per pulse value of the interference-to-noise ratio (INR) at the receiver input, e_0 is the probability for appearance of randomly arriving impulse interference with average length in the range cells. In the presence of a desired signal from a target, the elements of the test resolution cell are independent random variables with the following distribution law:

(2) $$f_{\mathrm{p}}(x_{0i}) = \frac{1-e_0}{\lambda_0(1+s)}\exp\left(\frac{-x_{0i}}{\lambda_0(1+s)}\right) + \frac{e_0}{\lambda_0(1+r_j+s)}\exp\left(\frac{-x_{0i}}{\lambda_0(1+r_j+s)}\right), \quad i = 1,\ldots,L,$$

where s is the per pulse average signal-to-noise ratio, λ_0 is the average power of the receiver noise, r_j is the average interference-to-noise ratio, e_0 is the probability for the appearance of randomly arriving impulse interference with average length in the range cells.

Under conditions of binomial distribution of pulse interference, the background environment includes the interference-plus-noise situation, which may appear at the output of the receiver with probability $2e_0(1 - e_0)$, two interference-plus-noise situation with probability e_0^2 and the noise only situation with probability $(1 - e_0)^2$, where $e_0 = 1 - \sqrt{1 - t_c F}$, F is the average repetition frequency of pulse interference and t_c is the length of pulse transmission [7]. The distribution is binomial when the probability of the pulse interference is above 0.1-0.2 [8]. In these situations the outputs of the reference window are observations from statistically independent exponential random variables. Consequently, the probability density function (pdf) of the reference window outputs may be defined as:

(3) $$f_{\mathrm{b}}(x_i)=\frac{(1-e_0)^2}{\lambda_0}\exp\left(\frac{-x_i}{\lambda_0}\right)+\frac{2e_0(1-e_0)}{\lambda_0(1+r_j)}\exp\left(\frac{-x_i}{\lambda_0(1+r_j)}\right)+\frac{e_0{}^2}{\lambda_0(1+2r_j)}\exp\left(\frac{-x_i}{\lambda_0(1+2r_j)}\right),$$
$$i=1,...,K,$$

where λ_0 is the average power of the receiver noise and r_j/λ_0 is the per pulse average interference-to-noise ratio (INR).

The set of samples from the test resolution cell $\{x_{0i}\}_L$ is assumed to be distributed according to Swerling II case with pdf given by:

(4) $$f_{\mathrm{b}}(x_{0i})=\frac{(1-e_0)^2}{\lambda_0(1+s)}\exp\left(\frac{-x_{0i}}{\lambda_0(1+s)}\right)+\frac{2e_0(1-e_0)}{\lambda_0(1+r_j+s)}\exp\left(\frac{-x_{0i}}{\lambda_0(1+r_j+s)}\right)+$$
$$+\frac{e_0{}^2}{\lambda_0(1+2r_j+s)}\exp\left(\frac{-x_{0i}}{\lambda_0(1+2r_j+s)}\right),\quad i=1,...,L,$$

where s is the per pulse average signal-to-noise ratio. When the probability for appearance of impulse interference is small (up to 0.1), then $e_0{}^2\cong 0$ and the flow is Poisson distributed [7].

The probability of detection for a Hough processor with fixed threshold for homogeneous interference with known power is proposed in [1]. The probability of detection for CFAR BI processor in the presence of homogeneous interference and binomial distribution impulse interference with the binary rule M-out-of-L, is evaluated similarly to [16], where $K=N$:

(5) $$P_{\mathrm{d}}^{(*)}=\sum_{i=0}^{N}C_N^i e_0^{2i}\sum_{j=0}^{N-i}C_{N-i}^j\left(2e_0(1-e_0)\right)^j(1-e_0)^{2(N-i-j)}\left\{\frac{(1-e_0)^2}{\left(1+\frac{T_{\mathrm{BI}}(1+2r_j)}{1+r_j+s}\right)^i\left(1+\frac{T_{\mathrm{BI}}(1+r_j)}{1+r_j+s}\right)^j\left(1+\frac{T_{\mathrm{BI}}}{1+r_j+s}\right)^{N-i-j}}+\right.$$
$$+\frac{2e_0(1-e_0)}{\left(1+\frac{T_{\mathrm{BI}}(1+2r_j)}{1+r_j+s}\right)^i\left(1+\frac{T_{\mathrm{BI}}(1+r_j)}{1+r_j+s}\right)^j\left(1+\frac{T_{\mathrm{BI}}}{1+r_j+s}\right)^{N-i-j}}+$$
$$\left.+\frac{e_0^2}{\left(1+\frac{T_{\mathrm{BI}}(1+2r_j)}{1+2r_j+s}\right)^i\left(1+\frac{T_{\mathrm{BI}}(1+r_j)}{1+2r_j+s}\right)^j\left(1+\frac{T_{\mathrm{BI}}}{1+2r_j+s}\right)^{N-i-j}}\right\},$$

where T_{BI} is a predetermined scale factor that provides a constant false alarm rate P_{fa}.

The probability of detection for EXC CFAR BI processor in the presence of homogeneous interference and binomial distribution impulse interference [5], with the binary rule M-out-of-L, is

$$P_{\mathrm{d}}^{(*)}=\sum_{k=1}^{N}C_N^k P_{\mathrm{E}}^k\left(1-P_{\mathrm{E}}\right)^{N-k}\left\{\left(1-e_0\right)^2 M_V\left(\frac{T_{\mathrm{EXC}}}{\lambda_0\left(1+s\right)}k\right)+\right.$$

(6)

$$\left.+2e_0\left(1-e_0\right)M_V\left(\frac{T_{\mathrm{EXC}}}{\lambda_0\left(1+r_j+s\right)}k\right)+e_0{}^2 M_V\left(\frac{T_{\mathrm{EXC}}}{\lambda_0\left(1+2r_j+s\right)}k\right)\right\}$$

where $M_V(.)$ is the moment generating function and T_{EXC} is a predetermined scale factor that provides a constant false alarm rate P_{fa}.

The set of samples $\{x_i\}_N$ is applied to an excisor, which nullifies the samples exceeding a predetermined threshold B_{E}. The set of surviving nonzero samples at the excisor output is averaged (noise level estimate V) and multiplied by a predetermined scale factor T_{EXC} resulting in a pulse detection threshold $H_{\mathrm{d}}=V.T_{\mathrm{EXC}}$.

The probability of detection for API CFAR processor in homogeneous interference and Poisson distribution impulse interference for N resolution cells, according to [14], is:

$$P_{\mathrm{d}}^{(*)}=\sum_{k=1}^{N}\binom{N}{k}\left(1-e_0\right)^k e_0^{N-k}\sum_{l=1}^{L}\binom{L}{l}\left(1-e_0\right)^l e_0^{L-l}\sum_{i=0}^{l-1}\binom{k+i-1}{i}\frac{T_{\mathrm{API}}^i\left(1+s\right)^k}{\left(T_{\mathrm{API}}+1+s\right)^{k+i}}+$$

(7)

$$+\sum_{k=1}^{N}\binom{N}{k}\left(1-e_0\right)^k e_0^{N-k}e_0^L\sum_{i=0}^{L-1}\binom{k+i-1}{i}T_{\mathrm{API}}^i\left(1+r_j+s\right)^k\left(T_{\mathrm{API}}+1+r_j+s\right)^{-(k+i)}+$$

$$+\sum_{l=1}^{L}\binom{L}{l}\left(1-e_0\right)^l e_0^{L-l}e_0^N\sum_{i=0}^{l-1}\binom{N+i-1}{i}T_{\mathrm{API}}^i\left(\frac{1+s}{1+r_j}\right)^N\left(T_{\mathrm{API}}+\frac{1+s}{1+r_j}\right)^{-(N+i)}+$$

$$+e_0^N e_0^L\sum_{i=0}^{L-1}\binom{N+i-1}{i}T_{\mathrm{API}}^i\left(\frac{1+r_j+s}{1+r_j}\right)^N\left(T_{\mathrm{API}}+\frac{1+r_j+s}{1+r_j}\right)^{-(N+i)}$$

where T_{API} is a predetermined scale factor that provides a constant false alarm rate P_{fa}.

All indications for signal detection obtained from N range resolution cells and N_s scans are arranged in a matrix Ω of size $N\times N_s$ in the r-t space. In this space a stationary or constant radar velocity target pears as a straight line which consists of the nonzero elements of Ω. Let us assume that Ω_{ij}^{nm} is a set of such nonzero elements of Ω that constitute a straight line in (r-t) space that is i,j $\in \Omega_{ij}^{nm}$. This line may be represented in Hough parameter space as a point n, m. Denoting N_{nm} as the maximal size of Ω_{ij}^{nm}, the cumulative false alarm probability for a cell n, m is written according to [3]:

(8)
$$P_{\mathrm{fa}}^{nm}=\sum_{l=K}^{N_{nm}}\binom{N_{nm}}{l}\left(P_{\mathrm{fa}}\right)^l\left(1-P_{\mathrm{fa}}\right)^{N_{nm}-l},$$

where K is a linear trajectory detection threshold.

The total false alarm probability in Hough parameter space is equal to one minus the probability that no false alarm occurred in any of the Hough cells. For independent Hough cells this probability is:

(9) $$P_{\text{FA}} = 1 - \prod_{N_{nm}=K}^{\max(N_{nm})} \left[1 - P_{\text{fa}}^{nm}\right]^{W(N_{nm})} ,$$

where max (N_{nm}) is the accessible Hough space maximum and $W\left(N_{nm}\right)$ is the number of cells from Hough parameter space whose values are equal to N_{nm}.

The cumulative probability of target detection in Hough parameter space P_{d} cannot be written in the form of a simple Bernoulli sum. As the target moves with respect to the radar, the SNR of the received signal changes depending on the distance to the target and the probability of a pulse $P_{d}\,(j)$ changes as well. Then the probability P_{d} can by calculated by Brunner's method. By means of Brunner's method [3] a matrix of size 20×20, the elements of which are the primitive probability of detection from k-th time slice, is formed. Using (5), (6), (7) it is possible to obtain all the $P(i,\, j)$ needed to calculate P_{d}. For N_{s} scans of the radar, the following is valid:

(10) $$P_{\text{D}} = \sum_{i=M}^{N_{\text{s}}} P_{\text{d}}^{(*)}(i, N_{\text{s}}) .$$

After N_{s} radar scans a two-dimensional r–t space of data is formed at the output of the signal detector. The coordinates of these cells in r-t space, where the detection is indicated, are Hough transformed according to [1]. In this way Hough parameter space is formed. Each cell in Hough parameter space is intersected by a limited set of sinusoids obtained by the Hough transform. If the number of intersections in any of the cells in the parameter space exceeds a fixed threshold T_{M}, target detection and linear trajectory detection are indicated. Target and linear trajectory detection is carried out for all the cells of Hough parameter space.

The general structure of an adaptive Hough detector, which is used in the algorithms of binary integration of the data in Hough parameter space, is shown in Fig. 1. There are not many cases in practice when a radar is equipped with a Hough detector working under conditions of randomly arriving impulse interferences. In such situations it would be appropriate to know the Hough losses depending on the parameters of the randomly arriving impulse interference, for rating the behaviour of the radar. The ratio between the two values of the signal-to-noise-ratio (SNR), measured in dB is used for the calculation of Hough detector losses [12].

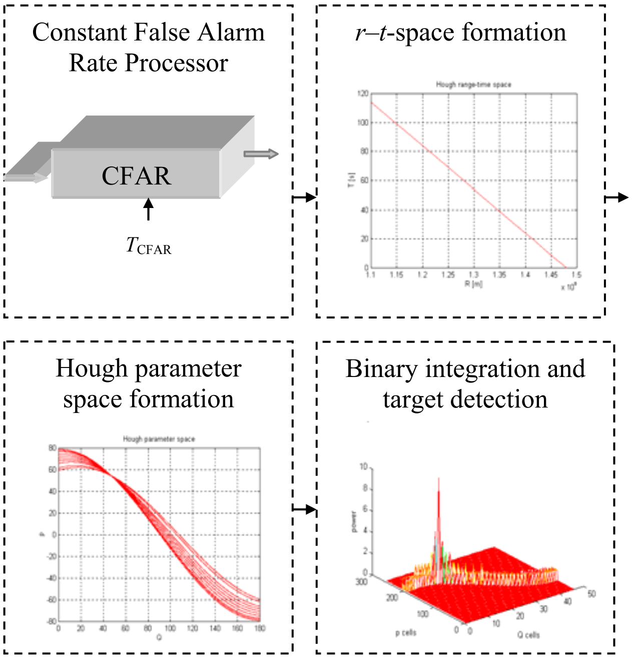

Fig. 1. The general structure of an adaptive Hough detector

The general structure of an adaptive Hough detector, which is used in the algorithms of binary integration of the data in Hough parameter space, is shown in Fig. 1. There are not many cases in practice when a radar is equipped with a Hough detector working under conditions of randomly arriving impulse interferences. In such situations it would be appropriate to know the Hough losses depending on the parameters of the randomly arriving impulse interference, for rating the behaviour of the radar. The ratio between the two values of the Signal-to-Noise-Ratio (SNR), measured in dB is used for the calculation of Hough detector losses [12].

3. Numerical results

In order to analyze the quality of the Hough detector, a radar with parameters, similar to those in [1], is considered in the paper. Carlson's approach, using Brunner's method for calculating the probability of detection in Hough parameter space is further developed in order to maintain constant false alarm probability at the output of the Hough detector. The suitable scale factor is iteratively chosen. The influence of the threshold constant on the required signal-to-noise ratio is studied. The investigation is performed for one value of probability of detection ($P_D = 0.5$) and different values of the probability of appearance of randomly arriving impulse interference with average length in the cells in a range. In order to achieve a constant value of the probability of a false alarm P_{FA}, the values of the threshold constants, which guarantee that, are determined for different numbers of observations in the reference window, an average interference-to-noise ratio (INR) and a probability for the appearance of randomly arriving impulse interference with average length in the cells in a range.

3.1. Hough detector with a Binary Integration (BI) CFAR processor

The efficiency of a Hough detector with a Binary Integration (BI) CFAR processor is studied under conditions of randomly arriving impulse interference. The block diagram of a CFAR processor with binary integration is shown in Fig. 2.

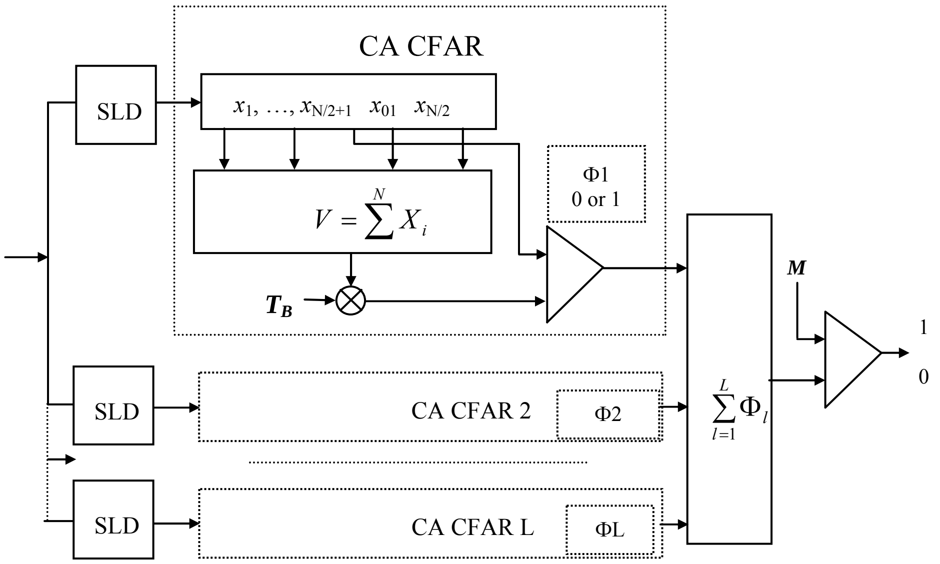

Fig. 2. Block-diagram of a CFAR processor with binary integration

Table 1 presents the results for threshold values for a Hough CFAR BI detector which provides for three different values of probability of a false alarm P_{FA}, for two values of the decision rule for binary processor (M/L=10/16, M/L=16/16), average interference-to-noise ratio (INR=30 dB) and different values of the probability for appearance of randomly arriving impulse interference with average length in the cells in range. All the results are obtained for a threshold value of the Hough parameter space T_M=2.

In order to determine the threshold in the Hough parameter space, the authors in [1] use the approach proposed by Barton. They assume T_M = 7 as an optimal threshold in the binary integration and apply it to the Hough parameter space. In this paper, after an iterative analysis, the optimal threshold in the Hough parameter space is determined also to be T_M = 7 for a value of the probability of appearance of randomly arriving impulse interference with average length in the range cells e_0 = 0.1.

Table 1

e_0	P_{FA}	M/L = 10/16	M/L = 16/16
0.1	10^{-4}	0.1496	0.00049418
	10^{-6}	0.38521	0.0009749
	10^{-8}	0.73887	0.0025098
0.2	10^{-4}	0.20417	0.00037994
	10^{-6}	0.3338	0.005598
	10^{-8}	0.333805	0.055537
0.3	10^{-4}	0.1892	0.010345
	10^{-6}	0.276352	0.04529
	10^{-8}	0.3738	0.083025
0.4	10^{-4}	0.17294	0.029303
	10^{-6}	0.240684	0.057873
	10^{-8}	0.31467	0.088442
0.5	10^{-4}	0.16088	0.037538
	10^{-6}	0.218037	0.06226
	10^{-8}	0.279565	0.08862
0.6	10^{-4}	0.152435	0.041592
	10^{-6}	0.202945	0.063945
	10^{-8}	0.256548	0.087723
0.7	10^{-4}	0.14655	0.043923
	10^{-6}	0.19227	0.064821
	10^{-8}	0.24004	0.08696
0.8	10^{-4}	0.142317	0.045606
	10^{-6}	0.184154	0.065622
	10^{-8}	0.227193	0.086655
0.9	10^{-4}	0.1390595	0.047159
	10^{-6}	0.1775	0.066622
	10^{-8}	0.216512	0.086801

Fig. 3 shows the average detection thresholds of a Hough detector for two different values of probability of pulse interference and the threshold constant in the Hough parameter space for binary rule M-out-of-L = 16/16.

Fig. 4 gives the benefits from using an optimal threshold in the Hough parameter space for different binary rules and one value of pulse interference appearance probability. Fig. 5 shows the profits of using the Hough detector with CFAR BI processor, calculated for values – $T_M = 2$ and $e_0 = 0.1$, compared to the Hough detector with CA CFAR processor [18], for value of probability of false a alarm $P_{FA} = 10^{-4}$. For comparison, Fig. 6 presents the profits of using the Hough detector with CFAR BI processor compared to the CFAR BI processor's working as an independent detector with a binary rule M-out-of-L = 16/16. This is done following the approach described in [12].

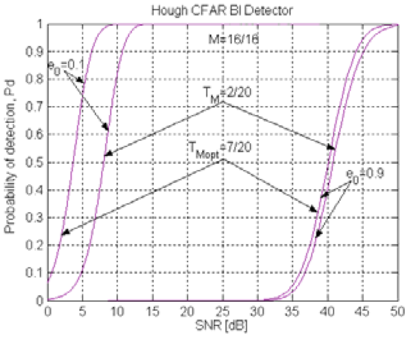

Fig. 3. Average detection threshold of the Hough detector compared to the optimal detection threshold in the Hough parameter space

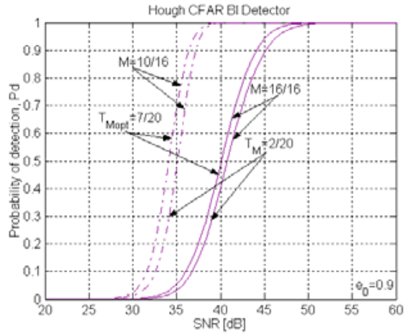

Fig. 4. Profits of the Hough detector (dashed line), for value of T_M=2 and for an optimal value T_M=7, for e_0=0.9, compared to the Hough detector (solid line) for another binary rule

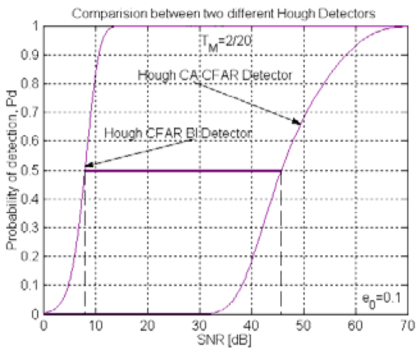

Fig. 5. Profits of using the Hough detector with CFAR BI processor, compared to using the Hough detector
with CA CFAR processor

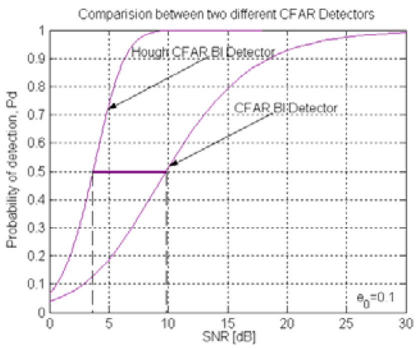

Fig. 6. Profits of using the Hough detector with CFAR BI processor, compared to using the CFAR BI processor

3.2. Hough detector with an Excision Binary Integration (EXC BI) CFAR processor

The efficiency of a Hough detector with an Excision Binary Integration (EXC BI) CFAR processor under conditions of randomly arriving impulse interference is investigated as well. Fig. 7 shows the block diagram of the EXC CFAR processor with binary integration.

A detailed performance analysis of an EXC Hough CFAR detector is presented in the paper. Its behaviour has been studied for different values of the threshold constant and for different values of the probability for appearance of impulse interference in the Hough parameter space. The experimental results are obtained for the following input parameters: average power of the receiver noise λ_0 = 1; average Interference-to-Noise Ratio (INR) r_j=30 dB; probability for the appearance of impulse interference with average length in the range cells from 0.1 to 0.9; number of reference cells N = 16; number of test cells L = 16; probability of a false alarm P_{FA}=10^{-4}, excision threshold B_e=2, number of scans N_s = 20, optimal values of Hough detection threshold T_M = 7, T_M = 13 and binary rules M-out-of-L = 10/16, M-out-of-L = 16/16.

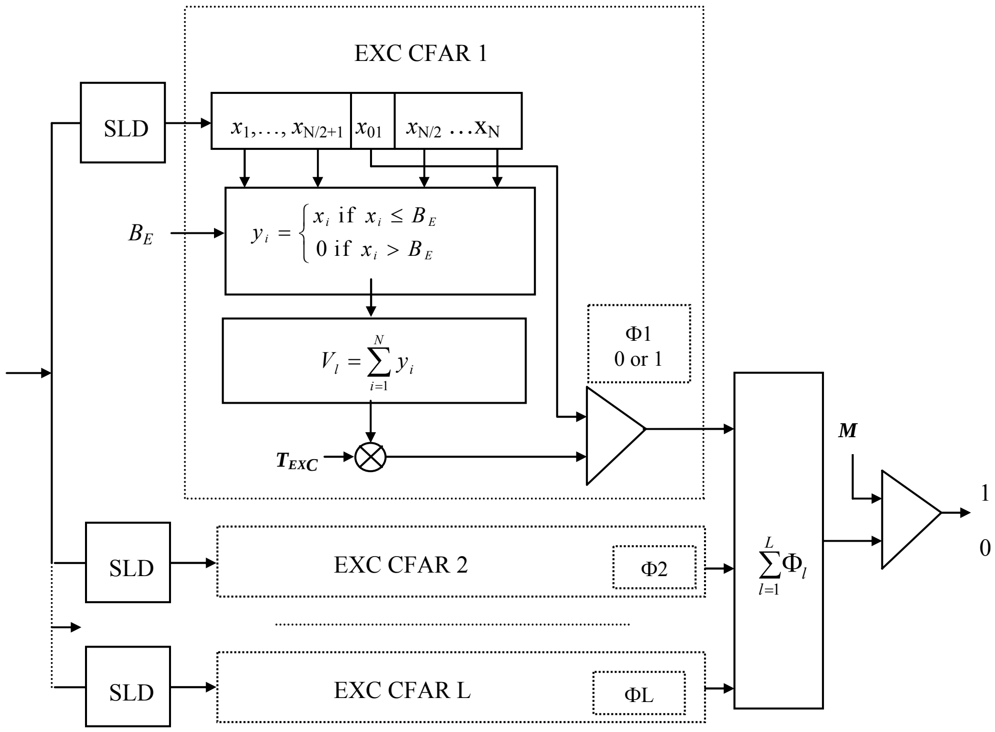

Fig. 7. Block diagram of an EXC CFAR processor with binary integration

According to [1] the value of the threshold constant should be $T_M = 7$, which corresponds to an optimal BI detector. The present research shows that the value $T_M = 7$ is suitable for the probability for appearance of impulse interference $e_0 = 0.9$. When the probability for appearance of impulse interference is $e_0 = 0.1$, the optimal threshold is $T_M = 13$.

Tables 2 and 3 present the results for average detection threshold (ADT) of EXC Hough CFAR BI detector for two different values for binary rule *M*-out-of-*L* = 10/16 (Table 2) and *M*-out-of-*L* = 16/16 (Table 3), with probability of false alarm ($P_{FA} = 10^{-4}$), excision constant B_e=2, for number of observations in the reference window ($N = 16$), an average interference-to-noise ratio (INR=30 dB) and different values for probability of appearance of randomly arriving impulse interference with average length in the cells in range.

Table 2

e_0	T_{EXCBI} for $T_{M\,opt} = 7/20$	ADT	T_{EXCBI} for $T_{M\,opt} = 13/20$	ADT
0.1	1.676	– 1.5538	1.193	– 1.8860
0.2	2.075	– 0.7683	1.397	– 1.3571
0.3	2.97	0.9698	1.744	– 0.6958
0.4	71.5	19.6977	2.505	0.8911
0.5	365.8	31.1839	30.52	15.0913
0.6	613.5	31.9899	267.98	30.8470
0.7	836.3	32.0907	472.8	32.0907
0.8	1 025.7	33.9043	628.3	34.9345
0.9	957.0	36.5726	524.8	38.4044

Table 3

e_0	T_{EXCBI} for $T_{\mathrm{M\ opt}} = 7/20$	ADT	T_{EXCBI} for $T_{\mathrm{M\ opt}} = 13/20$	ADT
0.1	0.3161	3.2028	0.157	4.2051
0.2	0.3465	3.3879	0.1706	4.2947
0.3	0.3885	3.5390	0.189	4.3703
0.4	0.4495	3.6902	0.215	4.4458
0.5	0.5445	3.8413	0.2535	4.5214
0.6	0.7072	4.2947	0.3125	4.8992
0.7	1.022	5.5793	0.393	6.4106
0.8	1.474	10.8690	0.324	12.5582
0.9	1.474	13.9573	0.324	15.1259

The probability of detection of the EXC Hough CFAR BI detector is shown in Fig. 8, for optimal values of the detection threshold T_M = 13, with value for probability of appearance e_0 = 0.1 and T_M = 7 for two values of probability of appearance – e_0 and binary rule – M-out-of-L=10/16. Fig. 9 shows the same characteristics for detection at different values for a binary rule M-out-of-L=16/16.

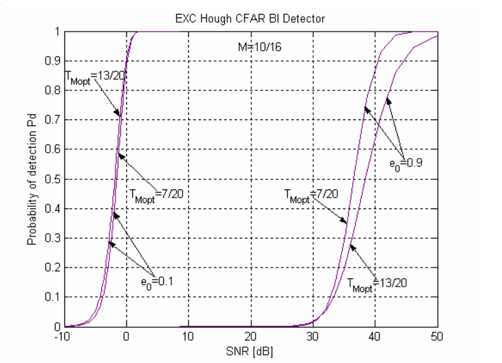

Fig. 8. Probability of detection of a Hough detector with EXC CFAR BI processor, for T_M=7/20 and T_M=13/20

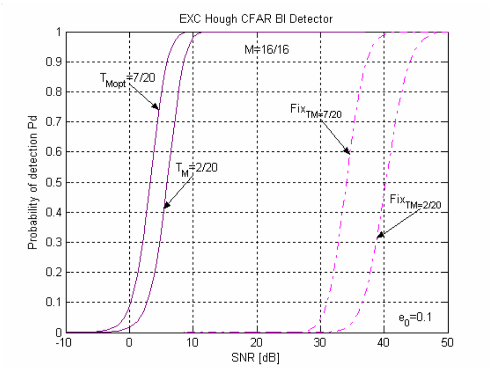

Fig. 9. Probability of detection of a Hough detector with EXC CFAR BI processor, for T_M=2/20, T_M=7/20 and e_0=0.1

The average decision threshold (ADT) for an EXC Hough CFAR BI detector in binomial distributed impulse interference is shown in Fig. 10. The results for the ADT are received using the signal-to-noise ratio (SNR) required for the adjustment of the detection probability P_d = 0.5. The EXC Hough CFAR BI detector with a binary rule M-out-of-L=10/16 are better (more efficient) in cases of lower values of the probability for appearance of impulse interference up to 0.5. For higher values of the probability for appearance of the impulse interference, above 0.5, the application of the binary rule M-out-of-L=16/16 results in lower losses.

If binary integration is applied in consequence, the benefit for the EXC Hough CFAR BI detector with the binary rule M-out-of-L=16/16 and the optimal threshold T_M = 7 is 35 dB and 40 dB for the optimal threshold values T_M = 13, in both cases e_0 = 0.9. For comparison, the results for the EXC Hough CFAR BI detector with a binary rule M-out-of-L=10/16, are shown as well. Here the benefits are lower by 20 dB approximately.

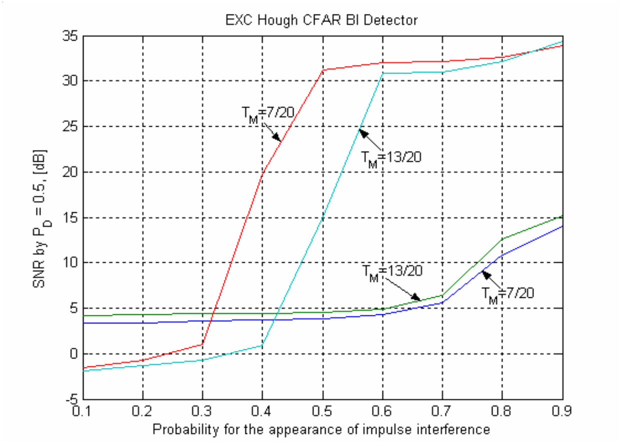

Fig. 10. ADT of an EXC Hough CFAR BI detector for optimal values of detection threshold T_M=7 and T_M=13 for binary rules M-out-of-L = 16/16 (solid line) and M-out-of-L = 10/16 (dashed line)

3.3. Hough detector with an Adaptive censoring Post detection Integration (API) CFAR processor

The efficiency of a Hough detector with an Adaptive censoring Post detection Integration (API) CFAR processor under conditions of randomly arriving impulse interference is investigated. The efficiency of the Hough API CFAR detector is studied by the method described in [12], i.e. the sensibility towards randomly arriving impulse interference. Fig. 11 shows the block diagram of an adaptive post detection integration (API) CFAR processor. Before pulse-to-pulse integration we can censor the elements of the reference window and the test resolution cells in order to form the adaptive threshold (Fig. 11). According to this algorithm, all elements with high intensity of the signal are removed from the reference window and the test resolution cell.

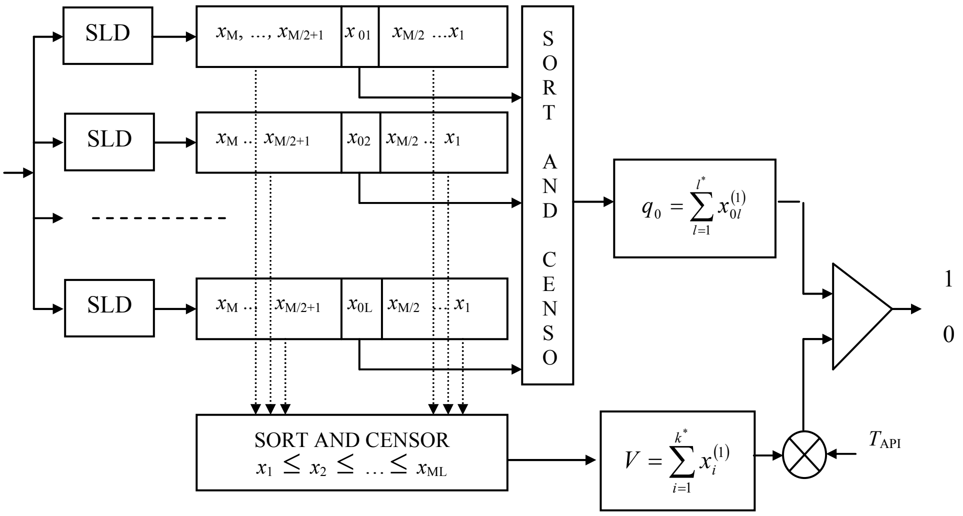

Fig. 11. Block diagram of active API CFAR processor

Table 4 presents the results for threshold values for API Hough CFAR detector obtained for three different values for probability of a false alarm P_{fa}, average interference-to-noise ratio (INR = 30 dB) and different values for probability for the appearance of randomly arriving impulse interference with average length in the cells in range. The API Hough CFAR detector's behaviour at different values of the test resolution cells L was thoroughly analyzed. All results are obtained for a threshold value of the Hough parameter space $T_M = 2$.

Table 4

e_0	P_{FA}	$L = 16$	$L = 32$
0	10^{-4}	5.2	9.3
	10^{-6}	7.7	13.4
	10^{-8}	10.9	19.0
0.01	10^{-4}	5.4	9.6
	10^{-6}	8.05	14.1
	10^{-8}	11.6	20.4
0.033	10^{-4}	5.8	10.4
	10^{-6}	9.0	15.9
	10^{-8}	13.6	24.0
0.066	10^{-4}	6.55	11.8
	10^{-6}	10.6	19.1
	10^{-8}	17.4	31.5
0.1	10^{-4}	7.5	13.6
	10^{-6}	13.0	23.6
	10^{-8}	23.5	43.5

Table 5 presents the results for an average detection threshold for API Hough CFAR detector with a value for probability of a false alarm (P_{FA} = 10^{-4}), for a number of observations in the reference window ($N = 16$), for a number of the test resolution cells L = 16, average interference-to-noise ratio (INR = 30 dB) and two different values for a probability for appearance of pulse jamming with average length in the cells in range.

Table 5

T_M	T_{API} for $e_0 = 0$	T_{API} for $e_0 = 0.1$	ADT for $e_0 = 0$	ADT for $e_0 = 0.1$
2	5.2	7.5	3.4950	4.6196
3	3.2	4.05	0.8848	1.5315
4	2.57	3.135	–0.5161	0.2821
5	2.24	2.68	–1.0680	–0.6175
6	2.03	2.40	–1.4747	–1.0277
7	1.87	2.19	–1.8741	–1.3548
8	1.75	2.035	–2.1562	–1.5919
9	1.648	1.905	–2.3174	–1.8539
10	1.56	1.795	–2.4584	–2.0276
11	1.482	1.70	–2.5592	–2.1562
12	1.414	1.612	–2.5793	–2.2569
13	1.35	1.535	***–2.5995***	–2.2788
14	1.29	1.462	***–2.5995***	***–2.2972***
15	1.235	1.395	–2.5390	–2.2327
16	1.18	1.33	–2.4584	–2.1562
17	1.127	1.265	–2.3174	–1.9908
18	1.073	1.20	–2.1159	–1.8136
19	1.0193	1.135	–1.7330	–1.4912
20	0.957	1.065	–1.2494	–0.9862

After an iterative analysis, the optimal threshold in the Hough parameter space is determined to be $T_M = 13$ at values of the probability for the appearance of randomly arriving impulse interference with average length in the range cells, $e_0 = 0$ and $T_M = 14$ for $e_0 = 0.1$.

Paper [14] discusses the comparison between the efficiency of Adaptive censoring Post detection Integration Constant False Alarm Rate (API CFAR) detector and Hough detector with non-coherent integration in randomly arriving impulse interference. The results show that Hough detector with API CFAR processor is most efficient under condition of decreasing randomly arriving impulse interference.

The probabilities of detection of API Hough CFAR detector and of Hough detector with fixed and adaptive thresholds are shown in Fig. 12, for optimal values of the detection threshold $T_M = 14$, at a value for probability of appearance $e_0 = 0.1$.

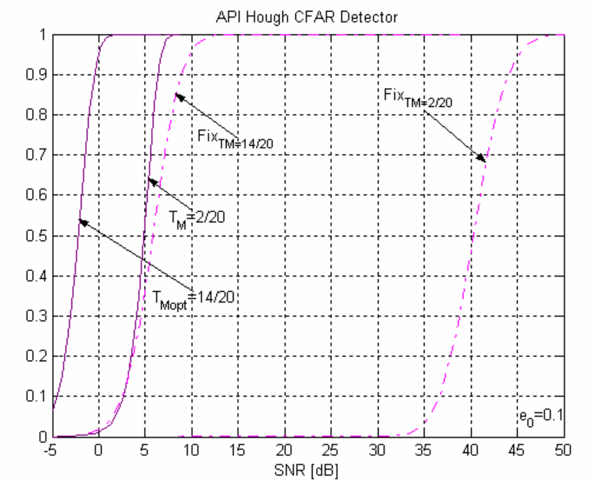

Fig. 12. Probability of detection of a Hough detector with API CFAR processor for T_M=2/20, T_M=14/20 and e_0=0.1

Fig. 13 shows different values of detection threshold in Hough parameter space T_M. Fig. 14 shows the profits of using the API Hough detector calculated for an optimal value of the detection threshold $T_M = 13$ and for a value $T_M = 2$, compared to the API CFAR detector, for number of test resolution cells L=16 and value for probability of a false alarm $P_{FA} = 10^{-4}$.

Under conditions of a strong flow from impulse interference, the usage of a fixed threshold detector or CFAR requires ADT of about 60 dB. Adding binary integration diminishes the ADT to 15 dB. The use of an API CFAR detector requires ADT up to 5-6 dB. Additional Hough transform diminishes the ADT to –2.5 dB.

Figs. 15 and 16 present the probability characteristics for the Hough detector with a fixed threshold and Hough detectors with CFAR BI, EXC CFAR BI and API CFAR processors, with different values of the threshold in the Hough parameter space T_M.

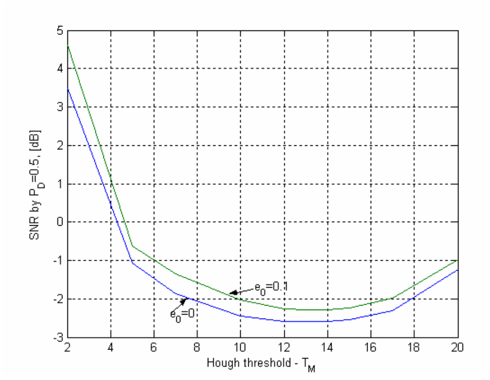

Fig. 13. Average detection threshold of the Hough detector compared to the optimal detection threshold in Hough parameter space

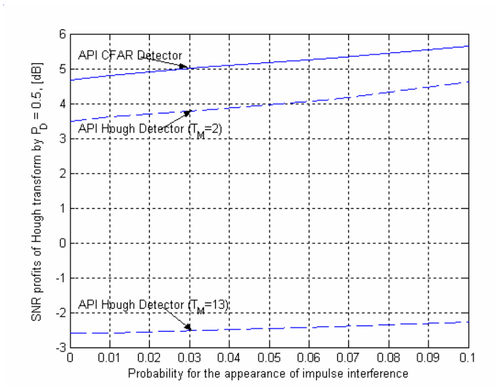

Fig. 14. Profits of the API Hough detector (dashed line), for optimal value of detection threshold T_{M}=13 and for value T_{M}=2, compared to the API CFAR detector (solid line) for L=16

The two-dimensional CFAR processors shown on the figures, have SNR losses about 1-4 dB, for $P_{\mathrm{d}} = 0.5$, because they detect no single pulses, but packets. In this case the Hough detector with API CFAR processor is better, because the amplitude integration is more efficient than the binary one. When there is impulse interference, increasing the number of integration cells in the Hough space $T_{M\mathrm{opt}}/N_{\mathrm{s}}$=2/20, 7/20 and 13/20, naturally leads to losses diminishing in the necessary SNR=(–12.5) – (–6.5) dB, for P_{d} = 0.5, where the optimum is with a value T_{M} = 7/20. The results obtained discover that the Hough detector with the API CFAR algorithm is the most efficient.

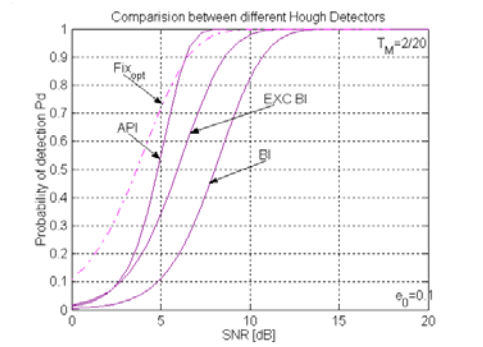

Fig. 15. Probability of detection for Hough detectors with two-dimensional CFAR processors, at a value $e_0 = 0.1$

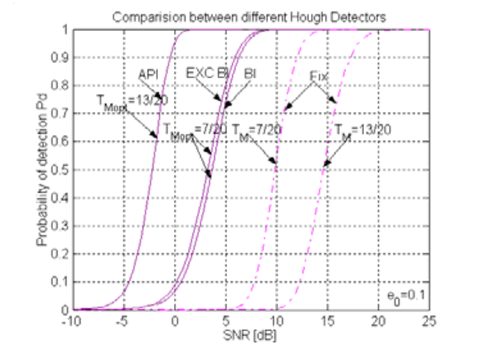

Fig. 16. Probability of detection for Hough detector with two-dimensional CFAR processors, at different values of $T_{M\mathrm{opt}}/N_{\mathrm{s}}$

The paper has investigated several types of Hough detectors with two dimensional CFAR processors in order to choose the most efficient one in the presence of randomly arriving impulse interference. The Hough detector with a fixed threshold is compared to three other types of Hough detectors.

4. Conclusions

The experimental results show the influence of the interference on the detection process, when having constant false alarm rate in pulse jamming. A method for losses estimation, which allows choosing the optimal detector parameters, is developed. The estimates of the efficiency of a Hough detector (with three different two-dimensional CFAR processors – CFAR BI, EXC CFAR BI and API CFAR) at randomly arriving impulse interference, are obtained for different stream characteristics.

Using Matlab, the average detection threshold for the three types of Hough detectors for a highly fluctuating target, Swerling II type target model detection under conditions of randomly arriving impulse interference, is calculated according to the approach presented in [12]. Using this approach, it is very easy to precisely determine the energy benefit when using a given detector. The results show that the Hough detector with two-dimensional processors is most efficient under conditions of decreasing randomly arriving impulse interference.

The optimal threshold values for different input conditions are estimated. The value of the test resolution cell and the probability of a false alarm over the average detection threshold are studied. The application of censoring techniques in the detection algorithm improves the Hough detectors efficiency.

The statistical analysis of the performance of a Hough detector with two-dimensional signal processors and its structure, working under conditions of randomly arriving impulse interference, is an addition to the results presented in paper [20], where a Hough detector with one-dimensional signal processors is considered. With the purpose to choose the best Hough detector architecture for noisy environment, detailed research of several different Hough detector structures, is presented. In the variety of different architectures, the Hough detector proves to be robust in tremendously worsened noisy environment with very high probability for appearance of impulse interference. The most stable algorithm proved to be the one with a Hough detector, using an Adaptive censoring Post detection Integration (API CFAR) processor, proposed by the authors of this paper in [13].

The final conclusion is that the results achieved in the paper confirm once again the necessity for synthesis of new algorithms for moving targets detection assuring robustness and higher efficiency of the radar systems. The results obtained in this paper could practically be used in radar and communication networks.

References

1. Carlson, B., E. Evans, S. Wilson. Search Radar Detection and Track with the Hough Transform. Part I. – IEEE Trans., Vol. **AES-30**, 1994, No 1, 102-108.
2. Carlson, B., E. Evans, S. Wilson. Search Radar Detection and Track with the Hough Transform. Part II. – IEEE Trans., Vol. **AES-30**, 1994, No 1, 109-115.
3. Carlson, B., E. Evans, S. Wilson. Search Radar Detection and Track with the Hough Transform. Part III. – IEEE Trans., Vol. **AES-30**, 1994, No 1, 116-124.
4. Goldman, H., I. Bar-David. Analysis and Application of the Excision CFAR Detector. – In: IEE Proc., Vol. **135**, Pt.F, (6), 1988, 563-575.

5. G o l d m a n, H. Performance of the Excision CFAR Detector in the Presence of Interferers. – In: IEE Proc., Vol. **137**, 1990, Pt.F, (3), 163-171.
6. B e h a r, V., C h r. K a b a k c h i e v. Excision CFAR Binary Integration Processors. – Compt. Rend. Acad. Bulg. Sci., Vol. **49**, 1996, No 11/12, 45-48.
7. A k i m o v, P., F. E v s t r a t o v, S. Z a h a r o v. Radio Signal Detection. Moscow, Radio and Communication, 1989, 195-203 (in Russian).
8. G a r v a n o v, I., C. K a b a k c h i e v. Sensitivity of CFAR Processors Toward the Change of Input Distribution of Pulse Jamming. – In: Proc. of IEEE – International Conference on Radar – RADAR, 2003, Adelaide, Australia, 2003, 121-126.
9. K a b a k c h i e v, C., I. G a r v a n o v, L. D o u k o v s k a. Excision CFAR BI Detector with Hough Transform in Presence of Randomly Arriving Impulse Interference. – In: Proc. of the International Radar Symposium – IRS'05, Berlin, Germany, 2005, 259-264.
10. D o u k o v s k a, L. Hough Detector with One-Dimensional CFAR Processors in Randomly Arriving Impulse Interference. – In: Proc. of Distributed Computer and Communication Networks, International Workshop, Sofia, Bulgaria, 2006, 241-254.
11. K a b a k c h i e v, C., L. D o u k o v s k a, I. G a r v a n o v. Cell Averaging Constant False Alarm Rate Detector with Hough Transform in Randomly Arriving Impulse Interference. – Cybernetics and Information Technologies, Vol. **6**, 2006, No 1, 83-89.
12. R o h l i n g, H. Radar CFAR Thresholding in Clutter and Multiple Target Situation. – IEEE Trans., Vol. **AES-19**, 1983, 4, 608-621.
13. B e h a r, V., C. K a b a k c h i e v, L. D o u k o v s k a. Adaptive CA CFAR Processor for Radar Target Detection in Pulse Jamming. – Journal of VLSI Signal Processing, **26**, 2000, 383-396.
14. K a b a k c h i e v, C., I. G a r v a n o v, L. D o u k o v s k a. Adaptive Censoring CFAR PI Detector with Hough Transform in Randomly Arriving Impulse Interference. – Cybernetics and Information Technologies, Vol. **5**, 2005, No 1, 115-125.
15. K a b a k c h i e v, C., L. D o u k o v s k a, I. G a r v a n o v. Hough Radar Detectors in Conditions of Intensive Pulse Jamming. – Sensors & Transducers Magazine, Special Issue Multisensor Data and Information Processing, August 2005, 381-389.
16. D o u k o v s k a, L., C. K a b a k c h i e v. Performance of Hough Detectors in Presence of Randomly Arriving Impulse Interference. – In: Proc. of the International Radar Symposium – IRS'06, Krakow, Poland, 2006, 473-476.
17. B e h a r, V., L. D o u k o v s k a, C. K a b a k c h i e v. Target Detection and Parameter Estimation Using the Hough Transform, Numerical Methods and Applications – NM&A, Springer "Lectures Notes and Computer Science", LNCS 4310, 2007, 525-532.
18. D o u k o v s k a L. Hough Detector with Binary Integration Signal Processor. – Compt. Rend. Acad. Bulg. Sci., Vol. **60**, 2007, No 5, 525-533.
19. M u s t a k e r o v, I., D. B o r i s s o v a. Technical Systems Design by Combinatorial Optimization Choice of Elements on the Example of Night Vision Devices Design. – Compt. Rend. Acad. Bulg. Sci., Vol. **60**, 2007, No 4, 373-380.
20. D o u k o v s k a, L. Moving Target Hough Detector in Pulse Jamming. – Cybernetics and Information Technologies, Vol. **7**, 2007, No 1, 67-76.