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# Identification and Extremum Seeking Control of the Anaerobic Digestion of Organic Wastes

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**Abstract:** The principle of extremum seeking control has been applied to  $2^{nd}$  and  $4^{th}$  order models of anaerobic digestion. In the case of variations of the inlet organics the maximum biogas flow rate was obtained. Laboratory experiments have been provided with step and impuls changes of acetate addition (new control input). Based on the dynamical responses of the biogas flow rate, non-linear optimization and simulations some of the model coefficients have been estimated more precisely. Input-output static characteristics, optimal steady-state and some constraints have been derived analytically.

*Keywords:* Anaerobic digestion, non-linear dynamical models, acetate addition, identification, steady-state analysis, extremum seeking control.

## 1. Introduction

During anaerobic digestion (AD) of organic wastes the organic matter is mineralized by microorganisms into biogas (methane and carbon dioxide) in the absence of oxygen. The biogas is an additional energy source and the methane is a greenhouse gas.

A lot of models of the AD are known. However, because of the very restrictive on-line information their coefficients estimation is a very difficult problem (B a s t i n, D o c h a i n [1]; S i m e o n o v [7]; N o y k o v a [5]).

The task of extremum seeking control is to find the operating set-points that maximize or minimize an objective function. Recently some new results concerning the stability analysis of extremum seeking of nonlinear systems have been obtained (W a n g et al. [8]; K r s t i c, W a n g [2]; M a r c o s, G u a y, D o c h a i n [4]).

In the present paper, in order to maximize the biogas productivity of the AD, we apply extremum seeking control using the dilution rate as a control action and the biogas flow rate as a measured output. As a benchmark for our demonstration, we use two non-linear process models after precision of some of their coefficients using an impuls like additional signal (acetate addition).

# 2. Process model and parameter estimation

#### 2.1. Experimental studies

Laboratory experiments are carried out in continuously stirred tank bioreactor with cattle wastes at mesophillic temperature and addition of acetate in low concentrations. The added acetate is mixed with the effluent organics and pH of this mixure is kept in the admissible range. (L u b e n o v a et al. [3]). The responses of Q are obtained for step and impuls changes of the acetate addition.

#### 2.2. Mathematical modelling of the process

The 2nd order model used in this paper is build on one-stage reaction scheme (S i m e o n o v [7]):

(1) 
$$\frac{dX}{dt} = \mu X - DX,$$

(2) 
$$\frac{dS}{dt} = -k_1 \mu X + D(S_{0i} - S),$$

$$(3) Q = k_2 \mu X ,$$

S (g/l) is the substrate concentration, X (g/l) – biomass concentration,  $Q(l.d^{-1})$  – biogas flow rate,  $\mu(d^{-1})$  is the Monod type specific growth rate.

The 4th order model for AD, used in this paper, is based on the two-stage reaction scheme:

(4) 
$$\frac{dX_1}{dt} = (\mu_1 - D)X_1,$$

(5) 
$$\frac{dS_1}{dt} = -k_1 \mu_1 X_1 + D_1 S_{1in}^{\circ} - DS_1,$$

(6) 
$$\frac{dX_2}{dt} = (\mu_2 - D)X_2,$$

(7) 
$$\frac{dS_2}{dt} = k_3 \mu_1 X_1 - k_2 \mu_2 X_2 + D_1 S_{2in}^0 + D_2 S_{2in} - DS_2,$$

$$(8) Q = k_4 \mu_2 X_2,$$

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(9) 
$$\mu_1 = \frac{\mu_{1\max}S_1}{k_{S_1} + S_1}, \qquad \mu_2 = \frac{\mu_{2\max}S_2}{k_{S_2} + S_2},$$

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where  $X_i(g/l)$ ,  $S_i(g/l)$ ,  $\mu_i(d^{-1})$ , i = 1, 2, are the bacterial concentrations, the associated substrate concentrations and the Monod type specific growth rates for the acidogenic (i=1) and methanogenic (i=2) stages, respectively;  $Q(l.d^{-1})$  is the biogas flow rate; the coefficients  $\mu_{maxi}(d^{-1})$  and  $k_{s_2}(g/l)$  represent the maximum specific growth rates and saturation constants for both microorganisms groups, respectively.  $k_1$ ,  $k_2$ ,  $k_3$  and  $k_4$  are yield coefficients;  $D_1(d^{-1})$ , is the dilution rate for the inlet soluble organics with concentration  $S_{1in}^0$ . We assume that the second biodegradation stage starts in the

bioreactor and therefore the concentration of acetate in the inlet substrate is  $S_{2in}^0 = 0$ ;

 $D_2$  (d<sup>-1</sup>), is the dilution rate for the input acetate concentration  $S_{2in}$  (g/l).  $D = D_1 + D_2$  is the total dilution rate. The state vector is:  $\mathbf{x} = \begin{bmatrix} S_1 & X_1 & S_2 & X_2 \end{bmatrix}$ . The parameter vector

is:  $\mathbf{p} = \begin{bmatrix} \mu_{1\max} & k_{s1} & k_1 & \mu_{2\max} & k_{s2} & k_2 & k_3 & k_4 \end{bmatrix}^T$ . The only measurable output is  $\mathbf{y} = Q$ . We assume that  $D_1$ , and are constants, the acetate concentration is a control input and the first acidogenic stage is in steady state with coordinates:

(10) 
$$S_{1}^{*} = \frac{Dk_{S_{1}}}{\mu_{1\max} - D};$$

(11) 
$$X_{1}^{*} = \frac{1}{k_{1}} \left( S_{1\text{in}} - \frac{k_{s_{1}}D}{\mu_{1\text{max}} - D} \right),$$

where all parameters are known. This allows us to simplify the dynamical model (4),  $\dots$ ,(8) as:

(12) 
$$\frac{dX_2}{dt} = \left(\mu_2 - D\right) X_2,$$

(13) 
$$\frac{dS_2}{dt} = C_1 - k_2 \mu_2 X_2 + D_2 S_{2in} - DS_2,$$

$$(14) Q = k_4 \mu_2 X_2$$

where  $C_1 = \frac{k_3}{k_1} DX_1^*$  is a constant.

In this simplified model the state vector is  $\mathbf{x} = \begin{bmatrix} S_2 & X_2 \end{bmatrix}$ ; input  $\mathbf{u} = S_{2in}$ , output  $\mathbf{y} = Q$ . The parameter vector is extended with the initial states values:  $\mathbf{p} = \begin{bmatrix} C_1 & \mu_{2max} & k_{s2} & k_2 & k_4 & X_2(0) & S_2(0) \end{bmatrix}^T$ . The initial value of  $\mathbf{p}$  is taken from (L u b e n o v a et al. [3]).

The simplified version (12), ..., (14) is very similar to the model (1), ..., (3), with the addition of the control action  $S_{2in}$ , which is very suitable for identification and control purposes.

2.3. Parameter estimation when only the biogas production rate Q is measurable.

S i m e o n o v [7] provided investigation of model parameters in (1), ..., (3). Here we use the estimated values from this work:  $k_1 = 6.7$ ,  $k_2 = 16.8$ ,  $\mu_{max} = 0.35$ ,  $k_s = 2.3$ . For the simplified model (12)-(14) S i m e o n o v et al. [6] investigated both

For the simplified model (12)-(14) S i m e o n o v et al. [6] investigated both theoretical and practical identifiability of the model parameters. After combining the results from both identifiability investigations (N o y k o v a [5]) we conclude that the most suitable parameter combination, which has to be estimated, is

$$\mathbf{p}_2 = \left[ X_2(0), \ \boldsymbol{\mu}_{2\max}, k_2 \right].$$

During the parameter estimation stage we had to minimise the following quadratic error functional:

(15) 
$$\operatorname{CRIT}(\mathbf{p}_2) = \sum_{i=0}^{N} \left( Q^i(\mathbf{p}_2) - Q_{\exp}^{i} \right)^2 .$$

We have used constrained non-linear optimisation, realised in the function **fmincon**, OPTIMIZATION toolbox 2.0 of MATLAB5.3. The initial values of the estimated parameters are:  $X_2(0)^0 = 0.01$ ,  $\mu_{2\max}^0 = 0.25$ ;  $k_2^0 = 4.2$ . The estimated parameter values are:  $X_2(0)^* = 0.9948$ ;  $\mu_{2\max}^* = 0.2561$ ;  $k_2^* = 5.5330$ . The value of the optimization criterion is CRIT = 1.8136. The graphical results from the parameter estimation (experimental and simulated data for Q and four impulses of acetate addition with amlitudes 0.5, 0.75, 1.0 and 1.5 (g/l), respectively) are shown on Fig.1. From these results it is clear that the model fits well the experimental data.



Fig. 1. Experimental and simulated data for  $Q_{\rm m}$  and  $Q_{\rm exp}$  (g/l) in time (d)

### 3. Extremum seeking control

#### 3.1. Problem statement

We assume that the goal of the AD process is production of biogas. As an optimization objective it is then natural to consider the maximization of the biogas flow rate  $Q(1.d^{-1})$ 

(16) 
$$Q \Rightarrow \max$$

In the next paragraph we show that the steady states of the AD process are characterized by a non – monotonic map relating the biogas flow rate Q (controlled output) to the dilution rate D, which is our control input. The purpose of the extremum seeking method is then to iteratively adjust the dilution rate in order to steer the process to the maximum of this map.

#### 3.2. Steady-states analysis of the open - loop system

In ideal stationnary conditions all the derivatives in the model (4)-(9) are equal to zero. The steady states  $S_1^*$  and  $X_1^*$  are given with (10) and (11). The other stationary values are:

(17) 
$$S_2^* = \frac{k_{S_2}D}{\mu_{2_m} - D},$$

(18) 
$$X_2^* = \frac{1}{k_2} \left( \frac{k_3}{k_1} \left( S_{0in} - \frac{k_{s_1} D}{\mu_{1max} - D} \right) - \frac{k_{s_2} D}{\mu_{2_m} - D} \right),$$

(19) 
$$Q^* = \frac{Dk_4}{k_2} \left( \frac{k_3}{k_1} S_{0in} - \frac{Dk_3 k_{s_1}}{k_1 (\mu_{1max} - D)} - \frac{Dk_{s_2}}{\mu_{2max} - D} \right)$$

From (10) and (17) is clear that in order to have positive substrate concentrations the following conditions have to be satisfied:

$$D < \mu_{1 \max}$$
 and  $D < \mu_{2 \max}$ .

From (11) we obtain the upper bound for dilution rate, such that does not go extinct:

(20) 
$$D_{\sup 1} = \frac{\mu_{1\max} S_{1\min}}{k_{S_1} + S_{1\min}}.$$

For the case when  $X_2^*=0$  we obtain from (18) a quadratic equation with respect to the dilution rate *D*. Next we analyze this function. We see that

$$\begin{aligned} X_2^*(\pm\infty) &= \frac{k_3}{k_1} \left( S_{1\text{in}} + k_{S_1} \right); \ X_2^*(0) = \frac{k_3}{k_1} S_{1\text{in}} ,\\ \lim_{D \uparrow \mu_1 \max} X_2^* &= -\infty ; \ \lim_{D \downarrow \mu_1 \max} X_2^* = +\infty ,\\ \lim_{D \uparrow \mu_2 \max} X_2^* &= -\infty ; \ \lim_{D \downarrow \mu_2 \max} X_2^* = +\infty .\end{aligned}$$
  
The first derivative  $\frac{\partial X_2^*}{\partial D}$  is always negative.

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Fig. 2. Graphical presentation of the function

The graphics of this function is shown on Fig. 2. In this case we assume that  $\mu_{1\max} < \mu_{2\max}$ . For the case when  $\mu_{2\max} < \mu_{1\max}$  the graphics is analogous because the function is symmetric with respect to the specific growth rates. When  $\mu_{1\max} = \mu_{2\max}$ there is only one point where the function is not defined.

We see that this function has two upper bounds  $D_{sup2}$  and  $D_{sup3}$  for dilution rate D, such that  $X_2$  does not go extinct. We assume  $D_{sup2}$  to be the smaller value. Next we compare the value  $D_{sup1}$  with  $D_{sup2}$  and  $D_{sup3}$ . For the case when  $\mu_{1max} < \mu_{2max}$  and  $D_{sup1} < \mu_{1max}$ . we can conclude that  $D_{sup1}$  lies in the first branch of this curve. We replace (20) into (18) and obtain the following expression

(21) 
$$X_{2}^{*}(D_{\text{supl}}) = \frac{k_{3}k_{s_{2}}\mu_{1\text{max}}S_{1\text{in}}}{k_{2}k_{1}\left(-k_{s_{1}}\mu_{2\text{max}} + \left(\mu_{1\text{max}} - \mu_{2\text{max}}\right)S_{1\text{in}}\right)}$$

and it is clear that  $X_2^*(D_{supl}) < 0$ , because

$$-k_{S_{1}}\mu_{2\max} + (\mu_{1\max} - \mu_{2\max})S_{1\min} =$$
$$= \frac{1}{k_{S_{1}} + S_{1\min}} (-\mu_{2\max} + D_{\sup}) < 0 .$$

In this case  $D_{sup2} < D_{sup1} < D_{sup3}$ . For the case when  $\mu_{2max} < \mu_{1max}$  there are two possibilities. We see from (21) that in the case when  $D_{sup1} < \mu_{2max} < \mu_{1max}$  the value  $X_2^*(D_{sup1}) > 0$ . When  $\mu_{2max} < D_{sup1} < 0$  $\mu_{1\text{max}}$  the value  $X_2^*(D_{\text{sup1}}) < 0$ . In both cases  $D_{\text{sup2}} < D_{\text{sup1}} < D_{\text{sup3}}$ .

Hence we conclude that  $D_{\sup 2} = \inf [D_{\sup(i)}], i = 1, 2, 3$ .

The analytical results, obtained from steady state investigation of the open loop system, could be summarized as follows:

Theorem 1. Consider the above presented dynamical model of the AD. There exists a nontrivial equilibrium for this model if and only if  $D < D_{sup^2}$ . This equilibrium is unique whenever it exists.



Fig. 3. Graphical presentation of the function

Our numerical example (model (4), ..., (9)) is in agreement with the general results from Theorem 1. We obtain the following values for  $D_{\sup(i)}$ :  $D_{\sup} = 0.192$ ,  $D_{\sup^2} = 0.187$ ,  $D_{\sup^3} = 0.229$ . Next we provide similar analytical investigations of the function  $Q^* = Q(D)$ ,

Next we provide similar analytical investigations of the function  $Q^*=Q(D)$ , described by (19). The graphics of this function is shown on Fig. 3. In this case we assume that  $\mu_{1\max} < \mu_{2\max}$ .

assume that  $\mu_{1\max} < \mu_{2\max}$ . Further we investigate only in the first branch when  $\mu_{1\max} > D$  and  $\mu_{2\max} > D$ , otherwise washout will appear.

For the maximal equilibrium state we obtain the following values for all variables:

$$D_{\max} = 0.14, \ S_{1\max}^* = 0.7; \ X_{1\max}^* = 1.015; \ S_{2\max}^* \ 1.05; \ X_{2\max}^* = 0.73, \ Q_{\max} = 0.4456$$



Fig. 4. Input-output characteristics of Q (l/d) and COD (g/l) in time (d<sup>-1</sup>)

On Fig.4 input-output characteristics Q=Q(D) (for  $S_{lin}^0$  at 5-11 (g/l)) and COD=f(D) are presented.

The second order model (1), ..., (3) is a particular case with:  $D_{sup} = 0.267$ ,  $D_{max} = 0.17957$ ,  $S_{max} = 2.43$ ,  $X_{max} = 0.742$ ,  $Q_{max} = 2.238$ . Full investigation of this model is presented in S i m e o n o v [7].

3.3. A peak seeking control via the dilution rate

The peak seeking feedback scheme is shown on Fig. 5. Its basic idea is to employ

periodic excitation signal  $a\sin\varphi t$ , wich is added to the signal  $\vec{B}$ . If this exitation signal is slow, then the AD process appears as a static map Q=Q(D) and its dynamics

do not interfere with the peak seeking scheme. If  $\vec{B}$  is on either side of  $D_{\max}$ , the excitation signal  $a\sin\varphi t$  create a periodic response of Q, which is either in phase or out of phase with  $a\sin\varphi t$ . The high passe filter  $s/(s+\varphi_n)$  eliminates the "DC component" of Q. Thus,  $a\sin\varphi t$  and  $\{s/(s+\varphi_n)Q \text{ will be (approximately) two sinusoids, which are:$ 

in phase for  $\vec{B} < D_{\max}$  or out of phase for  $\vec{B} > D_{\max}$ . In either case, the product of two sinusoids will have a "DC component"  $\xi$ , that can be argued to be approximately the sensitivity function  $(a^2/2)[Q(D)](\vec{B})$ . Then the integrator  $\vec{B} = (k/s)\xi$  is approximately the gradient update law:

(22) 
$$\frac{d}{dt}\vec{B} = k(a^2/2)\frac{d}{dt}[Q(D)](\vec{D}),$$

driven by the sensitivity function, which tunes  $\vec{D}$  to  $D_{\text{max}}$ 



Fig. 5. The peak seeking feedback scheme

The tuning parameters in this scheme must be chosen as follows (W a n g, et al. [8]; K r s t i c, W a n g [2]):

(23) Speed of non-linear dynamics =  $O(1) \gg \omega \gg \omega_h$ , a, k.

Thus, the overall feedback system has three time scales:

1) fastest – the process (with the stabilizing controller);

2) medium – the periodic excitation signal;

3) slow – the filter in the peak seeking scheme.

As a result this peak seeking control is model – free and able to automatically tune the dilution rate in the right direction. The scheme shown on Fig. 3 guarantees the stability result outlined in the following:

**Theorem 2**. Consider the feedback system on Fig.3 and assume that the AD dynamic model has the following properties:

1) for *D* in the interval [D', D''] there is an isolated one – dimensional manifold of equilibria E(D) which depends smoothly on *D*;

2) each of the equilibria in E[D', D''] are exponentially stable with an O(1) rate of decay;

3) the equilibrium value of the output Q on E[D', D''] is a smoth function of D with a maximum at  $D = D_{max}$ ;

Then there exists a ball of initial conditions around the equilibrium corresponding to  $D = D_{\text{max}}$  and a positive constant  $\overline{\omega} << 1$  such that for all  $\omega \in (0, \overline{\omega})$  and all  $a, k, \omega_h << \omega$ , the solution converges to an  $O(\omega)$  neightrhood of that equilibrium.

This theorem is an interpratation for AD process of the more general result for continuous type of biotechnological processe (W a n g et al. [8]) with detailed proff in (K r s t i c, W a n g [2]).

#### 3.4. Simulation results

For the 2nd and 4th order models of the AD we know that the peak  $(Q_{\text{max}})$  occurs at the above presented values of  $D_{\text{max}}$ . Our purpose is to tune D to  $D_{\text{max}}$ . We implement the peak seeking scheme with the following choice of parameters:  $\omega_h = 0.04$ ;  $\omega = 0.12$ ; a = 0.01; k = 0.25. First, we start from an initial dilution rate lower than the optimum rate (D(0)=0.025). The time responses of the output Q are shown on Figs. 7 and 9 (for 2nd and 4th order models, respectively) and the time responses of the tuning parameter D are shown on Figs. 6 and 10. The maximum seeking process with initial dilution rate  $D_0 = 0.04$  (with 2nd order model) is shown on Fig. 8. In the scond simulation study we start from an initial dilution rate larger than the optimum value. In both cases the peak seeking approaches the appropriate peak.

The convergence to the peak can be made faster by tuning the parameters of the scheme and by introducing an appropriate phase shift in the form the perturbation sinusoid. However, if we choose parameters, which makes the convergence left side of the peak faster, they are too aggressive for the right side of the peak and may lead



Fig. 6. Time response of D (with 2nd order model) in time t (d)

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Fig. 7. Time response of Q (with 2nd order model) in time t (d)



Fig. 8. The maximum seeking process in the phase plane with initial dilution rate  $D_0 = 0.04$  (with 2nd order model)



Fig. 9. Time response of Q(l, d) (with 4th order model) in time t(d)

to instability. As we do not assume to know the location of the peak, the adaptation must proceed cautiously. In the thirld simulation study (with the 4th order model) we start from an initial dilution rate larger than the optimum value (D(0) = 0.15), and in the 600th day a step variation of  $S_{0i}$  occurs (from 7.5 at 9.0 (g/l), e.g. increase with 20%). By the same step cise variation in the 1100th day  $S_{0i}$  obtains its previous value. The time response of the output Q is shown on Fig. 11. In both cases of variation of the perturbation  $S_{0i}$  the peak seeking approaches the appropriate peak. In the 4th simulation study (with the 4th order model) we start from an initial dilution rate



Fig. 10. Time response of D (l/d) (with 4th order model) in the time t (d)



Fig.11. Time response of Q (l/d) with variation of  $S_{0i}$  (with 4th order model) in time t (d)

lower than the optimum rate (D(0) = 0.025), and in the 600th day a step variation of  $S_{0i}$  occurs (from 7.5 up at 6.0 (g/l), e.g. decrease with 20%). In the 1100th day by the same step variation  $S_{0i}$  obtains its previous value. In both cases of variation of the perturbation  $S_{0i}$  the peak seeking approaches the appropriate peak. In the 5th simulation study (with the 4th order model) we have the previous case, however with measurement noise (with variance 0.001) on the biogas flow rate Q. The time response of the output Q is shown in Fig.12.

## 4. Conclusion

Theoretical and experimental studies have proven that it is possible to obtain appropriate values for some coefficients of nonlinear models of the AD measuring only Q.

Theoretical and simulation studies have proven that with the extremum seeking control law (22) we can optimize the operation of anaerobic digesters (maximization



Fig.12. Time response of Q (l/d) with noise (with 4th order model) in time t (d)

the biogas production) in the realistic case of strong variations of the influent organic matter. The control law (22) doesn't depend of the process model and it is much more simple for practical realisation than the result in (M a r c o s, G u a y, D o c h a i n [4].

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