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Moving Target Hough Detector in Pulse Jamming*

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Abstract: *The Hough detector with two types of a Constant False Alarm Rate (CFAR) processors – a Cell Averaging (CA CFAR) processor and an Excision (EXC CFAR) processor in the presence of pulse jamming is investigated in the present paper. The detection probability and the average detection threshold of a Hough detector with these two types of CFAR processors are studied. The experimental results are obtained by numerical analysis. They reveal that the use of Hough detector allows reducing drastically detectability losses in comparison to the conventional CFAR detectors and that it is effective for small signal-to-noise ratios. The research work is performed in MATLAB computational environment. The obtained analytical results for Hough detector can be used in both, radar and communication receiver networks.*

Keywords: *Radar Detector, Hough detector, Pulse Jamming, Probability of detection, Probability of false alarm, Detectability profits (losses).*

1. Introduction

During the last few years, mathematical methods for extraction of useful data about the behavior of observed targets by mathematical transformation of received signals are being widely used in the design of new highly effective algorithms for processing radar information. Such a mathematical approach is the Hough Transform (HT). The concept of using the HT for improving the target detection in white Gaussian noise is introduced by Carlson, Evans and Wilson in [1, 2, 3]. This approach is used by Carlson in [3], for a highly fluctuating target – Swerling II type target model, and stationary homogeneous interference.

In modern radars, the target detection is declared if the signal value exceeds a preliminary determined adaptive threshold. The current estimation of the noise level in the reference window forms the threshold. To estimate the noise level for radar signal detection in clutter environment with unknown average power level the technique proposed by Finn and Johnson in [4] is often used. Averaging the outputs

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of the reference cells surrounding the test cell forms this estimate. The detection threshold is determined as a product of the noise level estimate in the reference window and a scale factor to achieve the desired probability of false alarm. Thus a constant false alarm rate is maintained in the process of detection. These CA CFAR processors (Cell Averaging Constant False Alarm Rate Processors) are very effective in case of stationary and homogeneous interference. The presence of pulse jamming in both – test resolution cell and the reference cells can cause drastic degradation in the performance of the CA CFAR processor [6]. Such type of interference is non-stationary and non-homogenous and is often caused by adjacent radar or other radio-electronic devices.

A technique, that may be used to alleviate this problem, is the excision of strong pulses before a cell-averaging procedure. This approach for an EXC CFAR detector is presented by Goldman and Bar-David and analyzed for multiple target situations in [8, 9]. The analytical expressions for the probability of detection and the probability of false alarm of EXC CFAR and EXC CFAR BI detectors in the presence of Poisson distribution pulse jamming and Raleigh amplitude distribution in both – test and reference windows, are presented in [10].

The signal model is based on the fact that the law of distribution of impulse interference changes from Poisson to binomial, with the increasing of the probability of appearance of randomly arriving impulse, greater than 0.1 [11]. The binominal model is more general than Poisson distribution model. Therefore all mathematical formulas for evaluation of both probability measures – the probability of detection and the probability of false alarm, should be derived for binominal distribution for a Hough detector.

In the present paper, one situation for a highly fluctuating target – Swerling II type target model detection in conditions of pulse jamming is studied. The effectiveness of Hough detector with CA CFAR or EXC CFAR processor in pulse jamming for value of probability of detection – $P_D = 0.5$ is researched. The effectiveness of Hough detector with the method from [13], i.e. the sensibility towards pulse jamming is estimated. These estimates allow the comparison of Hough detector towards one-dimensional CFAR processors and the comparison towards another patterns researched from other authors [12]. The present results show that Hough detector is effective in conditions of decreased pulse jamming.

2. Statistical analysis of Hough detector

Using Carlson's approach [1, 2, 3], we obtain a new result for detection performance in Hough space, for target model of the type Swerling II in conditions of pulse jamming, described with the probability density function (pdf) of the reference window output [5]:

$$f(x_i) = \frac{1-e_0}{\lambda_0(1+s)}\exp\left(\frac{-x_i}{\lambda_0(1+s)}\right) + \frac{e_0}{\lambda_0(1+r_j+s)}\exp\left(\frac{-x_i}{\lambda_0(1+r_j+s)}\right), \quad (1)$$

where s is the per pulse average signal-to-noise ratio, λ_0 is the average power of the receiver noise, r_j is the average interference-to-noise ratio, e_0 is the probability for the appearance of pulse jamming with average length in the range cells.

In conditions of binomial distribution of pulse interference the background environment includes the interference-plus-noise situation [11]. Consequently, the probability density function (pdf) of the reference window outputs may be defined by

$$(2)\quad f(x_i)=\frac{(1-e_0)^2}{\lambda_0}\exp\left(\frac{-x_i}{\lambda_0}\right)+\frac{2e_0(1-e_0)}{\lambda_0(1+r_j)}\exp\left(\frac{-x_i}{\lambda_0(1+r_j)}\right)+\frac{e_0{}^2}{\lambda_0(1+2r_j)}\exp\left(\frac{-x_i}{\lambda_0(1+2r_j)}\right),$$

where λ_0 is the average power of the receiver noise and r_j/λ_0 is the per pulse average interference-to-noise ratio (INR).

The probability of detection for CA CFAR processor for target of case Swerling II in pulse jamming [7] is

$$(3)\quad P_{d_{\mathrm{SW2}}} = \sum_{i=1}^{N} C_N^i e_0^i (1-e_0)^{N-i}\left\{\frac{e_0}{\left(1+\frac{(1+r_j)T_{\mathrm{CA}}}{1+r_j+s}\right)^i\left(1+\frac{T_{\mathrm{CA}}}{1+r_j+s}\right)^{N-i}}+\frac{1-e_0}{\left(1+\frac{(1+r_j)T_{\mathrm{CA}}}{1+s}\right)^i\left(1+\frac{T_{\mathrm{CA}}}{1+s}\right)^{N-i}}\right\},$$

where s is the signal-to-noise ratio, T_{CA} is the threshold constant and r_j is the average interference-to-noise ratio (INR).

The probability of false alarm for a CA CFAR detector for case Swerling II in pulse jamming [5] is obtained for value of the probability of detection P_{d}=0.

The probability of target detection of an EXC CFAR detector is evaluated by:

$$(4)\quad P_{d_{\mathrm{SW2}}} = \sum_{k=1}^{N} C_{\mathrm{N}}^k P_{\mathrm{E}}^k (1-P_{\mathrm{E}})^{N-k}\left\{(1-e_0)^2 M_{\mathrm{V}}\left(\frac{T_{\mathrm{EXC}}}{\lambda_0(1+s)},k\right)+ 2e_0(1-e_0)M_{\mathrm{V}}\left(\frac{T_{\mathrm{EXC}}}{\lambda_0(1+r_j+s)},k\right)+e_0{}^2 M_{\mathrm{V}}\left(\frac{T_{\mathrm{EXC}}}{\lambda_0(1+2r_j+s)},k\right)\right\},$$

where $M_{\mathrm{V}}(.)$ is the moment generating function, according to [11] and T_{EXC} is a predetermined scale factor that provides a constant false alarm rate (P_{fa}), $P_{d_{\mathrm{SW2}}}$ is the probability of pulse detection of an EXC CFAR detector in the presence of binominal distribution impulse interference. The probability of false alarm of an EXC CFAR processor is evaluated by (4), setting s=0.

All indications for signal detection obtained from N range resolution cells and N_s scans are arranged in a matrix Ω of size $N \times N_s$ in $r-t$ space. In this space stationary or constant radar velocity target pears as a straight line which consists of nonzero elements of Ω. Let us assume that Ω_{ij}^{nm} is a set of such nonzero elements of Ω that constitute a straight line in $r-t$ space that is $(i, j) \in \Omega_{ij}^{nm}$. This line may be represented in Hough parameter space as a point (n, m). Denoting N_{nm} as the maximal size of , the cumulative false alarm probability for a cell is written according to [3]:

$$(5)\quad P_{\mathrm{fa}}^{nm}=\sum_{l=K}^{N_{nm}}\binom{N_{nm}}{l}(P_{\mathrm{fa}})(1-P_{\mathrm{fa}})^{N_{nm}-l},$$

where K is a linear trajectory detection threshold.

The total false alarm probability in Hough parameter space is equal to one minus the probability that no false alarm occurred in any of the Hough cell. For independent Hough cells this probability is

(6)
$$P_{fa} = 1 - \prod_{N_{nm}=K}^{\max(N_{nm})} \left[1 - P_{fa}^{nm}\right]^{W(N_{nm})},$$

where $\max(N_{nm})$ is the accessible Hough space maximum and $W(N_{nm})$ is the number of cells from Hough parameter space whose values are equal to N_{nm}.

The cumulative probability of target detection in Hough parameter space P_D cannot be written in the form of a simple Bernoulli sum. As the target moves with respect to the radar, the SNR of the received signal changes depending on the distance to the target and the probability of a pulse $P_D(j)$ changes as well. Then the probability P_D can by calculated by Brunner's method. By means of Brunner's method [3] a matrix of size 20×20, the elements of which are the primitive probability of detection from the k-th time slice is formed. Using (3), (4) it is possible all the $P(i, j)$ needed to calculate P_D to be obtained. For N_s scans of radar the following is valid:

(7)
$$P_D = \sum_{i=M}^{N_S} P_{d_{SW2}}(i, N_S).$$

There are not many cases in practice when radar is equipped with a Hough detector working in pulse jamming. In such situations it would be desirable to know the Hough losses depending on the parameters of the pulse jamming, for rating the behavior of the radar. For the calculation of Hough detector losses is used the ratio between the two values of signal-to-noise-ratio (SNR), measured in dB. The comparisons are made and towards Hough detector with CA CFAR processor and Hough detector with EXC CFAR processor in pulse jamming.

3. Numerical results

In order to analyze the quality of the Hough detector we consider radar with parameters, similar to those in [1]: the search scan time is 6 s; the range resolution is δR = 3 n. mi (1 n. mi = 1852 m); the beam range-time space has 128 range cells and 20 time slices. In this analysis is considered straight line incoming target with a speed of Mach 3 and 1 m^2 radar cross section. In the analysis the SNR average value is calculated as $S=K/R^4$, where $K=0.16\times10^{10}$ is the generalized energy parameter of the radar and R is the distance to the target measured in nautical miles.

Carlson's approach, using Brunner's method for calculating the probability of detection in Hough parameter space is further developed in order to maintain constant false alarm probability at the output of the Hough detector. The suitable scale factor is chosen iteratively. The influence of the threshold constant on the required signal-to-noise ratio is studied. The investigation is performed for probability of detection (P_D=0.5) and different values of the probability for the appearance of pulse jamming with average length in the cells in range.

In order to achieve a constant value of the probability of false alarm (P_{fa}), the values of the threshold constants, which guarantee that, are determined for different numbers of observations in the reference window, an average interference-to-noise ratio (INR) and a probability for the appearance of pulse jamming with average length in the cells in range. The profits (losses) of the CA Hough detector in pulse jamming are determined towards the CA CFAR detector, following the algorithm proposed in [13], for probability of detection 0,5.

In table 1 are presented results for average detection threshold for CA Hough CFAR detector with the probability of false alarm ($P_{fa} = 10^{-4}$), for number of observations in the reference window ($N = 16$), average interference-to-noise ratio (INR=30 dB) and two deferent values for a probability for the appearance of pulse jamming with average length in the cells in range.

Table 1

T_M	T_{CA} for e_0=0	T_{CA} for e_0=0.1	ADT for e_0=0	ADT for e_0=0.1
2	1.14	672	7.3363	45.7179
3	0.57	225	5.1267	47.1774
4	0.401	93.5	4.3203	47.0781
5	0.315	28.9	3.9747	43.4509
6	0.2609	4.109	3.7442	34.8363
7	0.2225	1.186	***3.6290***	30.1511
8	0.193	0.472	***3.6290***	27.1285
9	0.1696	0.2195	3.7442	24.7103
10	0.150	0.1303	3.8594	23.5013
11	0.1334	0.0842	3.9747	22.7456
12	0.1188	0.0544	4.2051	22.1411
13	0.1059	0.0329	4.4355	20.6297
14	0.0942	0.0174	4.7811	18.6650
15	0.0836	0.00815	5.2419	15.9446
16	0.0737	0.00409	5.7028	13.6776
17	0.0645	0.00241	6.3940	12.3174
18	0.0557	0.001565	7.2005	***11.7128***
19	0.0470	0.001065	8.2373	11.8640
20	0.0384	0.000728	9.9654	12.7708

The authors in [3] use approach proposed by Barton to determine the threshold in Hough parameter space. They assume T_M=7 as optimal threshold in the binary integration and apply it in Hough parameter space. In this paper, after iterative analysis, the optimal threshold in Hough parameter space is also determined to be T_M=7 for the value of the probability of appearance of pulse jamming with average length in the range cells e_0=0.

Different values of the detection threshold in Hough parameter space – T_M are shown on Fig. 1. The optimal value for this threshold is T_M=7 of 20 scans (T_M=7/20) for values of the probability for the appearance of pulse jamming with average length in the range cells ε_0=0. For ε_0= 0.1, the optimal value for detection threshold in Hough parameter space is T_M= 18 of 20 scans (T_M=18/20).

The probabilities of detection of Hough detector with a CA CFAR processor are shown on Fig. 2 and Fig. 3, for value of the detection threshold T_M=2 and for optimal values of the detection threshold T_M= 7, for e_0=0 and for T_M= 18, for e_0= 0.1.

The profits of using a Hough detector with CA CFAR processor, calculated for the threshold value T_M=2 and for optimal values of the detection threshold T_M=7, for

e_0=0 and T_M=18, for e_0=0.1, compared to a CA CFAR processor, for the number of test resolution cells N=16 and the value for probability of false alarm $P_{fa} = 10^{-4}$, are shown on Fig. 4. The CA Hough detector with the optimal Hough rule T_M-out-of-N equal to 7 of 20 is better in cases of lower values of the probability for appearance of impulse interference, up to 0.06. For higher values of the probability for appearance of impulse interference, above 0.06, the usage of the optimal Hough rule T_M-out-of-N_s=18 of 20 scans, results in lower losses.

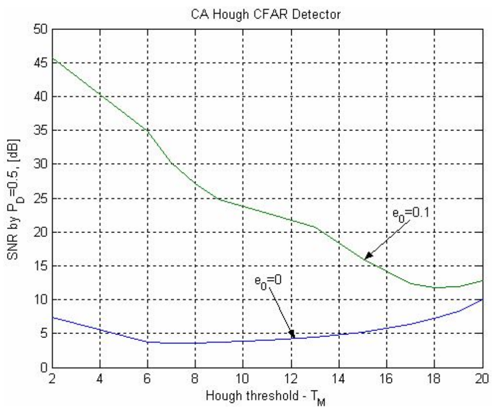

Fig. 1. Average detection threshold of a CA Hough CFAR detector

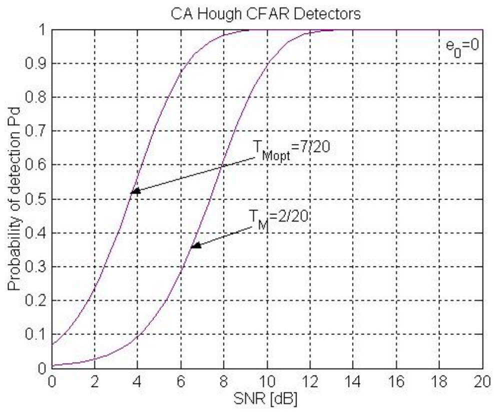

Fig. 2. Probability of detection of a Hough detector with CA CFAR processor in pulse jamming, for 20 scans, T_M=2, T_M=7 and e_0=0

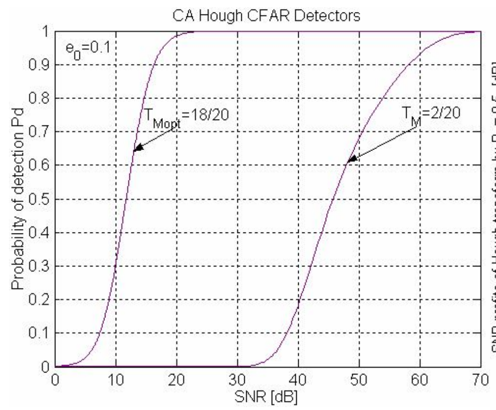

Fig. 3. Probability of detection of a Hough detector with CA CFAR processor in pulse jamming, for 20 scans, T_M=2, T_M=18 and e_0=0.1

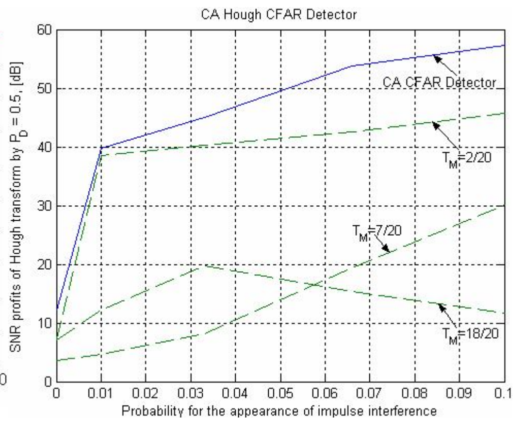

Fig. 4. Profits of a Hough detector (dashed line) with CA CFAR processor for 20 scans, T_M=2 and two optimal values of the detection threshold, T_M=7 for e_0=0 and T_M=18 for e_0=0.1, compared to a CA CFAR detector (solid line) for N=16

A detailed performance analysis of Hough detector with an EXC CFAR processor is presented in this paper. Its behavior has been studied for different values of the threshold constant and for different values of the probability for the appearance of impulse interference in Hough parameter space. The experimental results are obtained for the following input parameters: average power of the receiver noise λ_0=1, average interference-to-noise ratio (INR=30 dB), probability for appearance of impulse interference with average length in the range cells from 0.1 to 0.9, number of reference cells N=16 (or 32), number of test cells L=16, probability of false alarm $P_{fa}=10^{-4}$, excision threshold B_E=2, number of scans N_s=20, optimal values of Hough detection threshold T_M=7, T_M=13 and binary rules M-out-of-L=10/16, M-out-of-L=16/16.

In Table 2 are presented results for average detection threshold for EXC Hough CFAR detector with probability of false alarm ($P_{fa}=10^{-4}$), excision constant – B_E=2**,** for number of observations in the reference window ($N = 16$), an average interference-to-noise ratio (INR=30 dB) and two deferent values for probability of appearance of pulse jamming with average length in the cells in range.

The average detection threshold for EXC Hough CFAR detector in conditions of pulse jamming and for different values of the detection threshold in Hough parameter space – T_M is shown on Fig. 5.

The average detection threshold for Hough detector with an EXC CFAR processor for two different values of the number of reference window (N=16 and N=32) and for the value T_M=2 of the Hough detection threshold are shown on Fig. 6. The increasing of the reference window number leads to loss diminishing.

The probability of detection of the EXC Hough CFAR detector is shown on Fig. 7, for optimal values of the detection threshold T_M=13, by value for probability of appearance e_0=0.1. On Fig. 8 is shown the probability of detection of the EXC Hough CFAR detector, for two values of the detection threshold – T_M=2 and T_M=7, for value of probability of appearance e_0=0.9.

The ADT of Hough detector with an EXC CFAR processor for the optimal values of the detection threshold T_M=7 for e_0=0.9, T_M=13 for e_0= 0.1 and for the threshold value T_M= 2 are shown on Fig. 9.

For comparison on Fig. 10 the results obtained for the Hough detector with the two types of one-dimensional CFAR processors are shown – CA CFAR and EXC CFAR.

Table 2

T_M	T_{EXC} for e_0=0.1	T_{EXC} for e_0=0.9	ADT for e_0=0.1	ADT for e_0=0.9
2	21 880	53 880 880	40.4282	77.5882
3	8 200	375 500	38.2557	64.8929
4	4 790	62 600	37.3489	55.1448
5	3 120	24 300	36.6688	52.3111
6	2 065	13 500	35.7620	50.2708
7	1 317	9 055	34.0617	***49.3640***
8	747	6 655	31.9081	49.4773
9	287	5 175	27.2607	50.1574
10	7.55	4 147	8.1203	51.2909
11	4.885	3 380	6.3249	52.7645
12	3.868	2 785	5.8463	54.8048
13	3.225	2 305	***5.6952***	57.5252
14	2.743	1 905	5.7456	64.0995
15	2.353	1 565	5.9723	74.2481
16	2.025	1 270	6.2997	76.4547
17	1.735	1 009	6.7783	82.7465
18	1.472	774	7.5088	84.1152
19	1.227	555	8.5668	85.2997
20	0.9898	347	10.2292	85.4839

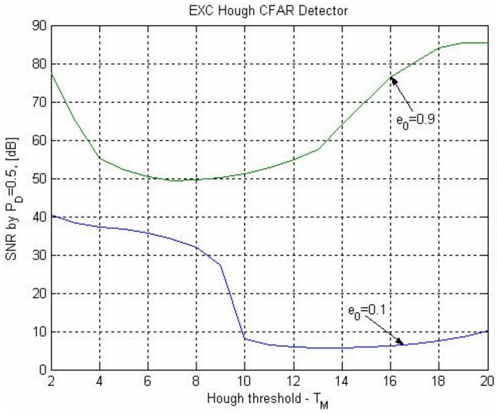

Fig. 5. Average detection threshold of an EXC Hough CFAR detector for two values of the probability of appearance

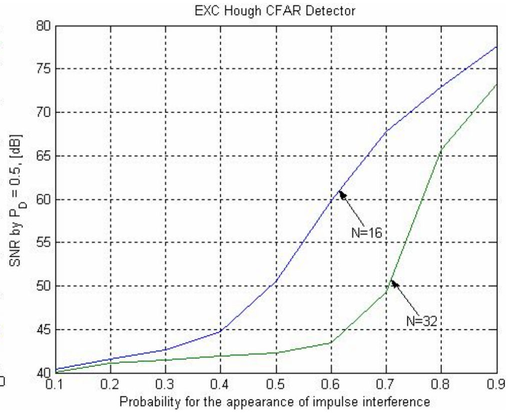

Fig. 6. Average detection threshold of an EXC Hough CFAR detector for two values of the number of test resolution cells

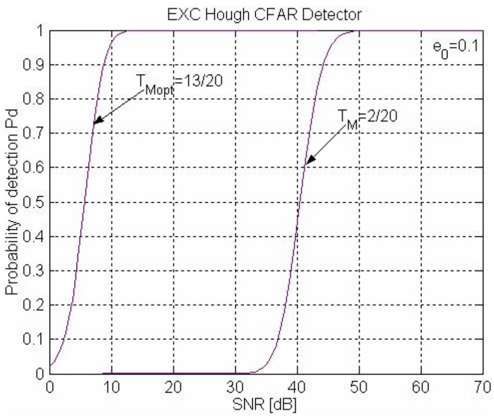

Fig. 7. Probability of detection of a Hough detector with EXC CFAR processor in pulse jamming, for 20 scans, T_M=2, T_M=13 and e_0=0.1

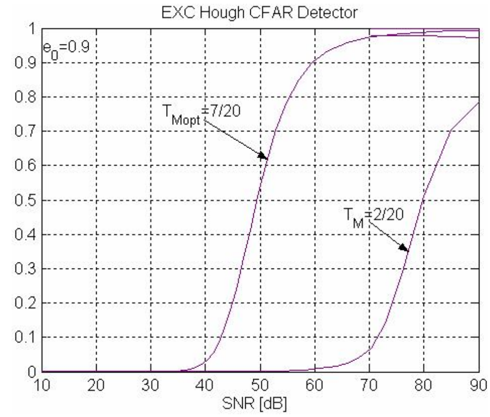

Fig. 8. Probability of detection of a Hough detector with EXC CFAR processor in pulse jamming, for 20 scans, T_M=2, T_M=7 and e_0=0.9

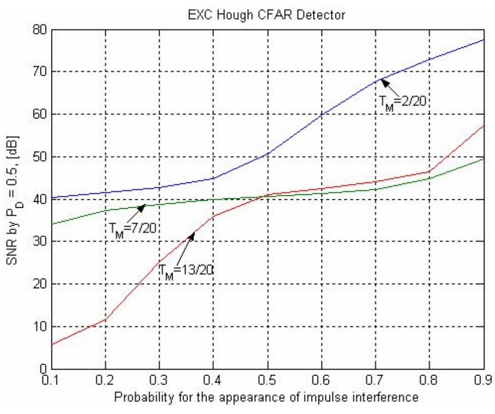

Fig. 9. Average detection threshold of an EXC Hough CFAR detector for two optimal values of detection thresholds T_M=7 for e_0 = 0.9, T_M=13 for e_0 = 0.1 and for threshold value T_M=2

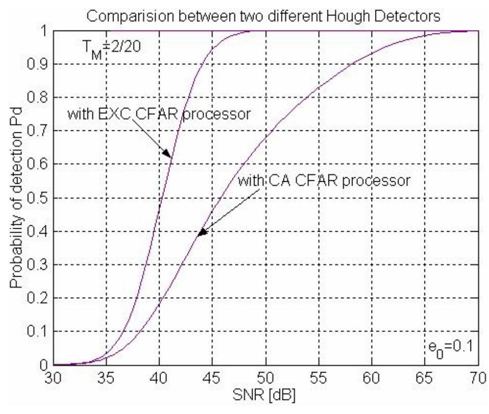

Fig. 10. Probability of detection for Hough detector with two one-dimensional CFAR processors

4. Conclusions

The experimental results reveal the influence of the interference on the detection process, when having constant false alarm rate in pulse jamming. A method for losses estimation, which allows choosing optimal detector parameters, is developed. The estimates of the effectiveness of a Hough detector (CA Hough CFAR or EXC Hough CFAR) in pulse jamming are received for different stream characteristics.

Using Matlab, the average detection threshold for the two types of Hough detectors for a highly fluctuating target, Swerling II type target model detection in conditions of pulse jamming, is calculated in accordance with the approach presented in [13]. Using this approach, it is very easy to precisely determine the energy benefit when using a given detector. The results achieved show that Hough detector with one-dimensional processor is effective in conditions of decreasing pulse jamming.

The performance of a Hough detector designed for non-homogeneous interference is studied. The optimal threshold values for different input conditions are estimated. The value of the test resolution cell and the probability of false alarm over the average detection threshold are studied. The results show losses in the signal to noise ratio of about 37 dB for the Hough detector with EXC CFAR processor and 42 dB for the Hough detector with CA CFAR processor with respect to the Hough detector with a fixed threshold [12]. Application of censoring techniques in the detection algorithm improves the Hough detectors effectiveness. The results obtained in this paper could practically be used in radar and communication networks.

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