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Analysis and Synthesis of IMC-Control Systems*

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Abstract: *This article is the second part of already published in the previous issue article under the same name. Here as a continuation of the first part, new and original method for equivalent transformation of the studied robust and predictor IMC-systems is presented. Based on this method, new class of IMC-control systems is developed. Their high effectiveness, compared to the effectiveness of the known up till now systems, in control of plants under uncertainty is due to combination of prediction and robust control considering both the irrational and the rational part in the plant's model. The method is applied to a numeric example.*

Keywords: *Internal Model Control, Smith-predictor and robust control systems; frequency method for equivalent synthesis; robust Nyquist-analysis.*

1. Introduction

This article is the second part of the presentation of this study. It is based on the proved in the first part *hypothesis of structural equivalence* of the studied systems with prediction control [5-7], robust systems with internal model [1-3] and robust systems with conditional feedback [2, 3], united in the class of IMC (Internal Model Control)-systems.

This study aims at determining principally new possibilities for development of the synthesis of the IMC-systems.

In order to achieve the goal the following tasks will be performed:

• development of generalized model for equivalent transformations connecting known design algorithm for IMC-systems;

• development of new type of IMC-systems, that could effectively control industrial plants under uncertainty in the rational as well as in the irrational part of the plant's model;

* Continued from [4], CIT No 2, 2006. Figs. 7, 11, 21, 22 are presented in [4], the formulae continue the enumeration from [4].

• synthesis, modeling and simulation of robust control system of the presented new type IMC-systems. The achieved results in this direction are subject to the second part of this study.

2. Offered solution

The three structures shown on Fig. 7, Fig.11, Fig. 21 (from [4]), are examined, where: y, y_0 are the controlled magnitude and its given value; ϑ , ς – the disturbances in the system; G^*, $\hat{G}^*$, $G^{\blacksquare}$ – the nominal model and its rational part and the “disturbed at upper limit” plant model G ; R_C^S, Q , R_C^F – the basic controllers in the systems in the generalized algorithms R_S , R_M, R_F .

Fig. 7

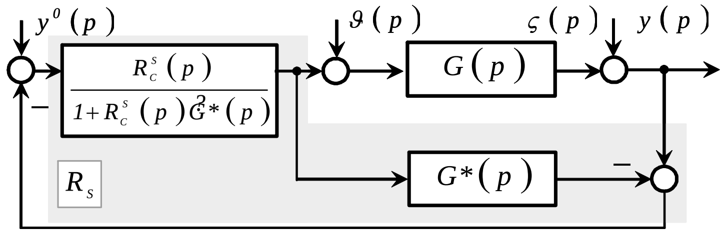


Fig. 11

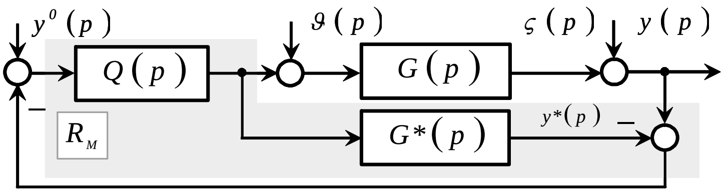
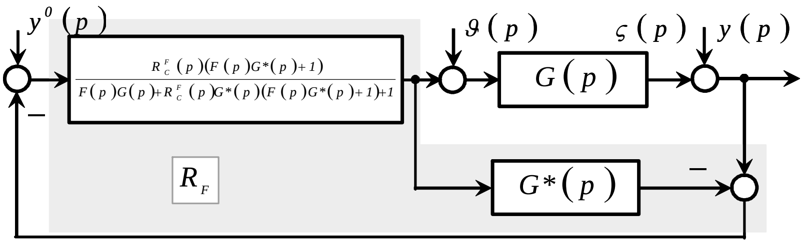


Fig. 21



Their identity determines the basic idea of the already proved hypothesis for structural equivalence. It is having in mind that the basic controller $R_C^S(p)$ is designed considering $\hat{G}^*(p)$, and correspondingly $R_C^F(p)$ is designed considering $G^*(p)$. And that the nominal model of the controlled plant $G^*(p)$ and its rational part $\hat{G}^*(p)$ are previously known (or given) in the designed process.

Based on the already proved *hypothesis for structural equivalence* of the studied IMC-systems, in this part *a method for equivalent structural transformation for design of IMC-systems* is presented.

Its basic analytical definitions are derived. They are based on the equivalence (18)-(20) – of the basic parts of the generalized algorithms from Fig. 7, Fig. 11 and Fig. 21, which are structurally “separated” from the used nominal model of the plant G^*;

$$Q(p) \equiv \frac{R_C^S(p)}{1+\hat{G}^*(p)R_C^S(p)}, \tag{18}$$

$$Q(p) = \frac{R_C^F(p)\left(F(p)G^*(p)+1\right)}{F(p)G(p)+R_C^F(p)G^*(p)\left(F(p)G^*(p)+1\right)+1}, \tag{19}$$

$$\begin{aligned} &\frac{R_C^S(p)}{1+\hat{G}^*(p)R_C^S(p)} \equiv \\ &\equiv \frac{R_C^F(p)\left(F(p)G^*(p)+1\right)}{F(p)G(p)+R_C^F(p)G^*(p)\left(F(p)G^*(p)+1\right)+1}. \end{aligned} \tag{20}$$

The purpose of the presented method (18)-(20) is the equivalent analytical transition between each, randomly chosen pair of the presented three typical IMC-structures.

Due to the presented *method for equivalent structural transformation*, this paper offers a well-grounded solution of the problem for:

- analytical connection of the known design algorithms for the presented IMC-systems, which connection makes the analytical implementation of the method created for design of one type of the studied systems applicable for the design of each of the others two types of IMC-systems;
- development of new type IMC-systems, that generalize the properties to control industrial plant under uncertainty with higher efficiency, in the rational as well as in the irrational part of the model of the plant due to the fact that they are prediction systems on structure, for example, but are designed considering complex criteria, methods and algorithms for design of prediction systems;
- confirmation of the applicability of the presented *method of the equivalent structural transformation* through synthesis, modeling and simulation of a robust system with internal model for control of industrial plants, designed through the method of the balance equation for stability.

3. Results and analysis

In this section results from the implementation of the *method of equivalent structural transformation* for solving the formulated problems are presented. They are given in two directions:

3.1. For the design of each of the IMC-systems (in equivalent initial conditions for G^*, $\hat{G}^*$, $G^{\blacksquare}$ and technological requirements – local performance criteria) the methods developed for design of each of the others IMC-systems could be used, based on the: analytical dependences (18)-(20) of the method for equivalent structural transformation and the following examples in solving the design problem:

- the controller $Q(p)$ in the robust system (Fig. 11) could be designed analytically using the dependence (21), if the already known controller $R_C^{S\lozenge}(p)$ is used, determined through the corresponding method during the design of the hypothetic prediction control system for the same plant (Fig. 2)

$$(21)\qquad Q(p) \equiv \frac{R_C^{S\lozenge}(p)}{1+\hat{G}^*(p)R_C^{S\lozenge}(p)};$$

• the generalized algorithm $R_M(p)$ in the robust system (Fig. 11) could be designed analytically through the dependence (22), if the already known controller $R_C^{S\lozenge}(p)$ is used, determined by applying the corresponding method in the design of the hypothetic prediction system for the control of the same plant (Fig. 2)

$$(22)\qquad R_M(p) \equiv \frac{R_C^{S\lozenge}(p)}{\left(1+\hat{G}^*(p)R_C^{S\lozenge}(p)-G^*(p)R_C^{S\lozenge}(p)\right)};$$

• the controller $Q(p)$ in the robust system (Fig. 11) could be designed analytically through the dependence (23), if the already known controller $R_C^{F\lozenge}(p)$ (13) and filter $F^{\lozenge}(p)$ are used, known from the design (for example using the method of the balance equation of stability) of the hypothetic robust system (Fig. 15) for control of the same plant

$$(23)\qquad Q(p) \equiv \frac{R_C^{F\lozenge}(p)\left(F^{\lozenge}(p)G^*(p)+1\right)}{F^{\lozenge}(p)G(p)+R_C^{F\lozenge}(p)G^*(p)\left(F^{\lozenge}(p)G^*(p)+1\right)+1};$$

• the generalized algorithm $R_M(p)$ in the robust system (Fig. 11) could be designed analytically through the dependence (24), if the already known controller $R_C^{F\lozenge}(p)$ (13) and filter $F^{\lozenge}(p)$ are used, known from the design (for example using the method of the balance equation of stability) of the hypothetic robust system (Fig. 15) for control of the same plant

$$(24)\qquad R_M(p) \equiv \frac{R_C^{F\lozenge}(p)\left(F^{\lozenge}(p)G^*(p)+1\right)}{F^{\lozenge}(p)G(p)+1}.$$

• the controller $R_C^{S0}(p)$ in the prediction system (Fig. 2) could be designed analytically through the dependence (25), if the already known controller $R_C^{F\lozenge}(p)$ (13) and filter $F^{\lozenge}(p)$ are used, known from the design (for example using the method of the balance equation of stability) of the hypothetic robust system (Fig. 15) for control of the same plant

$$(25)\qquad R_C^{S}(p) \equiv$$

$$\equiv \frac{R_C^{F\lozenge}(p)\left(F^{\lozenge}(p)G^*(p)+1\right)}{F^{\lozenge}(p)G^{\blacksquare}(p)+1+R_C^{F\lozenge}(p)\left(F^{\lozenge}(p)G^*(p)+1\right)\left(G^*(p)-\hat{G}^*(p)\right)}$$

• the controller $R_C^{S}(p)$ in the prediction system (Fig. 2) could be designed analytically through the dependence (26), if the already known controller $Q^{\lozenge}(p)$ is used, known from the design (for example using the method of the free parameter) of the hypothetic robust system (Fig. 11) for control of the same plant

$$(26)\qquad R_C^{S}(p) \equiv \frac{Q^{\lozenge}(p)}{\left(1-Q^{\lozenge}(p)\hat{G}^*(p)\right)};$$

• the controller $R_C^S(p)$ in the prediction system (Fig. 2) could be designed analytically through the dependence (27), if already known generalized algorithm $R_M^\lozenge(p)$ is used, known from the design (for example using the method of the free parameter or (H_2/H_∞)-optimal controller) of the hypothetic robust system (Fig. 11) for control of the same plant

(27) $$R_C^S(p) \equiv \frac{R_M^\lozenge(p)}{\left(1 + R_M^\lozenge(p) G^*(p) - R_M^\lozenge(p) \overset{?}{G}{}^*(p)\right)};$$

• the filter $F(p)$ in the robust system (Fig. 15) could be designed analytically through the dependence (28), if already known generalized algorithm $R_M^\lozenge(p)$ is used, known from the design (for example using the method of the free parameter or (H_2/H_∞)-optimal controller) of the hypothetic robust system (Fig. 11) for control of the same plant

(28) $$F(p) \equiv \frac{R_C^{F\lozenge}(p) - R_M^\lozenge(p)}{R_M^\lozenge(p) G^{\blacksquare}(p) - R_C^{F\lozenge}(p) G^*(p)};$$

• the filter $F(p)$ in the robust system (Fig. 15) with already designed $R_C^{F\lozenge}(p)$ (13) in the structure of the robust system (Fig. 15) could be designed analytically through the dependence (29), if already known controller $Q^\lozenge(p)$ is used, known from the design (for example using the method of the free parameter or (H_2/H_∞)-optimal controller) of the hypothetic robust system (Fig. 11) for control of the same plant

(29) $$F(p) \equiv \frac{1 - Q^\lozenge(p)\left(1 + R_C^{F\lozenge}(p) G^*(p)\right)}{Q^\lozenge(p) G^{\blacksquare}(p) + R_C^{F\lozenge}(p) G^*(p)\left(Q^\lozenge(p) G^*(p) - 1\right)};$$

• the filter $F(p)$ in the robust system (Fig. 15) with already designed $R_C^{F\lozenge}(p)$ (13) in the system (Fig. 15) could be designed analytically through the dependence (30), if already known controller $R_C^{S\lozenge}(p)$ is used, known from the design (using the corresponding algorithm) of the hypothetic prediction system (Fig. 2) for control of the same plant

(30) $$F(p) \equiv \frac{R_C^{F\lozenge}(p)\left(1 + \overset{?}{G}{}^*(p) R_C^{S\lozenge}(p)\right) - R_C^{S\lozenge}(p)\left(1 + R_C^{F\lozenge}(p) G^*(p)\right)}{R_C^{S\lozenge}(p) G^{\blacksquare}(p) + R_C^{F\lozenge}(p) G^*(p)\left(R_C^{S\lozenge}(p)\left(\overset{?}{G}{}^*(p) - G^*(p)\right) - 1\right)}.$$

3.2. For each of the studied IMC-systems, typical properties for the other IMC-systems could be given, when in the design of the corresponding generalized algorithm, synthesized generalized algorithm for other system is used, and afterwards is analytically equivalently transformed. Specific examples are the solutions of the problems given with the dependences (21), (22), (25), (26), (27), (30).

3.3. So that the applicability of the presented *method for equivalent structural transformation* could be proved, a robust system with internal model (Fig. 21) for control of a plant is modeled and simulated. The system is designed using the method of the balance equation of stability (used for the design of a system with conditional feedback – Fig. 15), through the dependence (24). The simulation results of the so designed system are visualized as follows:

• On Fig. 22 is shown the model of the designed robust system in MATLAB SIMULINK;

• On Fig. 23 are presented the step response (a) and the frequency (b), (c), (d) characteristics of the designed system;

• On Fig. 24 the results from the robust analysis are visualized. They prove the robust stability and the robust performance of the designed system using the: sensitivity function and the complementary sensitivity function of the close-loop system (Fig.24.a) and the Nyquist-robust analysis (Fig.24.b) – for the characteristics of the open-loop system.

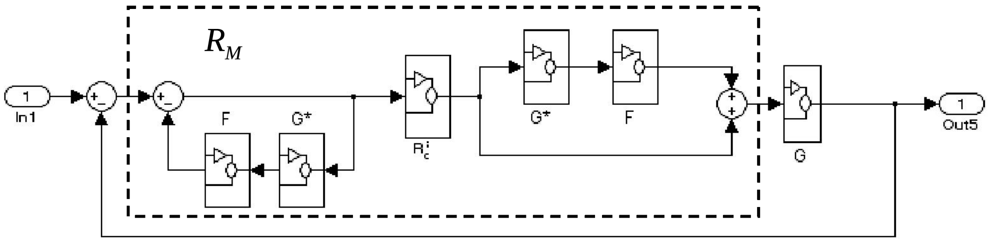
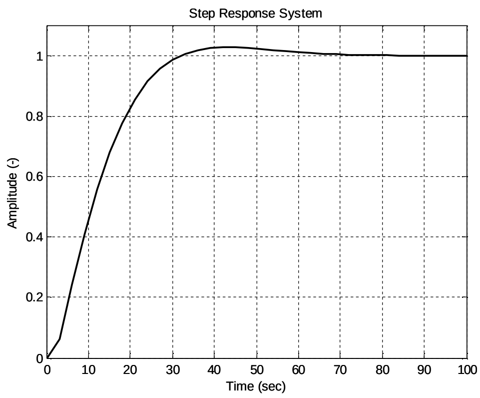
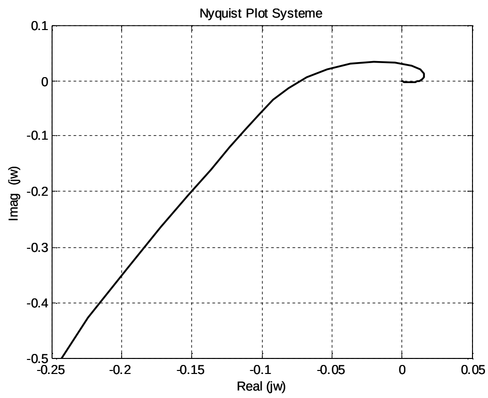


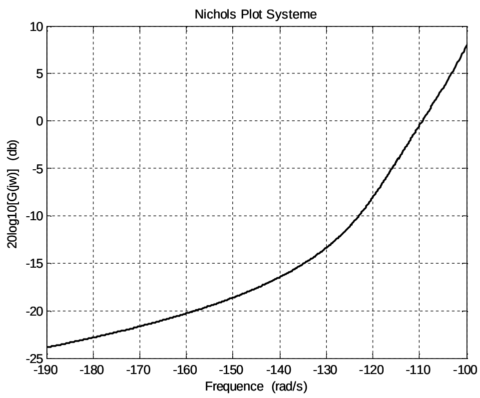
Fig. 22



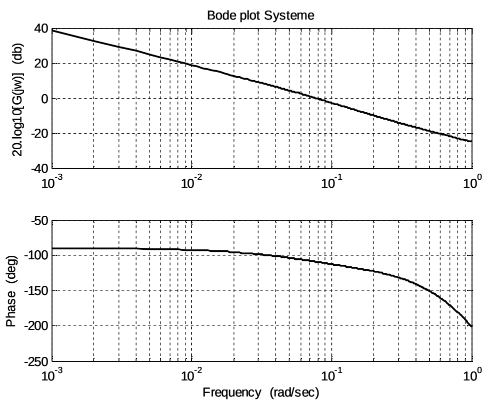
(a)



(b)

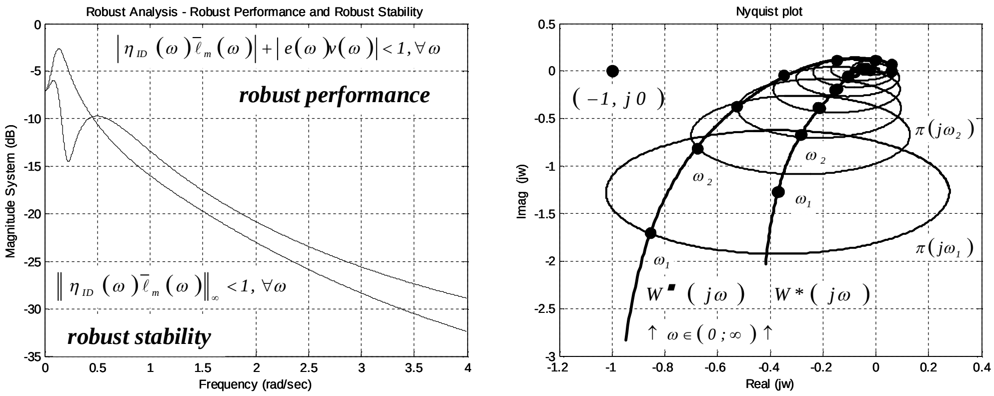


(c)



(d)

Fig. 23



(a) (b)

Fig. 24

4. Conclusion

Using the presented results from the implementation of the method of equivalent structural transformation on the studied IMC-systems, this paper:

• ***proves the common base*** of the basic structures of the studied three types of control systems (Fig. 2, Fig. 11, Fig. 15) – the classic control system of a plant and controller (Fig. 10; Fig. 14; Fig. 26);

• ***proves the structural equivalence*** between the studied systems (Fig. 2, Fig. 11, Fig. 15) in the class of IMC – it consists in that, that functionally all the three systems in their organization are based on one and the same type of internal model – the nominal model and the controlled plant (Fig. 7, Fig. 11 and Fig. 21);

• ***determines the analytical dependences*** between the nominal model of the plant and the generalized control algorithms of the prediction and robust IMC-systems;

• ***proves the hypothesis for structural equivalence of the prediction and robust IMC-systems***, using the common classic base and the structural equivalence of the studied IMC-systems;

• ***offers a method for equivalent structural transformation*** as a solution of the design problem for each of the IMC-systems (when equal initial conditions and technological requirements are given) through implementation of methods, developed for design of each of the others IMC-control systems for the same plant, and for the solution of the problem of giving to one of the IMC-systems typical properties of the others IMC-systems, when during the design of the corresponding generalized algorithm for the system, a design algorithm for other system should be used and afterwards an analytical equivalent transformation should be performed;

• ***approves the applicability of the method for equivalent structural transformation,*** through the design, modeling and simulation of a robust control system with internal model, designed using the method of the balance equation of stability.

The claims of this study are for the achieved original and new proofs for the common: base, structural equivalence and analytical dependences and the hypothesis for structural equivalence of the IMC-systems, as well as in the presented original method for *equivalent structural transformation*. The achieved results in this direction are applicable for the development of the design of the IMC-systems and for development of new type of IMC-systems, effective in the control of industrial plants under uncertainty.

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