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Time and Frequency Synchronization in OFDM Communication Systems

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Abstract: The paper investigates the influence of the frequency and time offset on the in-symbol and inter-symbol interference in OFDM communication systems.

A method is proposed for blind estimation of the offsets, which uses the cycle stationarity introduced in the information signal. A possibility is considered to accomplish the estimation, using the virtual subcarriers.

Keywords: synchronization, cycle stationarity, blind evaluation.

Introduction

The technology of OFDM communication provides significant positive qualities – high spectral efficiency, adaptability towards the changes of the channel, stability of the multibeam distribution, absence of inter-symbol interference at insignificant decrease of the transmission speed, which makes it very promising in wireless communications, communications, using channels with expressed frequency and time selectivity, and great depression. It is based on the principle of strict orthogonality between the separate subchannels.

The frequency efficiency is a result of the dense location of the frequency carriers, with overlapping spectra, reserving their orthogonality within the interval of each symbol. The efficiency, which can be really reached, is Mb per 1 Hz at *M*-th modulation of each carrier. OFDM enables maximum loading, practically up to the capacity of every channel, equalized or not equalized with respect to the large time dispersion, with multi-beam distribution. The first idea for this technology, expressed in [14], concerns the dispersion channel converting into a set of parallel dispersionless channels.

Fortunately, the strict requirements for orthogonality at generation and reception of the communication signals, are perfectly met by Direct and Inverse Fourier transforms [15, 16]. The unitary direct and inverse transforms, developed in the last years, particularly those with quick variants, allow the design of orthogonal information systems and of other bases. A similar system is discussed in [8].

OFDM is very sensitive towards asynchronous operation of the transmitter and receiver. The appearance of a frequency difference between the transmitter and the receiver, leads to decrease of the signal received for every subchannel and the occurrence of inter-subchannel interference, and at greater difference – change in the order of the elements in the symbol received. The time difference causes the appearance of inter-symbol interference.

The most significant element in OFDM communication systems is the synchronization. It is an object of many investigations, regarding its different aspects, and the different methods it uses with respect to the channel properties, the character of the communication system. Usually this is realized in two stages – initially a rough one, which enables the synchronizing at great time and frequency difference between the transmitter and the receiver and a current one, which is by nature with high accuracy and supports the initially established synchronization. The methods of each synchronization can be close, but they can also differ a lot, for example with the appearance of Doppler offset, and in communication with mobile objects.

In order to realize the synchronization, two methods are mainly used – assisted by the data transmitted and not assisted. In the first one, special signals are transmitted, the parameters of which are known of the two final points, specially chosen to achieve rapid synchronization. These can be continuously transmitted signals or periodically transmitted symbols, specified by their shape, auto-correlation function, spectrum and other specific parameters [12, 17]. The synchronism is established quickly and it is appropriate for common access systems, systems with packet transmission, local networks.

In the second approach, named blind synchronization, the natural non-stationarity of the information flow is used. It is entered by the transfer function of the channel, by the weight (window) function, which shapes the spectrum at the limit areas of the information signal, by the structure of the symbols transmitted and the basically entered protection interval (cyclic prefix) [3, 4, 7, 9, 10]. The channel and the window cause cyclic stationarity in the signals received, that is known to the receiver. The synchronization method in it is statistic and it requires sufficiently large sample in order to get a considerable estimation. The cyclic prefix introduces correlation between the separate sections of every symbol. Its use enables comparatively quick and efficient establishment of synchronism.

There exists a third approach, intermediate between the above mentioned. There is also separate apriori information in it, coordinated between the transmitter and the receiver, but it imposes too weak restrictions on the data transmitted. This can be multifold transmission of one and the same symbol without restriction on its nature [1], knowledge about the virtual (not modulated) subcarriers [18] and the use of differential OFDM with special choice of the first block of the data transmitted [2].

Principle of the OFDM modulation

The main procedures in the communication with OFDM modulation can be generally represented as follows.

The flow of primary binary symbols is organized in a flow of groups with a constant number of elements. Each group is transformed in the space of the received constellation into a symbol with a constant number of elements P (mapping). The k-th successive symbol is represented by its vector:

$$S(k) = \left| a_{k_1} \ \ a_{k_2} \ \ \ldots \ \ a_{k_P} \right| .$$

The elements a_k belong to a finite set of the constellation $A = \{a_1, ..., a_M\}$. We shall assume that they are equal in power σ_a^2, with a zero average and evenly distributed in symbols.

In order to receive a time-dependent signal of the symbol to be transmitted, N-dimensional Inverse Discrete Fourier Transform (IDFT) is accomplished:

$$X_0(k) = W_{N,1,P}^H S(k) .$$

Here $W_{N,1,P}^H$ is an $N{\times}P$ matrix of IDFT. It is composed of columns from I up to P of the complete $N{\times}N$ matrix. It is rectangular, due to the presence of virtual, not modulated frequencies, in which the zero element is also included. The vector $B(k)$, consisting of its last L elements in the form of a cyclic prefix (CP), is added to $X_0(k)$, in order to extinguish the inter-symbol interference:

(1) $$X(k) = \begin{vmatrix} B(k) \\ X_0(k) \end{vmatrix} .$$

As a result of the transforms executed, the signal for each packet accepts the form of a sum of P discrete harmonic signals, subcarriers with frequencies $p\Delta F = \dfrac{p}{NT_d}$ and amplitude a_p:

(2) $$x_{0_p}(n) = \sum_{p=1}^{P} a_p e^{j\frac{2\pi}{N}pn} .$$

Every symbol (2) is multiplied by the weight function (window) $g(n)$, which assures appropriate amplitude of the complex envelope, so that the required frequency characteristic is obtained within the limit regions of the spectrum

(3) $$y_0(n) - \sum_{p=1}^{P} \sum_{k=-\infty}^{\infty} a_{kp} e^{j\frac{2\pi}{N}p(n-kQ)} g(n-kQ) .$$

Here $Q = N+L$ is the duration of the symbol transmitted. All subcarriers of each packet have zero initial phases.

After passing through an interpolating filter, the spectrum of (3) is translated around the average frequency F_0 and transmitted along the channel provided.

We regard the information channel as a filter $h(n)$ with a finite pulse reaction L and a pure delay of $n_\varepsilon + \alpha$. The delay is measured by the number of discretes n_ε being its integer part, and $|\alpha| \le 0.5$ – its fraction part,

(4) $$h(n) = \sum_{i=0}^{L-1} h(i)\delta(n-i)\delta(n-n_\varepsilon-\alpha) = \sum_{i=0}^{L-1} h(i)\delta(n-n_\varepsilon-\alpha-i) .$$

The channel transfer function is represented in the frequency domain by the expression

(5) $$H(k) = e^{-j2\pi k(n_\varepsilon + \alpha)} H_0(k),$$

(5a) $$H_0(k) = \sum_{i=0}^{L-1} h(i) e^{-j2\pi ki}.$$

The spectrum of the channel received is offset at $F'_0 = F_0 + \Delta f_\varepsilon$ in its basic band and the cyclic prefix is removed Δf_ε – the difference between the high frequency carriers in the transmitter and the receiver. It is transferred with one and the same value on all the subcarriers in the receiver.

The signal in the basic band becomes (without the channel noise)

(6) $$y(n) = \sum S(i) H(i) G(i) e^{j\frac{2\pi}{N}(i+\varepsilon)n}$$

and the vector of the entire symbol is

$$\mathbf{y} = |y(0) \ \ldots \ y(N-1)|^{\mathrm{T}}.$$

Here $\varepsilon = N\Delta f$ is a frequency offset, reduced to the band of each subchannel, $\Delta f = \frac{1}{N}$, $G(i)$ is the Fourier image of the window., regarded as a linear filter;

(7) $$\mathbf{y} = W_{N,1,P}^{H} \Lambda(\varepsilon) H' G' S_0.$$

Here H, G and $\Lambda(\varepsilon)$ denote the diagonal matrices of the transfer functions of the channel and the window respectively, and of the introduced phase offset

$$\Lambda(\varepsilon) = \mathrm{diag}\left| e^{j\frac{2\pi}{N}\varepsilon} \ \ldots \ e^{j\frac{2\pi}{N}P} \right|.$$

The recovery of the initial vector S_0 becomes after DFT of $\boldsymbol{y}$.

(8) $$X = |X(1) \ \ldots \ X(P)|^{\mathrm{T}} = W_{N,1,P} W_{N,1,P}^{H} \Lambda(\varepsilon) H' G' S_0.$$

In order to simplify the exposed material, we assume that $G(i)$ is included in $H_0(i)$, and the noise components in the received signal, as non-correlated with the useful signal, are omitted, when this does not cause any wrong conclusions.

For i-th element of the signal received, the following expression is obtained;

(9) $$X(i) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=1}^{P} S(k) H(k) e^{j\frac{2\pi}{N}(k+\varepsilon)n} e^{-j\frac{2\pi}{N}in} = \frac{1}{N} \sum_{k=1}^{+I} S(k) H(k) \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k-i+\varepsilon)n}.$$

Applying the relations

$$\sum_{i=0}^{N-1} a^i = \frac{1-a^N}{1-a}, \quad \frac{1-e^{j2\alpha}}{1-e^{j\frac{2\alpha}{N}}} = e^{j\alpha\frac{N-1}{N}} \frac{\sin(\alpha)}{\sin(\frac{\alpha}{N})} \text{ and } \sin(l\pi + \alpha) e^{jl\pi} = \sin(\alpha)$$

for $\forall l \in Z$, expression (9) can be represented in the form:

(10) $$X(i) = \frac{1}{N} \sum_k S(k) H(k) \frac{\sin(\varepsilon\pi)}{\sin \frac{\pi(k-i+\varepsilon)}{N}} e^{-j\pi\frac{k-i+\varepsilon}{N}} e^{j\pi\varepsilon},$$

$$(11)\quad X(i)=\frac{1}{N}S(i)H(i)\frac{\sin\varepsilon\pi}{\sin\frac{\varepsilon\pi}{N}}e^{j\pi\varepsilon}+\frac{1}{N}\sum_{k\neq i}S(k)H(k)\frac{\sin\varepsilon\pi}{\sin(\frac{\pi(k-i+\varepsilon)}{N})}e^{-j\pi\frac{k-i+\varepsilon}{N}}.$$

The expression obtained contains the complete information (without the channel noise) about the dependence of the signal detected in the symbol, on the frequency and time offset.

Influence of the frequency offset

The information for the time offset is contained in $H(i)$, and more exactly in the argument of the exponential multiplier in (5). We assume that the delay $n_\varepsilon+\alpha$ is compensated. In this case (11) gives the dependence of the element detected, on the frequency offset.

The first term in (11) gives the signal received for i-th subchannel. It is determined by the signal transmitted, the channel transfer function and the attenuation due to the frequency mismatch ε. Its power is

$$|X(i)|^2=|S(i)|^2|H(i)|^2\frac{\sin^2\pi\varepsilon}{\pi\varepsilon}.$$

The second term gives the influence of all the remaining subchannels on the basic one, as a function of the frequency offset ε, the coefficient of channel transmission of the subsymbols in the signal transmitted.

The dependence of the module of the ratio “received signal/transmitted signal” for i-th subchannel, as a function of the frequency offset ε, is

$$(12)\qquad \left|\frac{X_i}{H(i)S(i)}\right|=\frac{\sin\pi\varepsilon}{N\sin\frac{\pi\varepsilon}{N}}=\varphi(\varepsilon)$$

and it is graphically shown in Fig. 1 for N=32 and $\varepsilon<16$.

It has the form of a slowly fading cosinosoid. At the beginning it is close to a sinc function, the attenuation being slower for $\varepsilon>N/4$. The dependence is even, for integer values of ε it has its zeroes and for $\varepsilon=1/2+i$, $i\in Z$, $i\neq 1$ – its alternating minima and maxima.

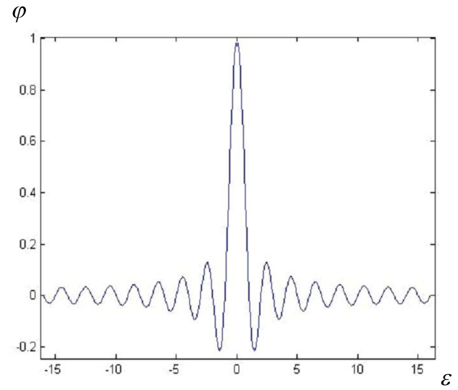

Fig.1

The separate components of the interfering part in (11) have the same dependence on ε as $\varphi(\varepsilon)$, but centered around the point $k = i - \varepsilon$. In this way, at integer values of the frequency offset ε, complete ortogonality is settled between the separate subcarriers, but the receiver is detecting the separate subchannels cyclically offset. This enables the study of the interference for integer values of ε and for fractions. $\varepsilon=\varepsilon_k+\varepsilon_p$, $\varepsilon_k \in Z$; $\varepsilon_p=0\div 0{,}5$.

For the entire estimation of the interference, it is necessary to know the statistics of the data transmitted, the frequency characteristic of the whole channel, the spectral composition of the disturbances in the channel, caused by other sources.

Assuming equal probability for the presence of the separate subsymbols $a(i)$ and $H(k)S(k)$ = const, $\forall\, k \in Z$, the expression for the interfering term in (11) becomes:

(13) $$IS_i = H\frac{X}{N}\sum_{\substack{k=-K\\k\neq i}}^{S}\frac{\sin\varepsilon\pi}{\sin\dfrac{\pi(k-i+\varepsilon)}{N}}e^{-j\pi\frac{k-i+\varepsilon}{N}}.$$

The averaged interfering power, regarded as a sum of independent sources, is

(14) $$\sigma_i^{\,2} = |X|^2|S|^2(\sin\varepsilon\pi)^2\frac{1}{N}\sum_{\substack{k=0\\k\neq i}}^{+P}\frac{1}{\left(\sin\dfrac{\pi(k-i+\varepsilon)}{N}\right)^2}.$$

The ratio signal/noise, determined from the interference due to the frequency offset ε in (11) and (13), becomes

(15) $$q_i = \frac{1}{\sin^2\varepsilon\dfrac{\pi}{N}}\frac{1}{\displaystyle\sum_{\substack{k=0\\k\neq i}}^{P}\frac{1}{\sin^2\pi\dfrac{k-i+\varepsilon}{N}}}.$$

We consider the boundary case of maximally dense frequency envelope $P=N-1$, when the interference is maximal.

The terms number in (15) is $N-2$, the separate components accepting argument values within the interval from $-\pi+\pi\dfrac{1+\varepsilon}{N}\pi$ upto $\pi-\dfrac{1+\varepsilon}{N}\pi$, by step π/N.

For $N>16$ and values of $\varepsilon < 0.5$, it can be accepted with sufficient precision, that $\sin\varepsilon\dfrac{\pi}{N} \approx \varepsilon\dfrac{\pi}{N}$ and from (15) it follows

(16) $$q_i = \frac{1}{\varepsilon^2\pi^2\displaystyle\sum_{k=1}^{N-1}\frac{1}{N^2\sin^2\pi\dfrac{k-1+\varepsilon}{N}}}.$$

The function $\varphi(z) = \dfrac{1}{N^2\sin^2\dfrac{\pi}{N}z}$ is even, periodical with a period of N and positively defined. The terms of the sum in (16) are sequentially located with a constant number of $N-2$. For the different subchannels i, the summing interval is left or right shifted, but it remains constant in duration, and fills the complete interval N. The influence of ε on the sum in (16) is not strong (ε<0.5) and if omitted, due to the periodicity of $\varphi(z)$, for every offset the summing interval, the dropped off terms are value-equal to the new ones, hence the interference is constant for all the subchannels.

A sufficiently exact solution of (16) for a real system N>16 can be obtained, if the summation is replaced by integration. Having in mind, that

(17) $$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\sin^2\theta} = \mathrm{ctg}\,\theta_1 - \mathrm{ctg}\,\theta_2,$$

$$A = \sum_{k=1}^{N-1} \frac{1}{N^2 \sin^2 \frac{\pi}{N}(k+\varepsilon)} \approx \frac{N}{\pi N^2} \int_{\frac{\pi}{N}+\frac{\pi}{N}\varepsilon}^{\pi\frac{N-1}{N}+\frac{\pi}{N}\varepsilon} \frac{d\theta}{\sin^2\theta} =$$

$$= \frac{1}{N\pi}[\mathrm{ctg}\frac{\pi}{N}(1+\varepsilon) - \mathrm{ctg}(\pi - \pi\frac{1-\varepsilon}{N})],$$

$$A = \frac{1}{N\pi} \frac{\mathrm{tg}\pi\frac{1+\varepsilon}{N} + \mathrm{tg}\pi\frac{1-\varepsilon}{N}}{\mathrm{tg}\pi\frac{1+\varepsilon}{N}\,\mathrm{tg}\pi\frac{1-\varepsilon}{N}},$$

$$A \approx \frac{1}{N\pi} \frac{2\frac{\pi}{N}}{(\frac{\pi}{N})^2(1-\varepsilon^2)} = \frac{2}{\pi^2(1-\varepsilon^2)}.$$

For the averaged interfering power it is obtained from (14) and (17)

(18) $$\sigma_i^2 = |H|^2 |X|^2 \sin^2 \pi\varepsilon \frac{2}{1-\varepsilon^2}.$$

For the interfering from the frequency offset ε, from (14) and (18) the ratio q_r signal/noise is obtained, accounting the presence of channel noise:

(19) $$q_r = \frac{\frac{\sin^2 \pi\varepsilon}{\pi^2\varepsilon^2}}{1 + \frac{2\alpha}{\pi^2(1-\varepsilon)} \sin^2 \pi\varepsilon} \alpha.$$

Here $\alpha = \frac{H^2 X^2}{\sigma_n^2}$ is the ratio signal/noise in the absence of frequency offset (ε =0) and presence of channel noise σ_n^2 only.

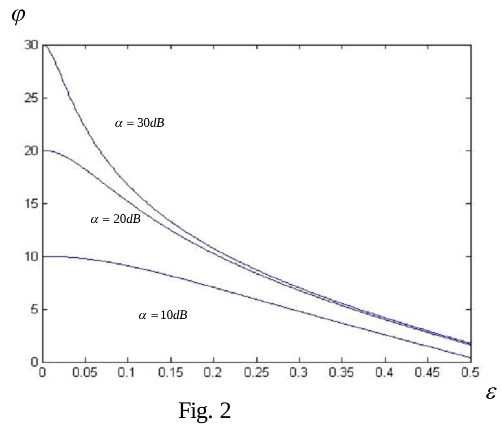

Fig. 2

Fig. 2 shows the degradation of the ratio signal/noise, determined by the channel noise only, depending on the relative frequency offset ε. It can be seen that q_r is rapidly decreasing with the rise of offset ε. The figures show the relations for ε< 0.5. At $|\varepsilon| > 0.5 + i$, the separate subchannels are dislocated.

Time offset

The time offset does not cause directly in-symbol (inter-subchannel) interference. At slight deviations it leads to significant phase deviations in the subcarriers received. This makes the weakening of the useful signal very dependent on the type of the modulation used. At large deviations, inter-symbol interference appears. The time offset gives the offset of the timing window on the signal being received, in which DFT is accomplished for a separate symbol.

The delay of the local clock causes directly inter-symbol interference, the greater the delay, the stronger it is.

OFDM system is tolerant towards outstripping smaller than the cyclic prefix. At greater outstripping, inter-symbol interference appears again.

Each offset, positive or negative, causes the decrease of the relevant signal due to the diminished number of real data in the window being processed.

The offset by τ introduces phase offset on k-th subcarrier with $\theta_{\tau_k} = 2\pi \frac{k}{Td}\tau$ ($T_d = \frac{1}{\Delta f}$ is discretization interval). The phase offset for every following, but higher subcarrier is increased by $\Delta\theta = 2\pi \frac{\tau}{Td} = 2\pi\Delta f\tau$.

In order to decrease the requirements towards the accuracy of the symbol synchronism, it is possible to enter, not at the beginning only, but also at the end of each transmitted symbol, a protection interval. This is necessary, especially for channels with well expressed time dispersion.

In channels with distinct frequency and time dispersion, such as the wireless connections between mobile objects, the entry of protection time intervals at the beginning and at the end of every symbol and of frequency protection intervals between the subcarriers, considerably facilitates the synchronization system.

Cyclo-stationary blind synchronization

The blind synchronization is very appropriate thanks to its high spectral efficiency, and entire separation of the subsystem for reception/transmission from the synchronization one. However, this approach is statistic and hence slower. In [3, 4] a method is proposed for blind synchronization investigation, that uses cyclic correlation of the input signal. The information flow is stationary by nature. The strict periodicity of the successively transmitted OFDM symbols naturally presumes easy entry of periodicity in the statistical indicators of the signal received. They are entered by the transfer function of most of the communication channels. Its use, however, is inappropriate in most of the cases, since it must be either known or currently estimated. In the larger part of the systems and particularly the ones using channels with time and frequency dispersion, the signal transmitted is processed by weight, which naturally makes the signal cyclo-stationary.

But the weight processing is not always realized. In simpler cases the influence of the time dispersion is removed just entering a protection interval – the weight function is a rectangular window.

Herein an approach for blind synchronization will be suggested, which applies the entry of cyclicity from the weight function, known at the two ends of the communication system. It is shown in [4] that the approach is applicable for a fading channel as well, represented by the dispersion power profile. The pulse reaction is composed from realizations, which are not correlated and for each cross section it is a Gauss variable with an average zero and power σ_l^2. Such a channel does not cause cyclic correlation. The simpler case with cyclicity, input by $g(t)$ only, will be discussed. There exist more sophisticated approaches, with weight processing of the receiving side as well, fittingly matched with $g(t)$ ([10], for BFDM – bi-orthogonal version of OFDM).

The signal received, after reducing to the basic band, sampled with K-fold higher frequency that the transmission one, gets the form

$$x(n) = \alpha(n) e^{j2\pi f_\varepsilon \frac{T_d}{K} n + \theta} \sum_{i=0}^{\left\lfloor \frac{n}{K} \right\rfloor} a(i) g(n - m_\varepsilon - ik) + v(n) . \tag{20}$$

The symbol $[A_q]$ here denotes the whole part of A_q, f_ε is frequency offset.

The auto-correlation function of the signal received is defined by the expression:

$$r(n,l) = E[x(n)x^*(n+l)] \text{ for } \forall n, l \in Z , \tag{21}$$

$$r(n,l) = r_\alpha(l) e^{-j2\pi f_\varepsilon \frac{T}{K} l} \sum a_i g(n - m_\varepsilon - ik) \sum_{j=0}^{\left\lfloor \frac{n}{K} \right\rfloor} a_j^* g(n + l - m_\varepsilon - jk) + r_v(l) .$$

The symbols set $[a_i]$ is evenly distributed, with equal powers and an average zero. This condition leads to $E[a_i a_j^*] = \sigma_a^2 \delta(i - v)$ and the double sum in [21] is reduced to terms with homonymous indices $i = j$,

$$r(n,l) = r_\alpha(l) e^{-j2\pi f_\varepsilon \frac{T}{K} l} \sigma_a^2 \sum_{i=0}^{\left\lfloor \frac{n}{K} \right\rfloor} g(n - m_\varepsilon - ik) g^*(n - m_\varepsilon - ik + l) + r_v(l) \,. \tag{22}$$

In order to improve the estimation, the averaging must be accomplished for several successive symbols. The shape of $r(n, l)$ is mainly determined by the determinated function $g(n)$. With the assumptions made, that for the averaging interval, the multiplier in front of the sum in (22) is independent on n, and for $n>>K$,

$$r_g(n,l) = \sum_{i=-\infty}^{\infty} g(n - m_\varepsilon - ik) g^*(n - m_\varepsilon - ik + l) = r_g(n + pK, l) \;\; \forall p \in Z \,.$$

The correlation function $r(n, l)$ is a periodical function of n for $\forall l \in [0, k-1]$ with a period K (equal to the duration of the separate symbol). This means that the value of $r(n, l)$ depends on the location in the symbol interval T_d, for which f_ε and εT_d are determined.

The periodicity of $r(n, l)$ and its shape carry significant information about the frequency and time offset. This is explicitly expressed when representing $r(n, l)$ in a Fourier's series;

$$r(n,l) = \sum b_k(l) e^{+j\frac{2\pi}{K}kn} \,,$$

$$b_k = \frac{1}{K} < r(n,l), e^{j\frac{2\pi}{K}} k, n > ,$$

$$b(l) = \frac{1}{K} r_\alpha(l) \sigma_a^2 e^{-j2\pi f_\varepsilon \frac{T}{K} l} \sum_{n=0}^{K-1} \sum_{i=-\infty}^{\infty} g(n - m_\varepsilon - ik) g^*(n - m_\varepsilon - ik + l) e^{-j\frac{2\pi}{K}kn} + r_v(l)\delta(k).$$

The equality $e^{-j\frac{2\pi}{K}kn} = e^{-j\frac{2\pi}{K}k(n-ik)}$ $\quad \forall i \in Z$ enables the replacement of double summing by a single one

$$b_k(l) = \frac{1}{K} r_\alpha(l) \sigma_a^2 e^{-j2\pi f_\varepsilon \frac{T}{K} l} \sum_n g(n - m_\varepsilon) g^*(n - m_\varepsilon + l) e^{-j2\pi k \frac{n}{K}} + r_v(l)\delta(k) \,.$$

The Fourier transform, as unitary, allows the replacement of the scalar product in the time domain by that in frequency domain (Parseval's equality).

$$< g(n - m_\varepsilon),\; g(n - m_\varepsilon + l) e^{-j2\pi \frac{k}{K} n} >=<$$
$$>=< G(f) e^{-j2\pi f \varepsilon T},\; G(f - \frac{k}{T}) e^{-j2\pi (f - \frac{k}{T})(\varepsilon - \frac{l}{K})T} >=< R(f, l, k),$$

(23) $$R(f,l,k)=\frac{K}{T}e^{j2\pi\frac{lk}{K}}e^{-j2\pi k\varepsilon}\int_{-(1/2)T}^{+(1/2)T}G(f)G^{*}(f-\frac{k}{T})e^{-j2\pi\frac{T}{K}lf}df.$$

Hence

(24) $$b_k(l)=\frac{1}{K}r_\alpha(l)\sigma_a^2(e^{-j2\pi f_\varepsilon\frac{T}{K}l})R(f,l,k)+r_v(l)\delta(k).$$

In (23) and (24) the frequency f_ε and time offset ε are independent separate values, which facilitates their determination. An important property of (24) is that the noises $V(n)$ influence the constant component ($k = 0$) only. The determination of the parameters sought for spectral lines $k>0$ will not be influenced by the channel noise.

The relations

(25) $$U(f,l,k)=\frac{K}{T_d}e^{j2\pi\frac{lk}{K}}\int_{-(1/2)T}^{+(1/2)T}G(f)G^{*}(f-\frac{k}{T_d})e^{-j2\pi\frac{T}{K}lf}df$$

contain components, set as essential ones in the design of the system. It can be accepted that there exists an inverse function $U^{-1}(f,l,k)$.. When multiplying (24) by $U^{-1}(f,l,k)$, the equation is obtained:

$$d_k(l)=\frac{1}{K}r_\alpha(l)\sigma_a^2e^{-j2\pi f_\varepsilon\frac{T}{K}l}e^{-j2\pi k\varepsilon}+U^{-1}(f,l,k)r_v(l)\delta(k).$$

It contains unknown parameters ε and f_ε,

(26) $$d_k(l)=\frac{1}{K}\Gamma_\alpha(l)\sigma_a^2e^{-j2\pi(f_\varepsilon\frac{T}{K}l+k\varepsilon)}\quad\text{for } k>0.$$

The parameters searched for ε and f_ε. are found in the argument of $d_k(l)$.

The frequency offset f_ε. depends linearly on the offset, for which the correlation function $r(n, l)$ is determined and does not depend on the order of the spectral component k. On the contrary, the time offset depends linearly on the spectral component k and does not depend on l. One simple solution is obtained defining $d_k(l)$ for two opposite in sign spectral lines and an arbitrary l;

$$f_\varepsilon=\frac{K}{4\pi lT}\arg d_k(l)d_{-k}(l).$$

The time offset can be defined for already known frequency offset f_ε, as

$$\varepsilon=\frac{1}{2\pi k}\arg[d_k(l)e^{j2\pi\frac{T_d}{K}lf_\varepsilon}]=\frac{1}{2\pi k}\arg d_k(l)-\frac{T_d}{K}lf_\varepsilon.$$

Potentially, ε can be determined independently on f_ε, using the power spectrum of the signal received $r(n, 0)$

$$\varepsilon=\frac{1}{2\pi k}\arg d_k(0).$$

The simultaneous determination of the two offsets is possible with the approach exposed.

At absence of offsets (ε=0, f_ε=0), $d_k(l)$ remains a value, independent on l and k, and $r(n, l)$ – a not modulated value.

The spectral coefficients $b_k(l)$ obtained are a direct transformation of the periodical correlation function $r(n, l)$. Its dependence on l can be estimated by Fourier transform only.

For the study discussed, it is more appropriate to use $d_k(l)$ instead of $b_k(l)$. This simplifies the expressions, freeing $b_k(l)$ from the necessary apriori information and reserving the character of a Fourier image of a periodical correlation function of the signal received.

The Fourier image of $d_k(l)$ with respect to the offset l for a fixed k, is regarded as Doppler's extension [6] of the spectrum. It may contain a continuous part, as well as separate spectral lines, which is already Doppler's offset,

$$\text{(27)} \qquad \mathrm{F}[d_k(l)] = D(k,\lambda) = \frac{1}{K}\sigma_a^2 e^{-j2\pi k\varepsilon} \sum_l r_\alpha(l) e^{-j2\pi(\lambda + f_e \frac{T}{K})l},$$

$$D(k,\lambda) = \frac{\sigma_a^2}{K} e^{-j2\pi k\varepsilon} R_\alpha(\lambda_k + \frac{T}{K} f_\varepsilon).$$

Here λ_k is Dopplers' frequency for k-th spectral line of $d_k(l)$; its extension or offset, if it contains a Dirak's function.

The extension is typical for fading channels. It is most strongly expressed in radio channels within the GigaHertz range. In the channels of an electrical network, it is connected with modulation of the transfer function by the power voltage.

The maximum extension is for the zero frequency. For frequency $\lambda_k + \frac{T}{K} f_\varepsilon = 0$, $D(\lambda, k)$ has its maximum.

In this way

$$\text{(28)} \qquad f_\varepsilon = \frac{K}{T} \arg\max D(\lambda, k), \ k \neq 0,$$

f_ε is determined from (28) with reference to the maximum for k-th spectral line of $D(\lambda, k)$. It is assumed that f_ε is frequency independent. The data, used to make the estimations, are restricted in number. The estimates obtained are not asymptotically displaced. However, the small number of data causes considerable dispersion of the estimates. In order to increase their validity, averaged estimates from the different spectral components $k \in [1, K-1]$, may be used.

An important advantage of the approach proposed is that it does not require knowledge of the channel pulse reaction. The channel pulse reaction is determined by its profile – the averaged and momentum power. In order to obtain a valid estimation with this method, a sufficient sample is needed, in order to reveal channel statistics.

Estimation of the frequency offset, using virtual frequencies

In most of the cases, the incomplete possible set of subchannels in OFDM is used, due to considerations either concerning the optimal loading of the frequency band disposed, or the number of subchannels being different from 2^m, a number preferred for use in the in the quick variant of DFT. The absence of subcarriers introduces considerable nonstationarity of the information signal, which can be used to establish frequency and time synchronism in the system.

In order to show this possibility, we derive from the established dependence of the recovered vector of the k-th symbol transmitted.

(29) $$X(k) = e^{j\varphi_k} W \Lambda(\varepsilon) W^H H_0^{'} S_0(k) .$$

Here $\varphi_k = 2\pi\Delta f_\varepsilon T_d (k-1)N + kL$ is the phase offset at the beginning of k-th symbol received.

In order to recover it completely, without mutual influences between the separate subcarriers, and to preserve the orhogonality among them, it is required that:

$W \Lambda W^H = I$, which can be satisfied only for $\Lambda = I$, i.e. $2\pi f_\varepsilon T_d = 2\pi i, i \in \mathrm{Z}$, Here I is a symbol of a single matrix.

The subfrequencies from P+1 up to N are virtual. The recovery of the orthogonalities is possible only with the input of a phase offset, equal in value and inverse in sign to the one, caused by the frequency offset. Analytically this can be expressed with the multiplication of the signal received $y(k)$ by the matrix $\tilde{\Lambda}^H$.

(30) $$\tilde{\Lambda} = \mathrm{diag} \left| e^{j\tilde{\Phi}_0} \; \ldots \; e^{j\tilde{\Phi}_0 (N-1)} \right| ,$$

$$\tilde{y}(k) = \tilde{\Lambda}^H y(k).$$

The symbol received is recovered as a result of DFT $\tilde{X}(k) = W \tilde{y}(k) \cdot$

After the recovery, at the output of the virtual subchannels only channel noise is expected. This noise can be averaged for the separate virtual subchannels, and for several successive blocks as well.

(31) $$\tilde{X}_i(k) = W_i^H \tilde{y}(k), \; W_i = \left| e^{+j\frac{2\pi}{N}i} \ldots \; e^{+j\frac{2\pi}{N}i(N-1)} \right|^{\mathrm{T}}, \; i = P+1, \ldots, N-1.$$

In [7] not the values, obtained for the separate symbols are summed, but their power (the square of their norm)

(32) $$\tilde{X}_i(k) \tilde{X}_i^*(k) = \left\| \tilde{S}_i(k) \right\|^2 = W_i^H \tilde{y}(k) \tilde{y}(k)^H W_i .$$

The summed power of $N - P$ virtual channels in K successive blocks is

(33) $$P_n = \sum_{k=1}^{k} \tilde{y}^H(k) W_{Np} W_{Np}^H y(k) .$$

Here $W_{Np} = \left| W_{p+1} \;\; W_{p+2} \;\; \ldots \;\; W_{N1} \right|$ is a $p \times N$ dimensional matrix.

The power of the virtual subchannels, expressed with the help of the matrix corrected (30), is

(34) $$P_n = \sum_{k=1}^{K} y^H(k)\, \tilde{\Lambda}\, W_{Np} W_{Np}^H \tilde{\Lambda}^H\, y(k).$$

The problem of subcarrier correction is reduced to finding $\tilde{\Lambda}_0$, which minimizes (34).

In the limit case of ignorable noises P_n=0.

We set $z = e^{j\tilde{\Phi}_0}$, $Z = \mathrm{diag}(1 \;\; Z \;\; \ldots \;\; Z^{N-1})$ and (34) gets the form of a polynom of power 2*N*–2 with respect to *z*.

Under the absence of noise, the determination of the compensating phase is reduced to finding the zeroes of (34), which are 2*N* – 2 in number. Under the absence of noise they should lie on the single circle. Since the roots are pair conjugated, to each root z_i there corresponds a root $z_i^{-1} = z^*$,

(35) $$P_n(z) = \sum y^H(k) Z^{-1} W_{Np} W_{Np}^H Z y(k).$$

That form gives the reason to define the zeroes of (34). Under the absence of noise this is equivalent. However, with noise presence, (34) is never nulled. It can be assumed, that the zeroes of (35) give an approximate solution of the problem for compensating matrix determination. It is indicated in [7], that this solution can be too far from the optimal value. The advantage in this approach is that the algorithm of polynomial roots determination can be directly used.

The use of the approach with maximal verification (MV) provides the best solution. The function (34) itself has a set of local maxima, connected with the polynom order. This makes the problem very source consuming, when using routine approaches. The convergence is too slow, depending on the network selected, on the initial point. Some simplified approximate solutions are possible, which make the approach operative.

Conclusion

The paper discusses the analytical relations, which determine the in-symbol and inter-symbol interference in an OFDM communication system. The blind estimation of the frequency and time offset is considered, using the cyclo stationarity introduced in the information signal.

References

1. M o o s e, P a u l H. A Technique for Orthogonal Frequency Division Multiplexing Frequency Offset Correction. – IEEE Trans. Communications, Vol. **42**, October 1994.

2. Z h i q i a n g, L i u, B i n w e i W e n g. Blind Full Range Frequency Offset Estimation for Differential OFDM. – In: IEEE Communication Society, 2004.
3. P a r k, B y u n g j o o n, H y n s o o C h e o n, E u n s e o k K o, C h a n g e o n K a n g, D a e s i k H o n g. A Blind OFDM Synchronization Algorithm Based on Cyclic Correlation. – In: IEEE Signal Processing Letters, Vol. **11**, February 2004, No 2, 83-85.
4. G i n i, F u l v i o, G. G i a n n a k i s. Frequency Offset and Symbol Timing Recovery in Flat-Fading Channels: A Cyclostationary Approach. – In: IEEE Trans. Communication, Vol. **46**, March 1998, No 3, 400-411.
5. N i k o l o v, Z., V. V a s s i l e v, L. N i k o l o v a. Frequency-Time Analysis of the Interferences in a 220 V Network. – ICC04, Vol. **1**, Paris, 2004, 118-122.
6. P r o a k i s, J. G. Digital Communication. 3rd Ed. New York, McCraw Hill, 1995.
7. G a o, F e i f e i, A. N a l l a n a t h a n. Blind Maxumum Likelihood CFO Estimation for OFDM System via Polynomial Rooting. – In: IEEE Signal Processing Letters, Vol. **13**, February 2006, No 2.
8. N i k ol o v, N., Z. Ni k o l o v. A Communication System with Wavelet Packet Division Multiplexing in an Environment of Gaussian Noise and Narrow-Band Interferences. – Cybernetics and Information Technologies, Vol. **5**, 2005, No 1.
9. J a n-J a a p v a n de B e e k, M. S a n d e l l, P e r O l a B o r j e s s o n. ML estimation of time and frequency offset in OFDM systems. – In: IEEE Transactions on Signal Processing, Vol. **45**, July 1997, No 7.
10. B o l c s k e i, H e l m u t. Blind Estimation of Symbol Timing and Carrier Frequency Offset in Wireless OFDM Systems. – In: IEEE Transactions on Communication, Vol. **49**, June 2001, 988-999.
11. L v, T i e j u n, H u a L i, J i e C h e n. Joint Estimation of Symbol Timing and Carrier Frequency Offset of OFDM Signals over Fast Time-Varying Multipath Channels. – In: IEEE Trans. Signal Processing, Vol. **53**, December 2005, No 12.
12. K e l l e r, T., L. P i a z z o, P. M a n d a r i n i, L. H a n z o. Orthogonal Frequency Division Multiplex Synchronization Techniques for Frequency-Selective Fading Channels. – In: IEEE J. on Selected Areas in Communications, Vol. **19**, June 2001, No 6, 999-1008.
13. P o l l e t, T., M. V a n B l a d e l, M. M o e n c l a e y. BER Sensitivity of OFDM Systems to Carrier Frequency Offset and Wiener Phase Noise. – In: IEEE Trans. Communications, Vol. **43**, February/March/April 1995, No 2/3/4.
14. C h a n g, R. W. Synthesis of Band-Limited Orthogonal Signal for Multichanel Data Transmission. – Bell Syst. Tech J., Vol. **45**, December 1966, 1775-1796.
15. D a r l i n g, S. On Digital Single – Sideband Modulators. – In: IEEE Trans Circuit Theory, Vol. **CT-17**, August 1970, 409-414.
16. W e i n s t e i n, S., P. E b e r t. Data Transmission by Frequency-Division Multiplexing Using the Discrete Fourier Transform. – In: IEEE Trans. Commun. Technol., Vol. **COM-19**, October 1971.
17. S c h m i d l, T. M., D. C. C o x. Robust Frequency and Timing Synchronization for OFDM. – In: IEEE Trans. Communications, Vol. **45**, December 1997, No 12, 1613-1621.
18. T u r e l i, U f u k, P. H o n a n, H u i L i u. Low Complexity Nonlinear Least Squares Carrier Offset Estimator for OFDM: Identifiability, Diversity and Performance. – In: IEEE Signal Processing, Vol. **52**, September 2004, No 9, 2441-2452.