BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **6**, No 2

Sofia • 2006

Analysis and Synthesis of IMC-Control Systems

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Abstract: *In this paper the hypothesis of structural equivalence of the systems with internal model is proved. The proof is based on determining common base and structural equivalence of basic structure of the type IMC (Internal Model Control)-control systems. These are predictive control systems, robust systems with internal model and robust systems with conditional feedback.*

Key words: *Internal Model Control, Smith prediction systems, robust control systems.*

1. Introduction

The realization of control with wanted qualities in conditions of considerable change in the plant's time delay and uncertainty, impose the search of new possibilities for the advance of the synthesis of known control systems [1-6], and also of real possibilities for development of systems of a new class. They must have the property of effective control of industrial plants under uncertainty in the rational as well as in the irrational part of the model of the plant. In the control theory are known [1-6] *IMC (Internal Model Control)* control systems. These are the predictive control systems [4-6], robust systems with internal model [1-3] and robust systems with conditional feedback [2, 3]. What puts them in one class is the fact that they all use internal model of the plant in their structure. For each one of them the control theory gives the relevant method of synthesis [1-6]. The common sign for their classification and other indications of "identity" suppose that there is a connection between those systems and therefore an eventual possibility for common analytical transformations of the known in the literature methods for their synthesis. The positive answer of such a question would lead to increasing the effectiveness of the design process of *IMC*-systems, would give basically new possibilities for development of their synthesis and would give real possibilities for developing new class of systems that could perform effective control of industrial plants under uncertainty in the rational as well as in the irrational part of the plant's model.

Based on this, this paper aims at checking the *hypothesis of structural equivalence of the systems with internal model*. As a basic method for verification of the hypothesis the method of equivalent structural transformations is used.

In achieving the so formulated goal, this paper must perform the following tasks: structural analysis of the class of IMC-systems; search and determining of the structural equivalence between them; determining of analytical connection between different control systems' structure.

2. Predictive control systems

Fig. 1 shows the multitude of the class of plants as a function of (τ/T), that need realization of "special" (anticipating the time delay) control algorithms. These plants are characterized by values of the relative time delay (τ/T) > 5 (where τ is "the time delay", and T is the dominant time-constant in the plant's model).

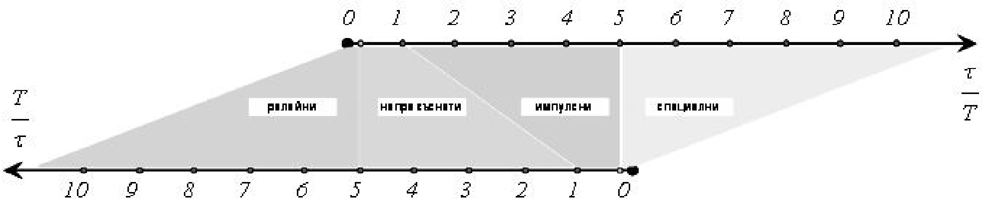

Fig. 1

The system (Fig. 2) with predictive control that realizes such control algorithms is well known (system with *Smith* R_S or *Reswick controller*) [4-6]. It is based on the "structural separation" of the delay in the internal model of the controlled plant G. So the system solves structurally the control problems of plants with considerable inertia and time delay, when the nominal model of the plant G* is supposedly previously known (or is given in the process of the system's design). In this paper the structure from Fig. 2 is transformed equivalently to that on Fig. 3.

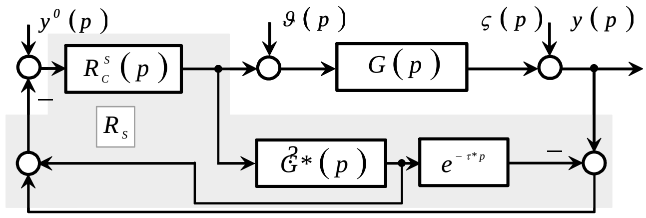

Fig. 2

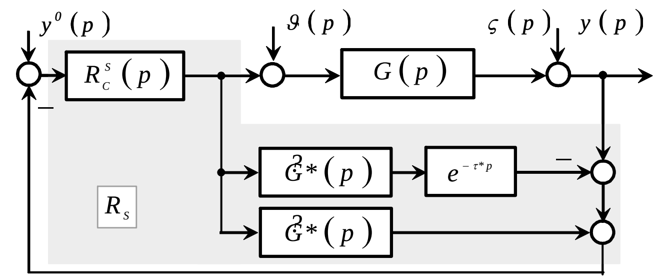

Fig. 3

When the dependence $G_{\text{nom}} \equiv G^* \equiv \hat{G}^* \,.\, e^{-p\tau^*}$ is taken in mind, the structure shown on Fig. 4 is reached.

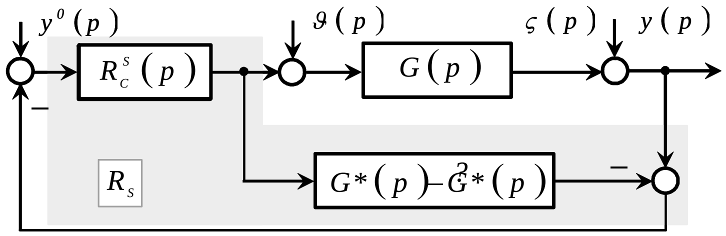

Fig. 4

Here $e^{-p\tau^*}$ is a part of the delay (the irrational part), and $\hat{G}^*$ is the rational part (basic linear dynamic of the nominal plant's model G^*). In this scheme the dependence (1) is used, where R_C^S could be controller of any type (2), optimally designed by known (corresponding) analytical methods (including engineering methods, when local quality criteria is given σ) to the hypothetic model of $\hat{G}^*$,

(1) $$\hat{G}^* - \hat{G}^* \,.\, e^{-p\tau^*} = \hat{G}^*\left(1 - e^{-p\tau^*}\right) = \hat{G}^* - G^* = -\left(G^* - \hat{G}^*\right),$$

(2) $$R_C^S \underset{\{\sigma = \text{const}\}}{\Leftrightarrow} \hat{G}^* \quad .$$

The following equivalent structural transformations are realized (Figs. 5-10),

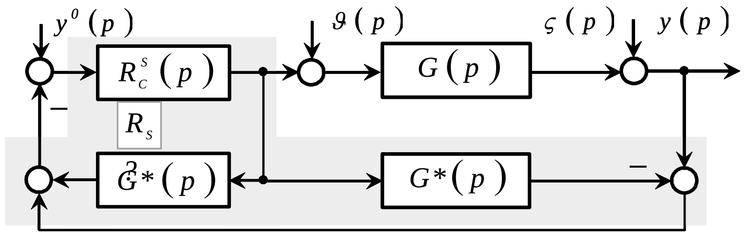

Fig. 5

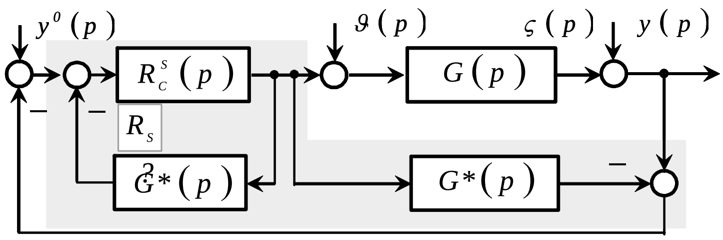

Fig. 6

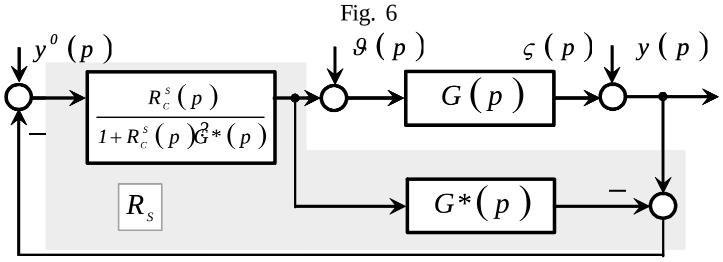

Fig. 7

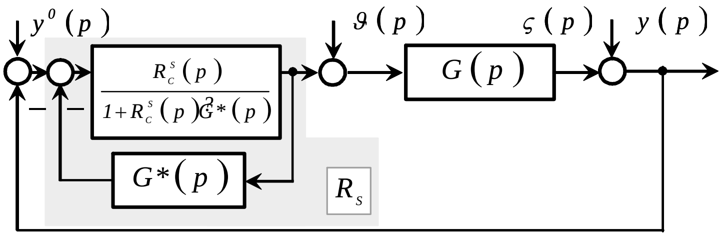

Fig. 8

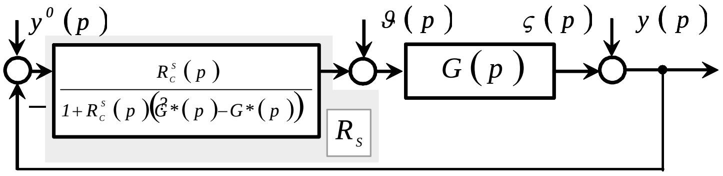

Fig. 9

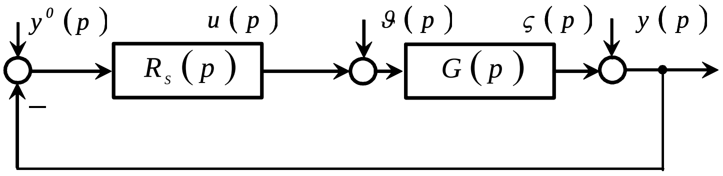

Fig. 10

from which the following dependences are derived (3)-(8):

(3) $$R_S(p)=\frac{\dfrac{R_C^S(p)}{1+\hat{G}^*(p)R_C^S(p)}}{1-\dfrac{G^*(p)R_C^S(p)}{1+\hat{G}^*(p)R_C^S(p)}}=\frac{R_C^S(p)}{1+\hat{G}^*(p)R_C^S(p)-G^*(p)R_C^S(p)},$$

(4) $$R_S(p)=\frac{R_C^S(p)}{1+R_C^S(p)\left(\hat{G}^*(p)-G^*(p)\right)}=\frac{R_C^S(p)}{1-R_C^S(p)\left(G^*(p)-\hat{G}^*(p)\right)},$$

(5) $$R_S(p)+R_S(p).R_C^S(p).\left(\hat{G}^*(p)-G^*(p)\right)=R_C^S(p),$$

(6) $$R_S(p)=R_C^S(p)-R_S(p).R_C^S(p).\left(\hat{G}^*(p)-G^*(p)\right),$$

(7) $$R_S(p)=R_C^S(p).\left(1-R_S(p).\left(\hat{G}^*(p)-G^*(p)\right)\right),$$

(8) $$R_C^S(p)=\frac{R_S(p)}{1-R_S(p)\left(\hat{G}^*(p)-G^*(p)\right)}=\frac{R_S(p)}{1+R_S(p)\left(G^*(p)-\hat{G}^*(p)\right)}.$$

These dependences define the functional connectivity between the generalized predictor algorithm R_S and the controller R_C^S with models $\hat{G}^*$ and G^*, as well as the dependence between R_S and R_C^S in the structure of the system (Fig. 2) with predictive control (a system with *Smith controller*).

3. Robust system with internal model

A known structure (Fig.11) of robust system with internal model is examined [1-3]. It is based on using previously known (or given during the system's design) nominal model of the plant G^* and controller Q in the structure of the robust controller R_M. Its purpose is control of plants with change in parameters and/or change in structure of the analytical model in operational conditions under uncertainty. Achieving robust properties of the control system (Fig.11) for plants in wide diapason of the multitude (Fig.1), is based on the design of the controller Q (or of R_M) as a function (9) of the nominal model G^*, when criteria for robust stability and robust performance of the control system are required [1-3] (given as requirements $\Omega(e, \eta)$ to the sensitivity functions of the system e, η) in a previously given functional multitude $\Pi(j\omega)$:

$$R_M \underset{\left\{\begin{smallmatrix}\sigma = \text{const} \\ \Omega(e,\eta)\,\le\, 1\end{smallmatrix}\right\}}{\Leftrightarrow} G^*, \Pi \quad , \tag{9}$$

$$\Pi(j\omega) = \left\{ \begin{matrix} \Delta G(j\omega) : \left| G(j\omega) - G^*(j\omega) \right| \le \bar{\ell}_a(\omega), (\omega \in [0;\infty)) \\ \Delta G(j\omega) : \dfrac{\left| G(j\omega) - G^*(j\omega) \right|}{\left| G^*(j\omega) \right|} \le \bar{\ell}_m(\omega), \left(\bar{\ell}_m(\omega) = \dfrac{\bar{\ell}_a(\omega)}{\left| G^*(j\omega) \right|} \right) \end{matrix} \right\}. \tag{10}$$

The multitude (10) is determined by the variations $\Delta G(j\omega)$ in the characteristics of the real plant $G(j\omega)$ near its nominal model $G^*(j\omega)$, due to additive $\bar{\ell}_a$ and/or multiplicative $\bar{\ell}_m$ disturbances.

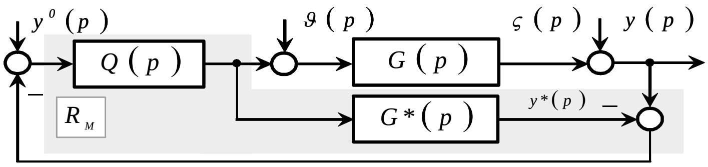

Fig. 11

The equivalent structural transformation of Fig. 11 gives the structures shown on Figs. 12-14, where the generalized robust controller R_M (Fig. 14) and the controller

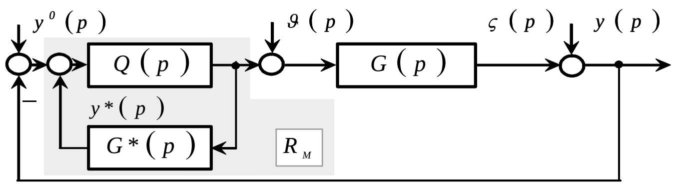

Fig. 12

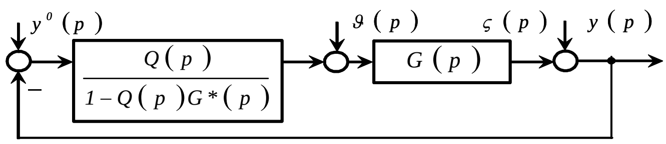

Fig. 13

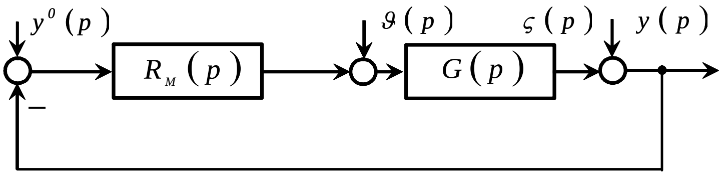

Fig. 14

Q are determined by (11), (12) as function of the nominal plant G^*, used as internal model in the robust control system (Fig. 11):

(11) $$R_M(p) = \frac{Q(p)}{\left(1 - G^*(p)Q(p)\right)},$$

(12) $$Q(p) = \frac{R_M(p)}{\left(1 + R_M(p)G^*(p)\right)}.$$

4. Robust system with conditional feedback

The robust control system with conditional feedback [2, 3] (Fig. 15) is based on using previously known (or given during system's design) nominal model of the plant G^*, controller R_C^F and dynamic filter F in the structure of the generalized robust algorithm R_F. The controller R_C^S could be any type of controller, optimally designed (13) using known (corresponding) analytical methods (including engineering methods when local quality criteria is given σ) to the nominal model G^* of the plant

(13) $$R_C^F \underset{\{\sigma = \text{const}\}}{\Leftrightarrow} G^*,$$

(14) $$F \underset{\left\{\substack{\sigma = \text{const} \\ \Omega(e,\eta) \le 1}\right\}}{\Leftrightarrow} G^*, \Pi, R_C^F.$$

Determining the dynamic of the filter F is the basic task of the design. It is connected with achieving robust properties of the control system (Fig. 15) for plants in wide range of the multitude (Fig. 1). Filter's design F is based on functional dependence (14) on the nominal model G^*, the multitude Π (10) and the controller R_C^F when criteria on robust stability and robust performance of the control system are required [2, 3] (given as requirements $\Omega(e, \eta)$ to the sensitivity functions of the system e, η) in previously given functional multitude $\Pi(j\omega)$ (10).

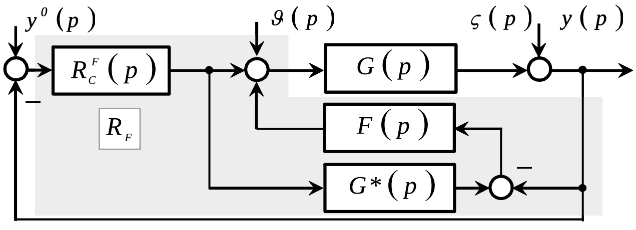

Fig. 15

The equivalent structural transformations of the scheme (Fig. 15) give the following results (Figs. 16-21), for which the dependences (15)-(17) are derived.

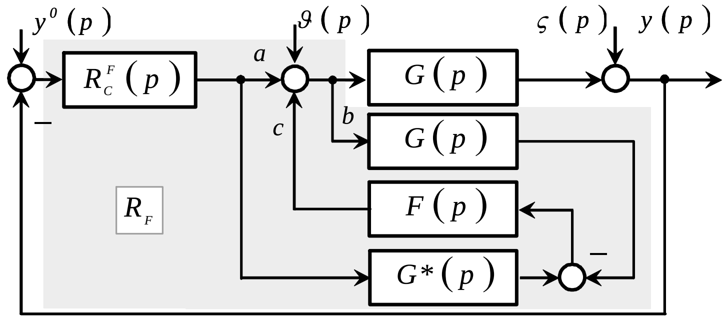

Fig. 16

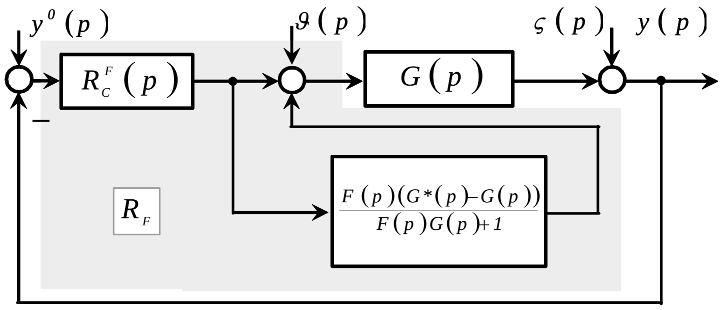

Fig. 17

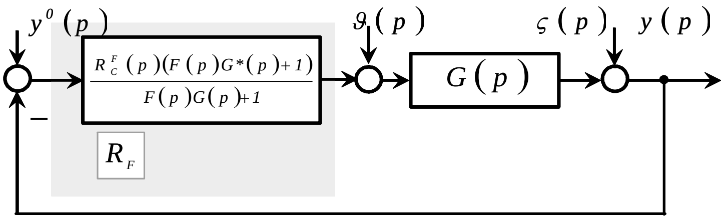

Fig. 18

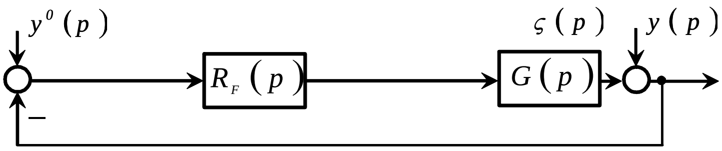

Fig. 19

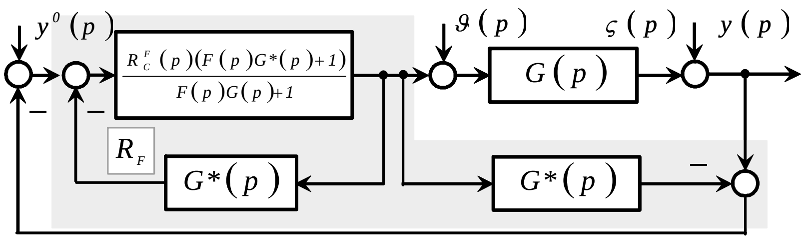

Fig. 20

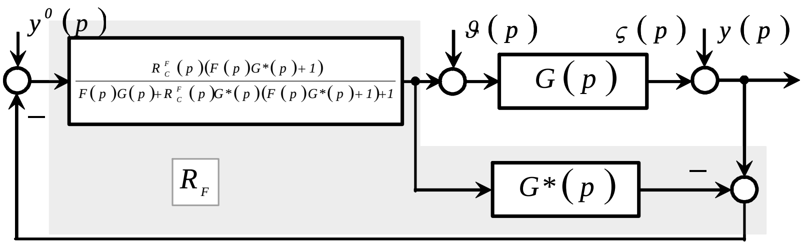

Fig. 21

In them the plant's model G is substituted equivalently with the "disturbed at upper limit" model $G^{\blacksquare}$, previously known (or given during the system's design). This method of equivalent substitution using $G^{\blacksquare}$ is the known [2, 3] "method of the balance stability equation" applied for the synthesis of F:

(15) $$R_F(p)=\frac{R_C^F(p)\left(F(p)G^*(p)+1\right)}{F(p)G(p)+1},$$

(16) $$F(p)=\frac{R_C^F(p)-R_F(p)}{R_F(p)G^{\blacksquare}(p)-R_C^F(p)G^*(p)},$$

(17) $$R_C^F(p)=\frac{R_F(p)\left(F(p)G^{\blacksquare}(p)+1\right)}{F(p)G^*(p)+1}.$$

The dependences (15)-(17) define the functional connections between the generalized algorithm R_F and the controller R_C^F with the filter F and the models G^* and $G^{\blacksquare}$, as well as the dependences between R_F and R_C^F in the structure of the examined robust system (Fig. 15) with conditional feedback.

5. Conclusions

Using the presented result from applying the method of equivalent structure transformations on the examined systems, this paper:

– *proves the common base* of the basic structure of the studied three types of control systems (Figs. 2, 11, 15) – this is the classic structure of a control system consisted of plant and controller (Figs.10, 14, 26);

– *proves the structure equivalence* between the examined systems (Figs. 2, 11, 15) from the class of *IMC* (*Internal Model Control*) – it consists in that that functionally all the three systems are based in their organization on same internal model – the nominal model G^* of the controlled plant (Figs. 7, 11, 24);

– *determines the analytical dependences* between the nominal model G^* and the generalized control algorithms of predictor and robust IMC-systems.

– *proves the hypothesis for structural equivalence of the predictor and robust* IMC-*systems*, using the common classic base and structural identity of the studied IMC-systems.

The claims of this first part of the study are in the achieved original and new proofs for the common: base, structural identity and analytical dependences. Due to them the hypothesis of structural equivalence was proved. It confirms the existence of the necessary and sufficient grounds to: search new possibilities for development of the synthesis of IMC-systems, develop new type of IMC-systems that could effectively control industrial plants under uncertainty in the rational and irrational parts of the plant's model. The achieved results in this direction will be presented in the second part of this study, which will be published in the next issue.

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