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Cell Averaging Constant False Alarm Rate Detector with Hough Transform in Randomly Arriving Impulse Interference*

Christo Kabakchiev, Lyubka Doukovska, Ivan Garvanov

Institute of Information Technologies, 1113 Sofia

Abstract: *In this paper we compare the efficiency of a Cell Average Constant False Alarm Rate (CA CFAR) detector and a Hough detector with average randomly arriving impulse interference. We assume that the target echo signal fluctuates according to a Swerling II signal model; the randomly arriving impulse interference is with a Poisson distribution of the probability for appearance and a Rayleigh distribution of the amplitudes. The profits (losses) are determined as a statistical estimation by means of the probability characteristics of both types of detectors, obtained in Matlab. The profits of the Hough detectors are calculated for different values of the probability for appearance of randomly arriving impulse interference with average length in the cells in range. Our results show that Hough transform is effective in conditions of decreasing randomly arriving impulse interference.*

Keywords: *Radar Detector, CA CFAR detector, Randomly arriving impulse interference, Probability of detection, Probability of false alarm, Detectability profits (losses).*

1. Introduction

In modern radar target detection is declared if the signal value exceeds a preliminary determined adaptive threshold. The threshold is formed by current estimation of the noise level in the reference window. The estimate proposed by Finn and Johnson in [5] is often used as an estimate of the noise level. Averaging the outputs of the reference cells surrounding the test cell forms this estimate. Thus a constant false alarm rate is maintained in the process of detection. CA CFAR (Cell Averaging Constant False Alarm Rate) processors are very effective in case of stationary and homogeneous interference. The presence of randomly arriving impulse interference in both, the test

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resolution cells and the reference cells can cause drastic degradation in the performance of the CA CFAR processor. Such type of interference is non-stationary and non-homogenous and is often caused by adjacent radar or other radio-electronic devices.

The detection performance of CA CFAR processors with post detection integrator is proposed by Hou in [4] for the case of homogeneous environment and chi-square family of fluctuating target models (Swerling I, II, III, IV).

During the last few years, mathematical methods for extraction of useful data about the behavior of observed targets by mathematical transformation of received signals are being widely used for design of new highly effective algorithms for processing of radar information. Such a mathematical approach is the Hough Transform (HT). The concept of using the HT for improving of target detection in white Gaussian noise is introduced by Carlson, Evans and Wilson in [1, 2, 3]. This approach is used by Carlson [3], for a highly fluctuating target – Swerling II type target model, and stationary homogeneous interference.

In our paper, we study the situation for a highly fluctuating target – Swerling II type target model detection in conditions in randomly arriving impulse interference. In [8, 9, 10] the detectability losses are calculated when compared to detectors in condition of pulse jamming and without pulse jamming.

The choice of the best effective pattern supposes a comparison towards a total model, for example the optimal detector [7, 11], or one towards another. In this paper we study the effectiveness of a CA Hough detector in randomly arriving impulse interference for P_D=0.5. We compare a CA Hough detector with a CA CFAR detector, using the approach presented in [11].

The losses (profits) of CA Hough detectors are calculated for different values of the false alarm probability, the number of observations in the reference window, the average interference-to-noise ratio (INR) and the probability for appearance of randomly arriving impulse interference with average length in the cells in range. The achieved results show that Hough transform is effective in conditions of decreasing randomly arriving impulse interference.

In conditions of randomly arriving impulse interference with INR=30 dB, probability of appearance 0.1 and false alarm probability $P_{fa}=10^{-4}$, the usage of CA CFAR causes losses in the Average Decision Threshold (ADT) of about 60 dB [6]. In the same conditions, adding binary integration with the rule M/N=16/16, the ADT diminishes to 15 dB [13]. If Hough transform is applied after the CA CFAR detector with optimal threshold T_M = 7, for values of the probability of appearance of randomly arriving impulse interference with average length in the range cells e_0=0, the ADT is reduced to 3.6 dB. Using an API CFAR detector instead of binary integration diminishes the ADT to 5-6 dB [13]. If Hough transform is applied after the API CFAR detector with an optimal threshold T_M = 13, the ADT is reduced to –2.5 dB [15].

2. Statistical analysis of a CA Hough detector

Using Carlson's approach [1, 2, 3], we obtain a new result for detection performance in Hough space, for target model of the Swerling II type in randomly arriving impulse interference described with the probability density function (pdf) of the reference window output [6]:

$$f(x_i)=\frac{1-e_0}{\lambda_0(1+s)}\exp\left(\frac{-x_i}{\lambda_0(1+s)}\right)+\frac{e_0}{\lambda_0(1+r_j+s)}\exp\left(\frac{-x_i}{\lambda_0(1+r_j+s)}\right), \quad (1)$$

where s is the per pulse average signal-to-noise ratio, λ_0 is the average power of the receiver noise, r_j is the average interference-to-noise ratio, e_0 is the probability for the appearance of pulse jamming with average length in the range cells.

The probability of detection for a CA CFAR detector for target of Swerling II case in randomly arriving impulse interference [6] is

$$P_{d_{\mathrm{SW2}}}=\sum_{i=1}^{N}C_N^i e_0^i(1-e_0)^{N-i}\left\{\frac{e_0}{\left(1+\frac{(1+r_j)T_{\mathrm{CA}}}{1+r_j+s}\right)^i\left(1+\frac{T_{\mathrm{CA}}}{1+r_j+s}\right)^{N-i}}+\frac{1-e_0}{\left(1+\frac{(1+r_j)T_{\mathrm{CA}}}{1+s}\right)^i\left(1+\frac{T_{\mathrm{CA}}}{1+s}\right)^{N-i}}\right\}, \quad (2)$$

where s is the signal-to-noise ratio, T_{CA} is the threshold constant and r_j is the average Interference-to-Noise Ratio (INR).

The probability of false alarm for a CA CFAR detector for Swerling II case in randomly arriving impulse interference [6] is obtained for the value of the signal-to-noise ratio $s = 0$:

$$P_{\mathrm{fa}}=\sum_{i=1}^{N}C_N^i e_0^i(1-e_0)^{N-i}\left\{\frac{e_0}{(1+T_{\mathrm{CA}})^i\left(1+\frac{T_{\mathrm{CA}}}{1+r_j}\right)^{N-i}}+\frac{1-e_0}{(1+(1+r_j)T_{\mathrm{CA}})(1+T_{\mathrm{CA}})^{N-i}}\right\}. \quad (3)$$

All indications for signal detection obtained from N range resolution cells and N_s scans are arranged in a matrix Ω of the size $N\times N_s$ in $(r - t)$-space. In this space a stationary or constant radar velocity target appears as a straight line, which consists of nonzero elements of Ω. Let us assume that Ω_{ij}^{nm} is set of such nonzero elements of Ω that constitute a straight line in $(r - t)$-space that is $(i, j)\in\Omega_{ij}^{nm}$. This line may be represented in Hough parameter space as a point (n, m). Denoting N_{nm} as the maximum size of , the cumulative false alarm probability for a cell is written according to [3]:

$$P_{\mathrm{fa}}^{nm}=\sum_{l=T_M}^{N_{nm}}\binom{N_{nm}}{l}(P_{\mathrm{fa}})(1-P_{\mathrm{fa}})^{N_{nm}-l}, \quad (4)$$

where T_M is a linear trajectory detection threshold in Hough parameter space.

The total false alarm probability in Hough parameter space is equal to one minus the probability that no false alarm occurred in any of the Hough cell. For independent Hough cells this probability is

$$P_{\mathrm{fa}}^{\mathrm{Hough}}=1-\prod_{N_{nm}=T_M}^{\max(N_{nm})}\left[1-P_{\mathrm{fa}}^{nm}\right]^{V(N_{nm})}, \quad (5)$$

where $\max(N_{nm})$ is the accessible Hough space maximum and $W(N_{nm})$ is the number of cells from Hough parameter space whose values are equal to N_{nm}.

The cumulative probability of target detection in Hough parameter space $P_{\mathrm{d}}^{\mathrm{Hough}}$ cannot be written in the form of a simple Bernoulli sum. As a target moves with respect to the radar, the SNR of the received signal changes depending on the distance to the target and the probability of a pulse $P_{\mathrm{d}}(j)$ changes as well. Then the probability $P_{\mathrm{d}}^{\mathrm{Hough}}$ can be calculated by Brunner's method. By means of Brunner's method we obtain a matrix of the size [20×20], the elements of which are the primitive probability of detection from the k-th time slice [3]. Using (2) we can get all the $P(i, j)$ needed to calculate $P_{\mathrm{d}}^{\mathrm{Hough}}$. For N_{s} scans we have:

$$P_{\mathrm{d}}^{\mathrm{Hough}} = \sum_{i=T_M}^{N_{\mathrm{s}}} P_{d_{\mathrm{SW2}}}(i, N_{\mathrm{s}}). \tag{6}$$

There are few cases in practice when radar is equipped with a CA Hough detector working in randomly arriving impulse interference. In such situations it would be desirable to know the CA Hough losses depending on the parameters of the randomly arriving impulse interference, for rating the behavior of the radar. For the calculation of CA Hough detector losses, we use the ratio between the two SNR, for a CA Hough detector and a CA CFAR detector, measured in dB, presented in the expression

$$\Delta = 10\log\frac{\mathrm{SNR}\big|_{\mathrm{CA\ Hough}}}{\mathrm{SNR}\big|_{\mathrm{CA\ CFAR}}} \text{ under } P_{\mathrm{fa}} = \mathrm{const},\quad P_{\mathrm{d}} = P_{\mathrm{d}}^{\mathrm{CA\,Hough}} = P_{\mathrm{d}}^{\mathrm{CA\,CFAR}} = 0.5. \tag{7}$$

The comparisons are made and towards a CA CFAR detector and a CA Hough detector in randomly arriving impulse interference.

3. Simulation results

In order to analyze the quality of the Hough detector we consider the following parameters, as in [3]: the search scan time is 6s; the range resolution is δR = 3 nmi (1 nmi = 1852 m); the beam range – time space has 128 range cells and 20 time slices, and the Hough space is 260 cells by 91 cells; the length of the reference windows in the CA CFAR detector is 16. We consider a straight line, incoming target with a speed of Mach 3 and 1 m^2 radar cross section. The calculation results are obtained for the following variants of the pulse jamming environment: e_0=(0; 0.01; 0.033; 0.066; 0.1), INR=30 dB. In the analysis, the SNR average value is calculated as $S=K/R^4$, where $K=0.16\times10^{10}$ is the generalized energy parameter of the radar and R is the distance to the target measured in nautical miles.

Carlson's approach, using Brunner's method for calculating the probability of detection in Hough parameter space, is further developed in order to maintain constant false alarm probability at the output of the Hough detector. The suitable scale factor is chosen iteratively. The influence of the threshold constant on the required signal-to-noise ratio is studied. The investigation is performed for probability of detection (P_{d}=0.5) and different values of the probability for the appearance of randomly arriving impulse interference with average length in the cells in range.

In order to achieve a constant value of the probability of false alarm (P_{fa}), the values of the threshold constants, which guarantee that, are determined for different numbers of observations in the reference window, an average interference-to-noise ratio (INR) and a probability for the appearance of randomly arriving impulse interference with average length in the cells in range. The threshold constant is obtained for each value of the false alarm probability $P_{fa} = 10^{-4}$, 10^{-6}, 10^{-8}, using (5).

The profits (losses) of the CA Hough detector in randomly arriving impulse interference are determined towards the CA CFAR detector, following the algorithm proposed in [11], for probability of detection 0.5.

Different values of the detection threshold in Hough parameter space – T_M are shown on Fig.1. The optimal value for this threshold is T_M=7 of 20 scans for values of the probability for the appearance of randomly arriving impulse interference with average length in the range cells ε_0=0. For ε_0=0.1, the optimal value for detection threshold in Hough parameter space is T_M is 18 of 20 scans.

The authors in [3] use approach proposed by Barton in [14] to determine the threshold in Hough parameter space. They assume T_M=7 as optimal threshold in the binary integration and apply it in Hough parameter space. In this paper, after iterative analysis, the optimal threshold in Hough parameter space is also determined to be T_M=7 for the value of the probability of appearance of randomly arriving impulse interference with average length in the range cells e_0=0.

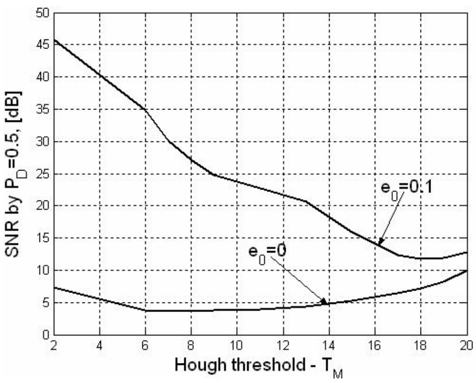

Fig. 1. Average detection threshold of a Hough detector compared to the optimal detection threshold in Hough parameter space

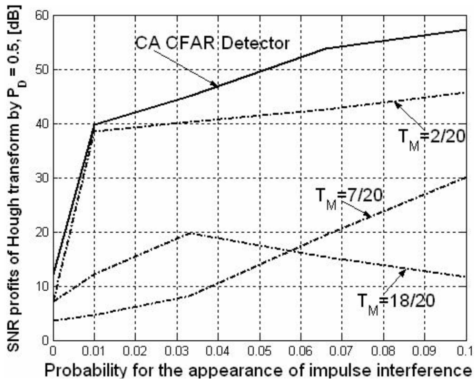

Fig. 2. Profits of a CA Hough detector (dashed line) with 20 scans, for T_M=2 and optimal values of the scans, for T_M=2 and optimal values of the e_0= 0.1, compared to a CA CFAR detector (solid line) for N=16

The profits of using a CA Hough detector, calculated for the threshold value T_M=2 and for optimal values of the detection threshold T_M=7, for e_0=0 and T_M=18, for e_0=0.1, compared to a CA CFAR detector, for the number of test resolution cells N=16 and the value for probability of false alarm $P_{fa} = 10^{-4}$, are shown on Fig. 2. The CA Hough detector with the optimal Hough rule T_M-out-of-N equal to 7/20 is better in cases of lower values of the probability for the appearance of impulse interference, up to 0.06. For higher values of the probability for the appearance of impulse interference, above 0.06, the using of the optimal Hough rule T_M-out-of-N_s=18/20 results in lower losses.

4. Conclusions

The experimental results reveal the influence of the interference on the detection process, when having constant false alarm rate in randomly arriving impulse interference. A method for losses estimation, which allows choosing optimal detector parameters, is developed. The estimates of the effectiveness of a CA Hough detector in randomly arriving impulse interference are received towards themselves, towards allow making a comparison towards other patterns researched from other authors.

Using Matlab, the average decision threshold of a CA Hough detector for a highly fluctuating target, Swerling II type target model detection in conditions of randomly arriving impulse interference, is calculated in accordance with the approach presented in [11]. The profits of a Hough detector are shown for different values of the probability of false alarm and for different numbers of observations in a reference window and average interference-to-noise ratio (INR). Using this approach, it is very easy to precisely determine the energy benefit when using a given detector. The achieved results show that Hough transform is effective in conditions of decreasing randomly arriving impulse interference.

The optimal threshold values for different input conditions are estimated. The value of the test resolution cell and the probability of false alarm over the mean detection threshold are studied. The profits of using a CA Hough detector with an optimal value of the detection threshold T_M=18, compared to a CA CFAR detector, are about 50 dB. Applying Hough transform reduces significantly the ADT compared to the detector considered in [6]. The results obtained in this paper could practically be used in radar and communication networks.

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