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A Theoretic-Experimental Model for Defining the Rate of the Air Flow through Capillary Channels*

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Abstract: *A model for defining the rate of the air flow through capillary channels is created by a theoretic-experimental approach. This model enables the formation of tables and nomograms for quick and easy determination of the flow rates in pneumatic lines with small cross sections.*

1. Introduction

The study and development of devices for generation, transformation and transfer of fluid information signals quite often imply the necessity to determine the features of the fluid flows through channels with small sections, which could be regarded as capillaries. The authors of the present paper have encountered a similar problem when designing and investigating a flow transducer of the air breathed out by a patient subjected to lungs examination [1, 2]. During the tests the requirement has appeared to use a flow measuring method, meeting the following requirements:

– a possibility for co-ordinated measuring of the flow rate in a set of points;
– a considerably large field of measurement;
– easy reading of the flow during its direct measurement;
– a possibility for easy and quick transformation in indirect measurement;
– sufficiently high accuracy of the measurements and results reproducibility;
– low cost of the realization.

It is known that the measuring of the flow rate of fluid running through pipelines or channels may be realized with the help of:

– an anemometer with an incandescing filament;

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– a volumetric flow meter;

– a fluid resistance.

Brief analysis of the possibilities of these methods observing the requirements above described indicates the following:

– the anemometer with an incandescing filament is a very precise instrument for measuring the flow rate, but it is not appropriate in cases when small flow values are measured in channels with small sections, and in addition it is a rather expensive method;

– the volumetric flow meter has a limited application area, at that it is calibrated for the measurement of fluid flows towards the atmosphere;

– the fluid resistors (FR) available on the market are of a comparatively high price and they cover a narrow range of the flow values measured, but the design of precise FR constructions is quite easy and at the same time rather cheap for realization.

Therefore, the prevailing part of the requirements above given will be satisfied, if FR be applied for flow measurement. In order to make this method technically accessible and financially efficient, it is necessary to design a sufficiently simple mathematical model that enables the quick and easy determination of the flow rate of fluids through capillary channels.

This paper presents in a concise form the results from the theoretic-experimental investigations connected with the formation of a model determining the air flow through capillary channels.

2. Theoretical investigations

The hydraulic resistance of a FR, determined according to the method of the electro-hydraulic analogy:

$$\overline{R} = \frac{\Delta P}{G}, \tag{1}$$

is a function of the pressure difference ΔP and the mass flow rate G. This function does not express any direct dependence of the resistance on FR geometric parameters. It is a formal application of Ohm's law for fluid movement in a FR.

Some authors [3, 4] recommend for gas flow formula (2), in which the mass flow and the gas pressure before and after FR are connected:

$$P_1^2 - P_2^2 = K_1 G + K_2 G^2, \tag{2}$$

where the constants K_1 (in (kg.s)/cm^4), and K_2 (in s^2/cm^4) are

$$K_1 = \frac{16 RT \mu L}{\pi n r^4}, \tag{3}$$

$$K_2 = \frac{17 RT}{6 g \pi^2 n^2 r^4}; \tag{4}$$

here R is the gas constant (in m/K); T – the room temperature (in K); μ – the gas viscosity (in (kg.s)/m^2); L – the capillary length (in mm); r – the capillary radius (in mm); n – the number of capillaries attached in parallel in a fluid line.

For air under normal conditions (T = 293 K; R = 29,27 m/K; μ = 0.185×10^{-5} (kg.s)/m^2; γ = 1.29 kg/m^3), equations (3) and (4) get the form (the measurements are the same as in (3) and (4)):

(5) $$K_1 = 0.81\frac{L}{nr^4},$$

(6) $$K_2 = 2.5\times10^6\frac{1}{n^2r^4}.$$

Equality (2), representing a quadratic equation with respect to the variable G, will possess only one solution if assumed that the mass flow of the air $G = 0$, when the pressure difference $\Delta P = P_1 - P_2 = 0$. This solution becomes

(7) $$G = \frac{-K_1 + \sqrt{K_1^2 - 4K_2\left(\Delta P^2 - 2P_2\Delta P\right)}}{2K_2}.$$

In case the geometric values r and L of the FR are known, the constants K_1 and K_2 can be determined; and changing the values of P_1 and ΔP with the help of a computer, some nomograms for quick and easy determination of the mass air flow through the capillaries could be obtained. In the cases when the capillary channels are without a circular section as most of the cases in fluidics, and also in the case of the authors' design of a flow transducer [1, 2], the solution of this problem is connected with high inaccuracy due to the fact that the parameter r is approximately defined, but included in equations (5) and (6) raised to forth power.

This has imposed the development of such a method for the determination of the air flow rate through capillaries, in which the defining of constants K_1 and K_2 does not require the use of parameter r. For this purpose expression (7) is offered as a more convenient and exact mathematical formula, representing the function G in MacLaurin's series, ending at the third term:

(8) $$G = \frac{2}{K_1}P_1\Delta P - \frac{1}{K_1}\Delta P^2 - \frac{4K_2}{K_1^3}P_1^2\Delta P^2 + \frac{16K_2^2}{K_1^5}P_1^3\Delta P^3 + \frac{4K_2}{K_1^3}P_1\Delta P^3.$$

This formula will be as follows in a general form:

(9) $$G = ax_1 + bx_2 + cx_3 + dx_4 + ex_5.$$

If a certain number of measurements on the FR studied are made, altering P_1, respectively ΔP and taking into account the corresponding values of the flow G, with the help of a computer we could interpolate the experimental points up to a curve of the type of equation (9) and in this way determine the values of constants K_1 and K_2. The following variables have been accepted in these investigations (refer to equations (8) and (9)):

(10) $$x_1 = P_1\Delta P;\ x_2 = \Delta P^2;\ x_3 = P_1^2\Delta P^2;\ x_4 = P_1^3\Delta P^3;\ x_5 = P_1\Delta P^3.$$

In order to determine these variables, the values of P_1, ΔP and G are entered in the regression program. After the coefficients K_1 and K_2 are determined, it is easy to obtain on a computer a table or a nomogram, in which the corresponding value of G is specified for given P_1 and ΔP values.

3. Experimental investigations

Fig. 1 shows the principal diagram of a FR composed of capillaries of stainless steel, and Fig. 2 – the connection scheme of a FR in the experimental study; Table 1 gives the data for the six FR investigated.

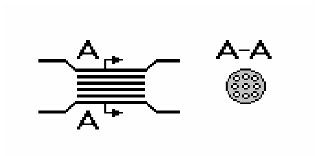

Fig. 1. The principal diagram of a FR composed of capillaries of stainless steel

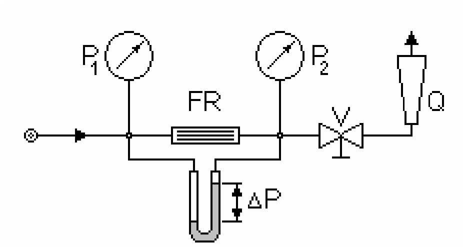

Fig. 2. The connection scheme of a FR in the experimental study

Table 1. Data for the fluid resistors studied

FR No	Number of the capillaries in FR	Capillary diameter, mm	Capillary length, mm
1	10	0.2	50
2	15	0.2	50
3	20	0.2	50
4	10	0.4	50
5	15	0.4	50
6	20	0.4	50

When implementing the investigations of the fluid resistors, the pressure difference ΔP is changed for each defined pressure P_1 of the supply air with the help of the needle valve (Fig. 2) and the flow rate is read by a rotameter.

Figs. 3 and 4 show the dependences of the flow rate on the pressure difference and supply, obtained after the investigations for two of the fluid resistors studied – the first one and the last one.

Using the values for ΔP and P_1 from the experiments for each FR considered, the coefficients of equation (9) are computed and presented in Table 2.

Table 2. Computed coefficients of equation (9) for the FR studied

No	*a*	*b*	*c*	*d*	*e*	Constants
1	5.3773×10^{-5}	-4.1231×10^{-5}	-1.6536×10^{-11}	0	2.3902×10^{-11}	–0.5713
2	9.0418×10^{-5}	-3.4413×10^{-5}	-5.1443×10^{-11}	2.1380×10^{-17}	0	–1.4739
3	1.2711×10^{-4}	-5.0642×10^{-5}	-7.9794×10^{-11}	4.4042×10^{-17}	0	–1.4870
4	2.4735×10^{-4}	-1.6970×10^{-4}	-1.6774×10^{-10}	4.0367×10^{-17}	1.3641×10^{-10}	5.9393
5	5.4828×10^{-4}	-2.9560×10^{-4}	-4.4805×10^{-10}	1.3991×10^{-16}	2.3925×10^{-10}	11.3067
6	5.9221×10^{-4}	-2.6185×10^{-4}	-5.5123×10^{-10}	2.1191×10^{-16}	2.0991×10^{-10}	11.3977

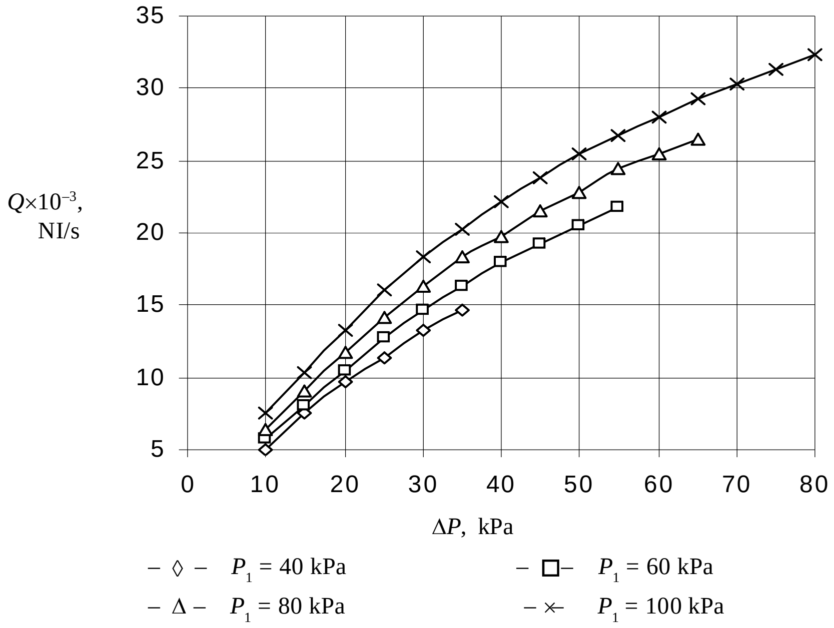

Fig. 3. Dependence of the flow rate on the pressure difference and supply for fluid resistor No 1

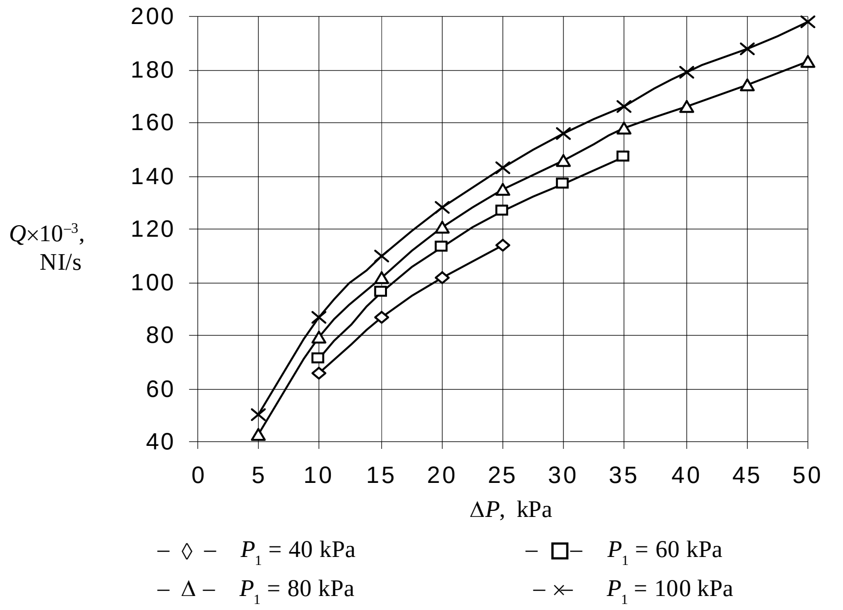

Fig. 4. Dependence of the flow rate on the pressure difference and supply for fluid resistor No 6

Figs. 5 and 6 could compare the results from the experimentally measured and the theoretically computed according to formula (9) values of the volumetric rate of the air flow through FR1 and FR6.

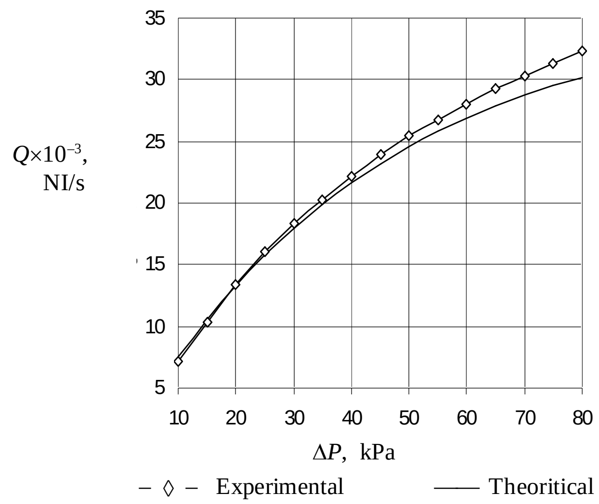

Fig. 5. Comparison between the experimental and the computed according to formula (9) values of the volumetric rate of the air flow through FR1 at input pressure P_1 = 100 kPa

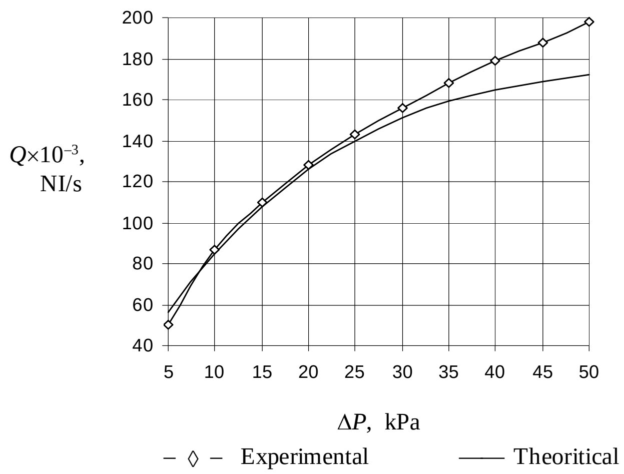

Fig. 6. Comparison between the experimental and the computed according to formula (9) values of the volumetric rate of the air flow through FR6 at input pressure P_1 = 100 kPa

4. Conclusion

The comparison of the experimentally obtained and the computed in accordance with formula (9) values of the volumetric rate of the air flow through fluid resistors made of a different number of capillaries shows good concurrence for a large range of the values of the supply pressure P_1 and the pressure difference ΔP. This is a good reason to affirm that the method applied for the determination of the flow rate of the air through fluid resistors, containing capillary channels, could be used in the design and study of fluid channels with small cross sections. The method proposed has the following advantages:

– only the values of the air pressure before and after the fluid resistor are necessary;

– quick and easy calculation of the flow rate values for different pressures by a computer is possible, and the formation of tables and nomograms as well;

– the use of the tables and nomograms obtained is very convenient and it shortens the time for the study accomplishment and the design of fluid devices.

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