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A Decision Support Method for Investment Preference Evaluation*

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Abstract: *The method enables public companies classification into three Investment Preference categories Risky, Satisfactory and Excellent. The IP concept is introduced as a qualitative criterion assumed to estimate the probability of companies' belonging to these categories. The method provides the necessary conditions for linear discriminant analysis application, performed by STATITICA™ 5.5.*

Keywords: *bankruptcy prediction, linear discriminant analysis, investment preference, incomplete data, decision making.*

1. Introduction

The techniques based on fundamental and technical analysis are widely applied in investment decision making on capital markets. The technical analysis on Bulgarian market, although already applied turned out not to be quite effective as it is not liquid enough, the market capitalization is low and free floats of listed companies are few. The fundamental analysis is preferred technique, but it is costly and hardly affordable for individual investors.

The incomplete public companies' empirical data available on Bulgarian Stock Exchange (BSE) – Sofia provoked the development of decision support method for Investment Preference (IP) evaluation. The homogeneity of that data has not allowed a clear discrimination between companies with positive and negative financial characteristics i.e. a sufficient information about bankrupted companies is not available. Such information is usually a prerequisite for successive application of the known data processing techniques, such as discriminant analysis, artificial neural networks or fuzzy sets as it is described in [2, 4, 5, 11].

As an attempt to propose a solution to part of the difficulties mentioned above, this paper presents a decision support method for qualitative evaluation of public

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companies prospects currently listed on BSE – Sofia. As an input for basic companies' classification, the use of seven popular Bankruptcy Prediction Models (BPM) is suggested.

The introduced method enables a categorization of public companies according to their financial stability. The IP concept herein proposed is a qualitative criterion assumed to estimate the probability of companies' belonging to the three IP categories Risky, Satisfactory and Excellent.

By analogy with the Decision Making Theory, the IP may be regarded as a proxy-variable.

Its introduction is twofold. Firstly, it allows the acquisition of the IP as something like a convolution of the objective estimate of BPMs into a grouping variable in linear Discriminant Analysis (DA). Therefore, the uncertainty of BPMs estimates under conditions of incomplete data about bankruptcy is substituted by the relative certainty of the expert decision. Secondly, it allows using DA for objective correction of the expert classification included IP.

The IP concept should be interpreted as a reference point in the initial stage of investment decision-making process. It helps to find out at which company the interest could be raised before further analysis performance.

The characteristics, scopes and verification of applicability of the seven BPMs used over a sample of Bulgarian companies were discussed in [10]. The seven selected BPMs are:

- Altman's Z-score [1]. It uses five variables and a two-level system for estimations of the company's status for prediction of bankruptcy. The two levels are "Bankrupt" and "No Bankrupt". Herein Z68 stands for this model.
- Z-score, revised [2]. It uses five variables and three-level system for estimation of the company's status for prediction of bankruptcy. The three levels are "Bankrupt", "No Bankrupt", and "Uncertainty" (U). Herein Z83 stands for this model.
- The "Mexican" Z-score [3]. It uses four variables and a two-level system for estimations of the company's status for prediction of bankruptcy. The two levels are "Bankrupt"(B) and "No Bankrupt" (N). Herein Z95 stands for this model.
- Fulmer's model [7]. It uses nine variables and a two-level system for estimations of the company's status for prediction of bankruptcy. The two levels are "Bankrupt" and "No Bankrupt". Herein F84 stands for this model.
- Springate's model [12]. It uses four variables and a two-level system for estimations of the company's status for prediction of bankruptcy. The two levels are "Bankrupt" and "No Bankrupt". Herein S78 stands for this model.
- R-Model [6]. It uses four variables and a five-level system for estimations of the company's status for prediction of bankruptcy. The five levels are "Bankrupt", "No Bankrupt", "Low Possibility", "Mid Possibility", and "High Possibility". Herein R99 stands for this model.
- Voronov–Maximov model as described by [9]. It uses four variables and three-level system for estimation of the company's status for prediction of bankruptcy. The three levels are "Bankrupt", "No Bankrupt" and "Mid Possibility" Herein VM stands for this model.

The objectives in this paper, based on a part of the existing BPMs are:

- Proposal of method for IP classification of public companies under conditions of incomplete data.
- Application of DA for verification or actualizing of classification.

• Elimination of subjectivity in evaluation of IP categories Excellent, Satisfactory and Risky;

• Specification of IP categories' classification functions.

To attain the objectives a linear DA was applied. An algorithm for specification of the ranges of IP categories Excellent, Satisfactory and Risky was suggested. The experimental sample is designed from three years (2001, 2002 and 2003) annual accounting data about thirty companies listed on BSE – Sofia. The companies in the sample are assumed to be stable. This assumption required introduction of only non negative IP categories: Excellent, Satisfactory and Risky.

2. Investment preference method application

The decision support method herein proposed is seven staged.

1. Scoring of the companies through the seven BPMs.
2. Determination of the statuses of each company for each of three years under consideration.
3. Transformation of BPMs estimates into scale of three aggregation groups (AGs): B, U and N.
4. Expert classification of companies' IP and determination of the three IP categories' intervals.
5. Objective determination of the three IP categories' intervals, eliminating the possible subjectivity of expert specification.
6. Updating of the DA classification function.
7. IP Ranges specification.

Stage 1. Scoring. The scoring of the companies is performed by estimation of financial ratios required by each BPM and the corresponding model's linear classification function. The data processing in this and in the next stage is performed by specially designed software module and database supported by MS ACCESS. The report of estimated scores of company A3 is set out in Table 1. The real names of the tested companies are replaced by alias.

Stage 2. Statuses determination. According to the estimated company scores and the intervals of the cut-off point (depending of the BPM) the statuses of each company for each of three years under consideration are determined. The applied BPMs scores' intervals and corresponding statuses are presented in Table 2. The corresponding statuses for company A3 are presented in Table 3.

Stage 3. AGs formation. To obtain a minimal reasonable diversity of BPMs outputs (scores and statuses) a three (at least more than two) level scale was accepted in [10]. This three level scale is represented by the AGs (Agregation Groups): B, U

Table 1. The BPMs' scores of A3 company

BPM	Scores for 2001	Scores for 2002	Scores for 2003
Z83	3.46	2.84	2.33
Z68	3.86	1.64	1.66
Z95	8.71	7.31	7.55
F84	4.86	4.43	3.72
S78	1.23	1.03	0.95
R99	1.89	1.57	1.54
VM	1.35	1.07	1.26

Table 2. The BPMs scores intervals, statuses and aggregation groups correspondence

BPM	Scores Intervals	Status	Aggregation Group
Z68	Score ≤ 2.678	Bankrupt	B
Z68	Score > 2.678	Non Bankrupt	N
Z83	Score ≤ 1.23	Bankrupt	B
Z83	1.23 < Score < 2.9	Uncertain	U
Z83	Score ≥ 2.9	Non Bankrupt	N
Z95	Score ≤ 1.1	Bankrupt	B
Z95	Score > 1.1	Non Bankrupt	N
F84	Score ≤ 0	Bankrupt	B
F84	Score > 0	Non Bankrupt	N
R99	Score ≤ 0	Bankrupt (90-100%)	B
R99	0 < Score < 0.18	Hp (60-80% probability)	B
R99	0.18 ≤ Score < 0.32	Mp (35-50% probability)	U
R99	0.32 ≤ Score < 0.42	Lp (15-20% probability)	N
R99	Score ≥ 0.42	N (<10% probability)	N
S78	Score ≤ 0.862	Bankrupt	B
S78	Score > 0.862	Non Bankrupt	N
VM	Score ≤ 0.38	Bankrupt	B
VM	0.38 < Score < 0.92	Mid Possibility	U
VM	Score ≥ 0.93	Non Bankrupt	N

Table 3. The corresponding annual BPMs' statuses of company A3

BPM	Annual status for 2001	Annual status for 2002	Annual status for 2003
Z83	N	U	U
Z68	N	B	B
Z95	N	N	N
F84	N	N	N
S78	N	N	N
R99	N	N	N
VM	N	N	N

and N. Each model assigns a definite annual status to a given company (see Table 2, third column). The correspondence between AGs and the BPMs statuses is based on the following subjective assumptions:

- The two-level models (Z68, Z95, F84 and S78) exert two statuses estimates "Bankrupt" and "Non Bankrupt" which in this work is accepted to coincide with respective AG B and N.
- The three-level models (see Z83 and VM) exert three output statuses estimates "Bankrupt", "Uncertain" or "Mid Possibility" and "Non Bankrupt" which in this work is accepted to coincide with respective AGs B, U and N.
- The five-level model R99 exert five output statuses estimates "Bankrupt" (90-100% probability of bankruptcy), "Hp" (60-80%), "Mp" (35-50%), "Lp" (15-20%) and "Non Bankrupt" (< 10%). In this work statuses "Bankrupt" and "Hp" is accepted to correspond to AG B with probability of bankruptcy > 50%. The status Mp is accepted to coincide with AG U. The statuses "Lp" and "Non Bankrupt" are accepted to correspond to the AG N.

This aggregation is based on the subjective expert judgment. A larger sample of companies would possibly lead to change the limits of percentage of bankruptcy probabilities which define AGs B, U and N. Each company is tested thought each of the BPMs. Each of the statuses' B, U or N is herein accepted to bring one point to the total sum of point in AGs B, U or N. The points from each model are added to the corresponding AG. Therefore the maximal sum of points L in AGs B, U and N equals number of years *x* number of models (in our case $L = 3 \times 7 = 21$). Shortly, the correspondence between the three AGs and the output of the models under consideration is as follows:

- Model Z68 has outputs corresponding to statuses B or N. In case of company status B, the record in the testing sample is filled in with 1 point in AG B, 0 point in AG U and 0 point in AG N;
- Model Z95 has outputs corresponding to statuses B or N. In case of company status N, the record in the testing sample is filled in with 0 point in AG B, 0 point in AG U and 1 point in AG N;
- Model F84 has outputs corresponding to statuses B or N. In case of company status N, the record in the testing sample is filled in with 0 point in AG B, 0 point in AG U and 1 point in AG N;
- Model S78 is similar to models above;
- Model Z83 has outputs corresponding to statuses B, U or N. In case of company status N, the record in the testing sample is filled in with 0 point in AG B, 0 point in AG U and 1 point in AG N;
- Model VM has outputs corresponding to statuses B, Mp or N. In case of company status Mp, the record in the testing sample is filled in with 0 point in AG B, 1 point in AG U and 0 point in AG N;
- Model R99 has outputs corresponding to statuses B, Hp, Mp, Lp or N. The statuses B and Hp are considered as AG B, Mp is considered as AG U and Lp and N as AG N. In case of company status Lp, the record in the testing sample is filled in with 0 point in AG B, 0 point in AG U and 1 point in AG N.

For company A3 the points assigned to each AG are presented in Table 4.

The testing sample for further analysis is also acquired in this stage. This testing sample contains thirty records (companies), one for each company with its Name, status B, status U, and status N.

Stage 4. Expert classification. In our previous research [10] it was shown that BPMs give contradictory assessments for one and the same companies' status. The contradictions concern both: the seven models' statuses for given company and the annual single model status for different years. This situation suggested the idea to use

Table 4. The points assigned to each AG for company A3

BPMs	Status B				Status U				Status N			
	2001	2002	2003	Σ	2001	2002	2003	Σ	2001	2002	2003	Σ
Z83	0	0	0	0	0	1	1	2	1	0	0	1
Z68	0	1	1	2	0	0	0	0	1	0	0	1
Z95	0	0	0	0	0	0	0	0	1	1	1	3
F84	0	0	0	0	0	0	0	0	1	1	1	3
S78	0	0	0	0	0	0	0	0	1	1	1	3
R99	0	0	0	0	0	0	0	0	1	1	1	3
VM	0	0	0	0	0	0	0	0	1	1	1	3
Σ	AG B = 2				AG U = 2				AG N = 17			

all seven annual BPMs' estimates for each of the three years as an expert starting point for initial companies' classification. In addition, the BPMs' estimates (expressed by number of points in each AGs B, U and N) concern currently listed public companies. None of them are being under procedure or bankrupt. Thus, it was accepted to introduce the concept of IP. It is described by non negative categories Excellent, Satisfactory and Risky. The AGs B, U and N do not coincide with the IP categories Excellent, Satisfactory and Risky.

Initially, the IP categories express which of the estimated statuses in AGs prevails. So, if for a given company the points in AG B prevail it could be assumed as a Risky company. Analogously, if for a given company the points in AG U prevail it could be assumed as a Satisfactory company. Finally, if for a given company the points in AG N prevail it could be assumed as an Excellent company. Consider the situation for company A3: B = 2, U = 2 and N = 17 (see Table 4). According to the assumption above the initial IP classification of such company could be Excellent.

According those assumptions, the expert classification of the companies' IP and specification of IP categories intervals (number of points in each AG) is acquired. The results are presented in Table 5.

Table 5. An Investment Preference Expert Classification of the Sample

Alias	B	N	U	Investment preference
A1	0	21	0	Excellent
A2	1	19	1	Excellent
A3	2	17	1	Excellent
A4	1	18	2	Excellent
A5	4	17	0	Excellent
A6	4	17	0	Excellent
A7	4	16	1	Excellent
A8	3	16	2	Excellent
A9	5	16	0	Excellent
A10	6	14	1	Excellent
A11	3	13	4	Excellent
A12	6	13	2	Excellent
A13	6	13	4	Excellent
A14	4	13	4	Excellent
A15	7	12	2	Satisfactory
A16	5	12	4	Satisfactory
A17	6	12	3	Satisfactory
A18	6	12	3	Satisfactory
A19	6	12	3	Satisfactory
A20	5	11	5	Satisfactory
A21	4	11	5	Satisfactory
A22	7	9	5	Satisfactory
A23	6	9	6	Satisfactory
A24	6	9	6	Satisfactory
A25	6	8	7	Satisfactory
A26	6	8	7	Satisfactory
A27	10	8	3	Satisfactory
A28	13	5	3	Risky
A29	14	5	2	Risky
A30	16	5	0	Risky

Stage 5. Objective determination of IP categories' intervals. In order to limit subjectivity in specification and interpretation of IP categories, DA [8] was applied over the testing sample. The software STATISTICA™ 5.5 was used for DA performance. This allows elimination of subjectivity in further classification by determination of the posterior probabilities of cases' IP classification. AGs B, U and N were used to design the DA grouping variable. Only two of them are independent, therefore, B and N were chosen as more clearly interpretable. The posterior probabilities of IP classification as they were determined by DA are presented in Table 6.

Stage 6. Classification functions updating. The updating of the DA function was performed by adding of ten new cases and taking into account the incorrect classifications of the testing sample. The objective is to improve the posterior probabilities of classification and to obtain more precise discrimination between categories Excellent, Satisfactory and Risky The procedure includes the following steps:

1. The cases (companies) classified incorrectly are referred to the group to which they are classified with maximal posterior probability (pp); if it is satisfactory large, for example pp > 80%, this case may be used for updating of the DA function.

Table 6. A Corrected Investment Preference Classification of the Sample. Posterior Probabilities (ranking system_n.sta). Incorrect classifications are marked with *

No	Alias	Observed	Excellent	Satisfactory	Risky
		Classification	p=0.30000	p=0.60000	p=0.10000
1	A1	Excellent	1.0000	0.0000	0.0000
2	A2	Excellent	0.9998	0.0002	0.0000
3	A3	Excellent	0.9984	0.0016	0.0000
4	A4	Excellent	0.9984	0.0016	0.0000
5	A5	Excellent	0.9889	0.0111	0.0000
6	A6	Excellent	0.9889	0.0111	0.0000
7	A7	Excellent	0.9288	0.0712	0.0000
8	A8	Excellent	0.9288	0.0712	0.0000
9	A9	Excellent	0.9288	0.0712	0.0000
10	A10	Satisfactory	0.2173	0.7827	0.0000
11	A11	Satisfactory	0.0389	0.9611	0.0000
12	A12	Satisfactory	0.0389	0.9611	0.0000
13	A13	Satisfactory	0.0389	0.9611	0.0000
14	A14	Satisfactory	0.0389	0.9611	0.0000
15	A15	Satisfactory	0.0059	0.9940	0.0001
16	A16	Satisfactory	0.0059	0.9940	0.0001
17	A17	Satisfactory	0.0059	0.9940	0.0001
18	A18	Satisfactory	0.0059	0.9940	0.0001
19	A19	Satisfactory	0.0059	0.9940	0.0001
20	A20	Satisfactory	0.0009	0.9983	0.0008
21	A21	Satisfactory	0.0009	0.9983	0.0008
22	A22	Satisfactory	0.0000	0.9717	0.0283
23	A23	Satisfactory	0.0000	0.9717	0.0283
24	A24	Satisfactory	0.0000	0.9717	0.0283
25	A25	Satisfactory	0.0000	0.8509	0.1491
26	A26	Satisfactory	0.0000	0.8509	0.1491
27	A27	Satisfactory	0.0000	0.8509	0.1491
28	A28	Risky	0.0000	0.0256	0.9744
29	A29	Risky	0.0000	0.0256	0.9744
30	A30	Risky	0.0000	0.0256	0.9744

2. A new case is added to the testing sample; DA is performed without including it into the grouping variable.

3. Assigning to the new case an IP category Excellent, Satisfactory or Risky according to the highest posterior probability acquired (see also Stage 1).

4. In case of correct classification, the new case (see also Stage 1) is entered into the grouping variable and the DA function is updated.

5. In case that the new case, although correctly classified, worsens the posterior probabilities of the other cases, it should be rejected.

6. The DA function may be regarded as a reliable one if a large enough number of new cases does not change it significantly, i.e. if the DA grouping variable remains unchanged after entering of the new cases.

7. The steps are repeated for all new cases.

Schematically the procedure is presented in Fig. 1.

This procedure acquires:

• either maximum match between an expert judgment and the actualized discriminant function – the classification process is convergent and corrected expert judgment is verified;

• or rejecting of the expert judgment in case of discrepancy between the discriminant function and the expert judgment.

Each new added case is used ones for classification and second for actualizing of the discriminant function.

Stage 7. IP ranges specification. For the limits of IP category ranges specification a simple algorithm is proposed. It allows minimizing or eliminating the DA classification errors.

Let the whole interval L of the Investment Preference estimates is divided into K elementary subintervals Δ, m is the number of intervals Δ included in the category Risky (length r_1), n – the number of Δ intervals within the category Satisfactory (length r_2) and p – the number of Δ intervals in the category Excellent (length r_3), and

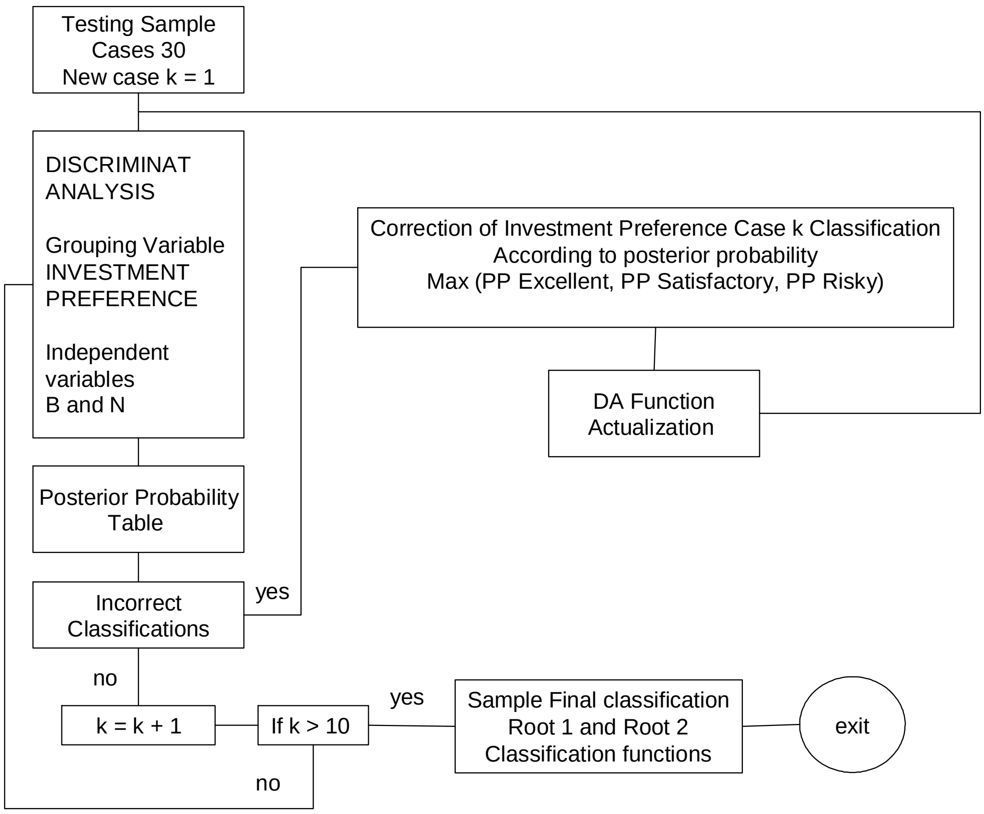

Fig. 1. Classification functions updating procedure

(1) $$\begin{aligned} &m + n + p = K, \\ &r_1 = m\Delta,\ r_2 = n\Delta,\ r_3 = p\Delta, \\ &L = K\Delta \to K = \frac{L}{\Delta}. \end{aligned}$$

The number T of all the combinations C (m, n, p) of the three IP intervals at fixed L and Δ is given by:

$$T = mnp.$$

Taking into account (1) the expression for T becomes

(2) $$T = mn\left(\frac{L}{\Delta} - m - n\right),$$

where m and n values are limited by the inequalities:

(3) $$1 \le m < K, 1 \le n < K, \quad m + n < K.$$

The total IP range (21 points in our case) should be divided into three subintervals, corresponding to Excellent, Satisfactory and Risky categories. Assume this division is performed by expert decision. It would inevitably introduce a subjective element in the decision for the limits of the intervals which define the categories Excellent, Satisfactory and Risky. To eliminate this subjective element the following procedure is further proposed:

a) calculation of the possible number of combinations $C(m, n, p)$ of subintervals within the total IP range;

b) for a given combination of subintervals each of the cases (companies) is referred as belonging to one of the categories Excellent, Satisfactory and Risky. Based on this belonging the DA grouping variable is designed and then DA is performed for each of the combinations. Thus the posterior probabilities assigned to the companies into consideration (30 in our case) after the DA classification are obtained;

c) choice is done of this/these combinations $C(m, n, p)$ of intervals which lead to the minimal classification error in order this/these combinations to be used to define the final structure of the DA grouping variable.

This procedure includes consequent and exhausted DA application for each of the combinations $C(m, n, p)$ defined by equations (2) and (3). Schematically it is presented in Fig. 2.

Thus, the expert decision about the limits that define the categories Excellent, Satisfactory and Risky is substituted by an objective classification procedure. The described algorithm may be additionally carried out for different combinations of L and Δ values. It can be easily generalized for arbitrary number of IP ranges.

DA is preformed (at fixed $C(m, n, p)$) as many times as the number of combinations $C(m, n, p)$ is. The criteria for comparison of the resulting classifications $C(m, n, p)$ are: the number of classification errors and the accuracy of the posterior probabilities. As a best combination $C(m, n, p)$ is accepted the one with the less number of incorrectly classified cases and/or with highest accuracy of the posterior probabilities (at least > 50%). These criteria ensure that such a combination provides best classification capabilities of the DA function. The selected combination $C(m, n, p)$ may be subsequently subjected to a learning procedure and updating of the DA function.

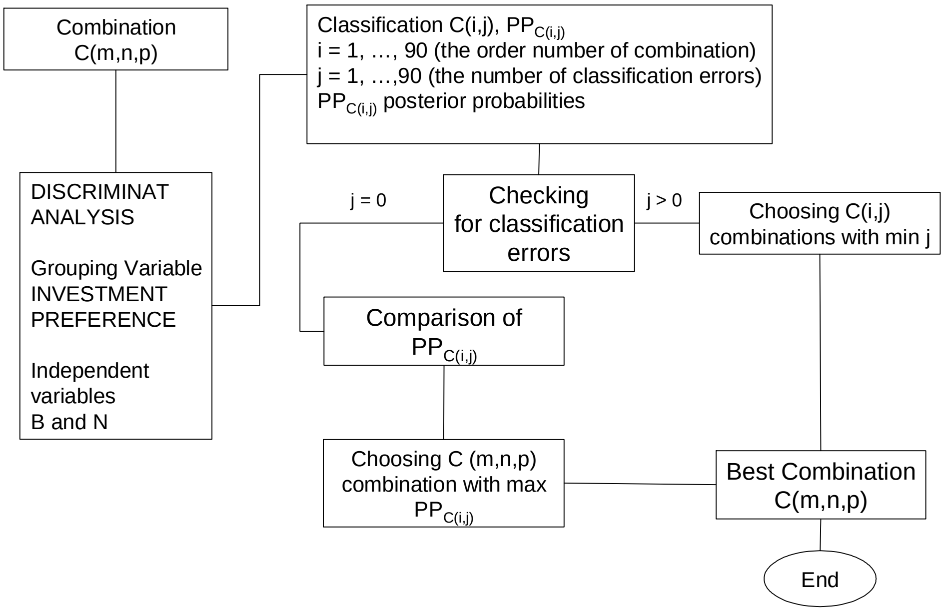

Fig. 2. IP ranges specification

3. Results and analysis

The performance of presented method is demonstrated on testing sample for 30 Bulgarian companies.

By the initial expert classification, acquired in stage 4, three companies are classified as Risky, thirteen companies are classified as Satisfactory and fourteen companies are classified as Excellent.

The application of DA on the above expert classification showed five misclassified cases (companies A10, A11, A12, A13, and A14). Their IP has changed form category Excellent to Satisfactory with probability rate of 78.11% for A10 and by 96.11% for the rest. The appropriate shift of the range limits of IP three levels are: for Satisfactory form 6-12 points to 6-14 points and for Excellent from 13-21 points to 15-21 points. The final corrected companies' classification by IP is shown in Table 5.

As it is known, the application of DA requires normality of the distribution of the dependent variables. The testing for normality of the variables B and N by means of the K-S test showed that N is approximately normally distributed at $p > 0.20$, while the hypothesis for normality of the B distribution was rejected at $p < 0.05$. Any way this result is possibly due to: a) the small sample size, b) the inadequate data about bankrupted Bulgarian companies or c) the strongly different estimates of the companies through the seven models used in this work. Anyway, taking into account that DA is robust to significant deviations of the variables from normal distribution we applied DA to the data available.

For objective determination of IP categories' interval include application of suggested algorithm. Therefore, the length L of the total interval of IP estimates, based on aggregation group N, is 21. The maximum length of each category Excellent, Satisfactory and Risky is $r_1 = r_2 = r_3 = 7$, then the number T of combinations $C(m, n, p)$ is 7^3=343. Herein it was accepted = 1. All combinations with length $r_1 = r_2 = 1$ and $r_1 = r_2 = 2$ were excluded, because such short intervals result in very large r_3 interval which is not consistent with the data. The repeating combinations (one and the same m, n and p values) were also excluded. In result, only 90 combinations were subjected to DA.

Thirty six of these 90 combinations turned out to lead to zero incorrect classifications. So the second criterion – the value of the posterior probabilities was used to select the best combination. The highest posterior probabilities were observed in combination $C(11, 3, 7)$. The range in the aggregation group "N" for Investment Preference Risky is from 1 to 11 points, for group Satisfactory: from 12 to 14 points and for group Excellent: from 15 to 21 points. The posterior probabilities are set out in Table 6 (the first 30 rows).

The DA function updating was based on the combination $C(11, 3, 7)$. The ten new cases were added one by one, the posterior probabilities were stored in order to trace the accuracy changes. The results are presented in Table 6 (form row 31 to 40). The analysis of the development of posterior probabilities showed that there is not a gradual increase in the classification accuracy with the increase of the number of added cases. The marked cells (in italic font) indicate the highest result achieved for a given case (company) in the course of adding of additional cases to the initial sample of 30 cases. The highest posterior probabilities by categories of the grouping variable Investment Preference were not observed within one and the same sample. In average: For the category Excellent, the highest posterior probabilities were observed for the sample of size 33, for category Satisfactory – the sample of size 38 and for category Risky – the sample of size 39 (Table 7).

Actually, the mean accuracy of 97.5% (averaged over the whole Table 7) is very high. By categories for Excellent: 98.32% (averaged over the Excellent cases in the whole Table 7), for Satisfactory (averaged over the Satisfactory cases in the whole Table 7) 97.695%, and for Risky (averaged over the Risky cases in the whole Table 7) 96.46%. A larger amount of cases would possibly lead to a saturation effect so that the best results for all categories of the grouping variable would be observed within the sample of largest size (the sample of 40 cases in our case).

The DA canonical scores (roots), which are weighted sums of B and N provide the possibility to visualize the IP separation of the companies (Fig. 3) in the plane (Root1, Root2):

$$\text{Root1} = 16.53093 - 0.45288\text{B} - 1.09646\text{N},$$

$$\text{Root2} = 6.907906 - 0.47788\text{B} - 0.32414\text{N}.$$

The companies (presented by alias) (A5, A6, B8), (A10, B5), (A12, A13), (A17, A18), (A23, A24), (A25, A26) are represented by coinciding points, because their scores B and N are identical. It seems that categories Excellent, Satisfactory and Risky are mostly discriminated by root1 than by Root2.

Another DA option is available through the DA classification functions. They can be used to determine to which category each case most likely belongs. Each function allows us to compute *classification scores Y* for each case for each category, by applying the formulas:

Table 7. Posterior Probabilities (%) of the selected testing samples

Alias	IP category	Case				
		30	33	38	39	40
A1	Excellent	100.00%	*100.00%*	100.00%	100.00%	100.00%
A2	Excellent	100.00%	*100.00%*	100.00%	100.00%	100.00%
A3	Excellent	99.99%	*99.99%*	99.99%	99.98%	99.97%
A4	Excellent	99.93%	*99.95%*	99.91%	99.84%	99.77%
A5	Excellent	99.98%	99.98%	99.97%	99.97%	99.95%
A6	Excellent	99.98%	99.98%	99.97%	99.97%	99.95%
A7	Excellent	98.15%	98.13%	97.05%	96.32%	95.20%
A8	Excellent	87.89%	88.12%	81.82%	76.37%	71.74%
A9	Excellent	99.74%	99.73%	99.59%	99.53%	99.36%
A10	Satisfactory	77.77%	84.22%	87.89%	88.27%	89.67%
A11	Satisfactory	98.40%	98.98%	*99.25%*	99.14%	98.89%
A12	Satisfactory	99.70%	99.83%	99.87%	99.88%	99.88%
A13	Satisfactory	99.70%	99.83%	99.87%	99.88%	99.88%
A14	Satisfactory	99.70%	99.77%	*99.86%*	99.84%	99.78%
A15	Satisfactory	99.87%	99.85%	*99.91%*	99.91%	99.87%
A16	Satisfactory	96.92%	97.26%	*97.76%*	97.52%	96.97%
A17	Satisfactory	99.41%	99.38%	*99.57%*	99.55%	99.38%
A18	Satisfactory	99.41%	99.38%	*99.57%*	99.55%	99.38%
A19	Satisfactory	99.41%	99.38%	*99.57%*	99.55%	99.38%
A20	Risky	64.60%	60.65%	66.11%	*70.29%*	69.01%
A21	Risky	90.82%	87.36%	91.18%	*93.02%*	91.84%
A22	Risky	99.51%	99.57%	99.80%	*99.85%*	99.77%
A23	Risky	99.91%	99.90%	99.96%	*99.97%*	99.96%
A24	Risky	99.91%	99.90%	99.96%	*99.97%*	99.96%
A25	Risky	100.00%	100.00%	100.00%	*100.00%*	100.00%
A26	Risky	100.00%	100.00%	100.00%	*100.00%*	100.00%
A27	Risky	98.66%	99.29%	99.66%	*99.70%*	99.59%
A28	Risky	100.00%	100.00%	100.00%	*100.00%*	100.00%
A29	Risky	99.99%	100.00%	100.00%	*100.00%*	100.00%
A30	Risky	99.82%	99.96%	99.99%	*99.99%*	99.98%
B1	Excellent		100.00%	100.00%	100.00%	100.00%
B2	Risky		99.99%	100.00%	100.00%	99.99%
B3	Risky		99.57%	99.80%	99.81%	99.73%
B4	Satisfactory			99.74%	99.80%	99.81%
B5	Satisfactory			87.89%	88.27%	89.67%
B6	Excellent			99.97%	99.97%	99.95%
B7	Risky			100.00%	100.00%	100.00%
B8	Excellent			99.97%	99.97%	99.95%
B9	Satisfactory				99.39%	99.47%
B10	Excellent					100.00%

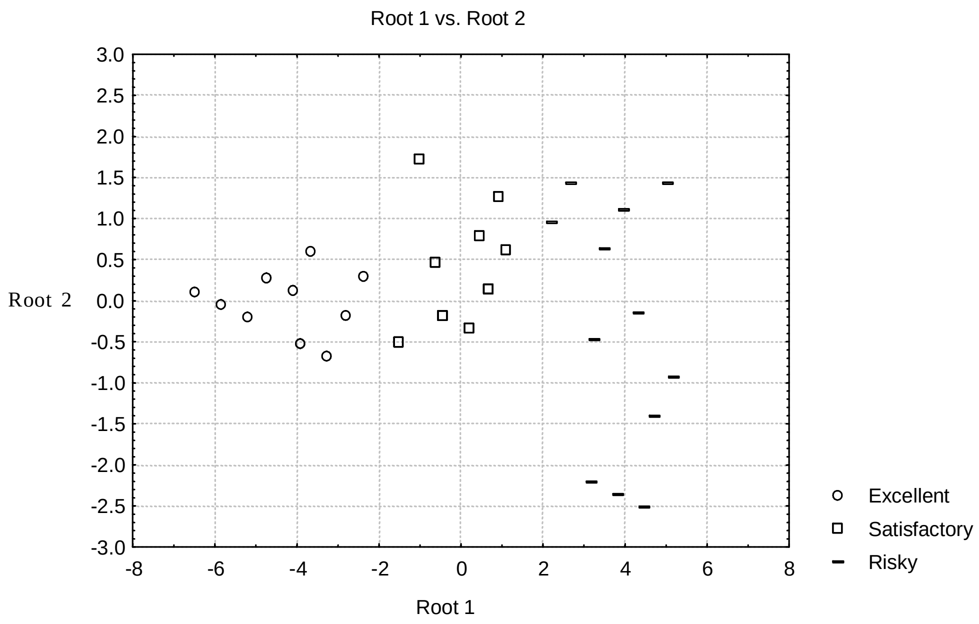

Fig. 3. Investment Preference Categories Visualization via Root 1 and Root 2

$$Y_{\text{Excellent}} = 12.74303B + 24.98364N - 240.248,$$
$$Y_{\text{Satisfactory}} = 10.68649B + 20.35277N - 160.916,$$
$$Y_{\text{Risky}} = 9.065868B + 16.08671N - 105.085.$$

A given case is classified as belonging to the category (Excellent, Satisfactory or Risky) for which it has the *highest classification score Y.*

4. Conclusions and open problems

The decision support method for IP evaluation afore presented enables a classification of Bulgarian public companies under conditions of incomplete data. A concept IP is proposed as a qualitative criterion assumed to estimate the minimal probability of bankruptcy expressed via three categories Risky, Satisfactory and Excellent.

The decision support method allows:

• BPMs scores conversion into three AGs – Bankrupted, Uncertain and Non bankrupted.

• A proxy – variable, derived from the results of seven BPMs to be designed and applied as a grouping variable in linear DA.

• Based on AGs B, U and N, elimination of the subjectivity of expert decision when determining the quantitative intervals of IP categories Excellent, Satisfactory and Risky.

• To actualized the discriminant function and to iteratively correct the expert decision when designing the grouping variable necessary for DA performance.

• Determination of optimal intervals of IP categories Excellent, Satisfactory and Risky with respect to the improvement of the classification accuracy, including minimal incorrect classifications and maximal posterior probabilities.

• Other BPMs to be added to the analysis.

The application and scope of decision support method were tested by use of seven selected popular BPMs over data about thirty listed Bulgarian companies.

The following constraints of the decision support method could be considered:

• It is incomparable with risk/return measures;

• Each considerable extension of the sample size could lead to change in the classification accuracy of DA functions;

• At this point, the IP evaluation is limited by the accuracy of the imbedded BPMs.

The validation of IP concept needs further application and verification over larger samples of companies and a larger number BPMs in current use.

The further studies may be aimed at formulation of the aggregation rules concerning the IP categories "Bankrupt", "No Bankrupt" and "Uncertain".

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