БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ . BULGARIAN ACADEMY OF SCIENCES

КИБЕРНЕТИКА И ИНФОРМАЦИОННИ ТЕХНОЛОГИИ · Том **4**, № 2
CYBERNETICS AND INFORMATION TECHNOLOGIES · Volume **4**, No 2

София . 2004 . Sofia

Synthesis of an Optimized Non-conflict Schedule Accounting the Direction of Messages Transfer in a Communication Node*

Kiril Kolchakov

Institute of Information Technologies, 1113 Sofia

Abstract: *The paper discusses some problems in designing a non-conflict schedule in communication nodes, realizing synthesis of an optimized non-conflict schedule. A block diagram of the synthesizing algorithm is presented.*

The basis for the synthesis of an optimized non-conflict schedule is the accounting of the direction of messages transfer in the communication node and the use of two special matrices-masks. The synthesis of two non-crossing sets of permitted connections enables the construction of a set of matrices with two parallel to the main diagonal neighbouring diagonals with permitted connections (satisfied requests). It is proved that in comparison with the approach not accounting the directions of messages movement, where the matrices of permitted connections are with one diagonal parallel to the basic one only, under equal other conditions (equal matrix of permitted connections), in the optimized non-conflict schedule the number of permitted connections (satisfied requests) is twice greater.

Keywords: *non-conflict schedule, matrices-masks, permitted connections.*

Introduction

When solving the problems connected with conflicts in the communication nodes of distributed information networks, the bi-directional character of messages transfer in the communication nodes is not accounted [1]. The use of the information about the direction of a message transfer, contained in its request for commutation, enables the optimization of non-conflict scheduling. The reserves consist in the possibility for bi-directional transfer of the messages between two devices simultaneously.

Some conflicts may occur in two cases:

* The investigations are sponsored by theme "Networks models for study and computer modeling of informatiion processes and systems" No 010058 of IIT – BAS, plan, team leader Prof. Hr. Radev.

– when a message source generates a request for connecting to two or more receivers of messages;

– when a receiver of messages is requested for connection to two or more sources of messages.

Synthesis of an optimized non-conflict schedule

Fig.1 presents the structure of a communication node with dimensions $N \times N$. The messages commutation is realized between the group of devices A_i, $i = 1, ..., N$, and the group of devices B_j, $j = 1, ..., N$, and vice versa.

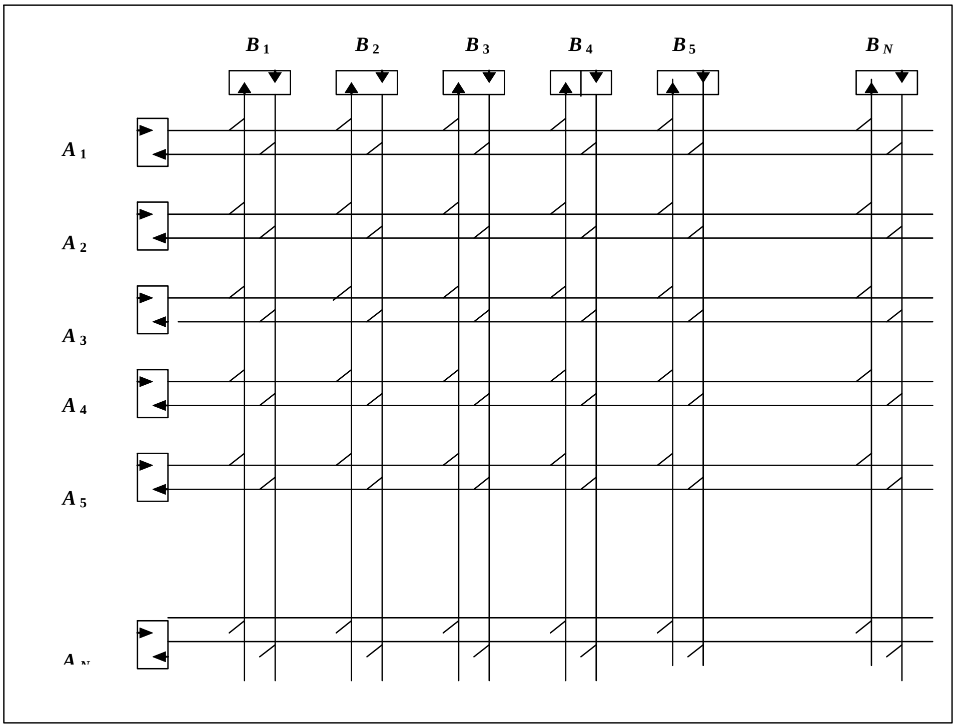

Fig. 1

Each device is supplied with a receiver and a transmitter. This enables one device to receive and to send messages at one and the same time. The generalized matrix of the requested connections (T) of the commutation node obtains the following form:

$$T = \left| \begin{array}{llll} \alpha_{11}, \beta_{11} & \alpha_{12}, \beta_{12} \ldots & \ldots & \alpha_{1j}, \beta_{1j} \\ \alpha_{21}, \beta_{21} & \alpha_{22}, \beta_{22} \ldots & \ldots & \alpha_{2N}, \beta_{2N} \\ \alpha_{31}, \beta_{31} & \alpha_{32}, \beta_{32} \ldots & \ldots & \alpha_{3N}, \beta_{3N} \\ \alpha_{41}, \beta_{41} & \alpha_{42}, \beta_{42} \ldots & \ldots & \alpha_{4N}, \beta_{4N} \\ \ldots & & & \\ \alpha_{i1}, \beta_{i1} & \alpha_{i2}, \beta_{i2} \ldots & \ldots & \alpha_{iN}, \beta_{iN} \\ \ldots & & \ldots & \alpha_{Nj}, \beta_{Nj} \\ \alpha_{N1}, \beta_{N1} & \alpha_{N2}, \beta_{N2} \ldots & \ldots & \alpha_{NN}, \beta_{NN} \end{array} \right|$$

When α_{ij} = 1, this means that there is a request for message transmission in a direction from A_i towards B_j and when β_{ij}=2 respectively, there is a request to send a message from B_j to A_i. A case is possible when α_{ij} =1 and β_{ij} = 2, without causing a conflict, unlike the approaches for synthesis of a non-conflict schedule which do not account the direction of messages transfer [1, 2, 3].

The request for sending a message in a direction from B_j towards A_i is denoted by the number 2, and the absence of a request – by null respectively, with the purpose to facilitate the presentation and avoid the ambiguity.

The generalized matrix of the connections requested is decomposed in two matrices, which contain the requests only for messages transfer in one direction T_α and T_β.

$$T_\alpha = \begin{vmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1j} & \cdots & \alpha_{1N} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2j} & \cdots & \alpha_{2N} \\ \alpha_{31} & \alpha_{32} & \cdots & \alpha_{3j} & \cdots & \alpha_{3N} \\ \alpha_{41} & \alpha_{42} & \cdots & \alpha_{4j} & \cdots & \alpha_{4N} \\ \cdots \\ \alpha_{i1} & \alpha_{i2} & \cdots & \alpha_{ij} & \cdots & \alpha_{iN} \\ \cdots \\ \alpha_{N1} & \alpha_{N2} & \cdots & \alpha_{Nj} & \cdots & \alpha_{NN} \end{vmatrix};$$

$$T_\beta = \begin{vmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1j} & \cdots & \beta_{1N} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2j} & \cdots & \beta_{2N} \\ \beta_{31} & \beta_{32} & \cdots & \beta_{3j} & \cdots & \beta_{3N} \\ \beta_{41} & \beta_{42} & \cdots & \beta_{4j} & \cdots & \beta_{4N} \\ \cdots \\ \beta_{i1} & \beta_{i2} & \cdots & \beta_{ij} & \cdots & \beta_{iN} \\ \cdots \\ \beta_{N1} & \beta_{N2} & \cdots & \beta_{Nj} & \cdots & \beta_{NN}. \end{vmatrix}.$$

The following approach is applied to each matrix of requested connections T_α and T_β, in which the generalized matrix of the requested connections is decomposed: the matrix of the requested connections is successively multiplied by a family of the so-called *matrices-masks*. The corresponding terms of the matrix of requested connections are multiplied by the respective terms of the matrix-mask, as a result of which a matrix of the permitted non-conflict connections is obtained [1].

The family of matrices-masks is formed in such a way that the conditions for avoiding a conflict are met. The matrices-masks consist of ones and zeroes only, like the matrix of connections [1].

As a result two sets of matrices of permitted non-conflict connections $R^k{}_\alpha$ are obtained for $k = 1,..., K$ and $R^k{}_\beta$, for $k = 1,..., K$:

(1) $$[R^k{}_\alpha] = [T_\alpha] \times [M^k], \ k = 1, ...,K.$$

$$R^k_\alpha = \begin{vmatrix} \alpha_{11}.\ M^k_{11} & \alpha_{12}.\ M^k_{12} & \ldots\ \alpha_{1N}.\ M^k_{1N} \\ \alpha_{21}.\ M^k_{21} & \alpha_{21}.\ M^k_{22} & \ldots\alpha_{2N}.\ M^k_{2N} \\ \ldots & & \\ \alpha_{i1}.\ M^k_{i1} & \alpha_{i2}.\ M^k_{i2} & \ldots\alpha_{iN}.\ M^k_{iN} \\ \ldots & & \\ \ldots & & \\ \alpha_{N1}.\ \mathrm{M}^k_{N1} & \alpha_{N2}.\ M^k_{N2} & \ldots\ \alpha_{NN}.\ M^k_{NN} \end{vmatrix};$$

(2) $$[R^k_\beta\ ] = [T_\beta\ ] \times [\ M^k],\ k = 1,\ K.$$

$$R^k_\beta = \begin{vmatrix} \beta_{11}.\ M^k_{11} & \beta_{12}.\ M^k_{12} & \ldots\ \beta_{1N}.\ M^k_{1N} \\ \beta_{21}.\ M^k_{21} & \beta_{21}.\ M^k_{22} & \ldots\beta_{2N}.\ M^k_{2N} \\ \ldots & & \\ \beta_{i1}.\ M^k_{i1} & \beta_{i2}.\ M^k_{i2} & \ldots\beta_{iN}.\ M^k_{iN} \\ \ldots & & \\ \ldots & & \\ \beta_{N1}.\ \mathrm{M}^k_{N1} & \beta_{N2}.\ M^k_{N2} & \ldots\ \beta_{NN}.\ M^k_{NN} \end{vmatrix}.$$

The number of matrices-masks (K) depends on N and is defined by the following equation [1]:

(3) $$K = 2N - 1.$$

Analytical form of the matrices-masks

The description of the matrices-masks is the following:

1) $[M^1]$ is a matrix-mask, for which $m_{ij}= 1$ for $i = N$ and $j = 1$; and $m_{ij} = 0\ \forall$ remaining values of i and j;

2) $[M^2]$ is a matrix-mask, for which $m_{ij}= 1$ for $i = N - 1,\ N$ and $j = 1,\ 2$; and $m_{ij} = 0\ \forall$ remaining values of i and j;

3) $[M^3]$ is a matrix-mask, for which $m_{ij}= 1$ for $i = N - 2,\ N - 1,\ N$ and $j = 1,\ 2,\ 3$; and $m_{ij} = 0\ \forall$ remaining values of i and j;

4) $[M^4]$ is a matrix-mask, for which $m_{ij}= 1$ for $i = N - 3,\ N - 2,\ N - 1,\ N$ and $j = 1,\ 2,\ 3,\ 4$; and $m_{ij} = 0\ \forall$ remaining values of i and j;

…

k) $[M^k]$ is a matrix-mask, for which $m_{ij}= 1$ for $i = N - (k-1),\ N - (k - 2), N - (k-3),\ N - (k - 4),\ \ldots,\ N - (k - k) = N$ and $j = 1,\ 2,\ 3,\ 4,\ \ldots,\ k$; and $m_{ij} = 0\ \forall$ remaining values of i and j;

...

N–1) $[M^{N-1}]$ is a matrix-mask, for which $m_{ij}= 1$ for $i = 2,\ 3,\ 4,\ 5,\ldots,\ N$ and $j = i - 1$; and $m_{ij} = 0\ \forall$ remaining values of i and j;

N) $[M^N]$ is a diagonal matrix, having ones only along the main diagonal, all the rest elements being zeroes, all terms $m_{ij}= 1$, for $i = j$ and $m_{ij}= 0$ for $i \neq j$;

N +1) $[M^{N+1}]$ is a matrix-mask, for which $m_{ij}= 1$ for $i = 1,..., N-1$ and $j = i + 1$; and $m_{ij} = 0$ $\forall$ remaining values of i and j;

N+g) $[M^{N+g}]$ is a matrix-mask, for which $m_{ij}= 1$ for $i = 1,..., N-g$ and $j = i + g$; and $m_{ij} = 0$ $\forall$ remaining values of i and j;

…

2N–1) $[M^{2N-1}]$ is a matrix-mask, for which $m_{ij}= 1$ for $i = 1$ and $j = N$; and $m_{ij} = 0$ $\forall$ remaining values of i and j.

On the basis of the description of the matrices-masks, the two following formulas (4a) and (4b) are derived, which describe them analytically.

(4a) $[M^k]$ is a matrix-mask, for which $m_{ij}= 1$, for $i = N-(k-1), N-(k-2), N-(k-3), N-(k-4), ..., N-(k-k) = N$ and $j =1, 2, 3, 4, ..., k$; and $m_{ij} = 0$ $\forall$ remaining values of i and j.

This formula is used for $k \le N$.

(4b) $[M^{N+g}]$ is a matrix-mask, for which $m_{ij}= 1$, for $i = 1,..., N-g$ and $j = i + g$; and $m_{ij} = 0$ $\forall$ remaining values of i and j.

This formula is used for $k > N$.

(5) $$[R^k] = [R^k_\alpha] + [R^k_\beta] \text{ for } k = 1,..., K-1; \; [R^K] = [R^K_\beta] + [R^1_\beta].$$

The algorithm of computations and obtaining the matrices with permitted connections is shown in Fig. 2.

In an extended form, formula (5) is like:

$$R^k = \left|\begin{array}{llll}
\alpha_{11} M^k_{11} + \beta_{11} M^{k+1}_{11} & \alpha_{12} M^k_{12} + \beta_{12} M^{k+1}_{12} & \ldots & \alpha_{1N} M^k_{1N} + \beta_{1N} M^{k+1}_{1N}\,, \\
\alpha_{21} M^k_{21} + \beta_{21} M^{k+1}_{21} & \alpha_{21} M^k_{22} + \beta_{21} M^{k+1}_{22} & \ldots & \alpha_{2N} M^k_{2N} + \beta_{2N}. M^{k+1}_{2N}\,, \\
\ldots & & & \\
\alpha_{i1} M^k_{i1} + \beta_{i1} M^{k+1}_{i1} & \alpha_{i2} M^k_{i2} + \beta_{i2} M^{k+1}_{i2} & \ldots & \alpha_{iN} M^k_{iN} + \beta_{iN} M^{k+1}_{iN}\,, \\
\ldots & & & \\
\alpha_{N1} M^k_{N1} + \beta_{N1} M^{k+1}_{N1} & \alpha_{N2} M^k_{N2} + \beta_{N2}. M^{k+1}_{N2} & \ldots & \alpha_{NN} M^k_{NN} + \beta_{NN} M^{k+1}_{NN}\,.
\end{array}\right|$$

At $k = K-1$, and for $k = K$, the extended form of formula 5 is as follows:

$$R^K = \left|\begin{array}{llll}
\alpha_{11} M^K_{11} + \beta_{11} M^1_{11} & \alpha_{12} M^K_{12} + \beta_{12} M^1_{12} & \ldots & \alpha_{1N} M^K_{1N} + \beta_{1N} M^1_{1N}\,, \\
\alpha_{21} M^K_{21} + \beta_{21} M^1_{21} & \alpha_{21} M^K_{22} + \beta_{21} M^1_{22} & \ldots & \alpha_{2N} M^K_{2N} + \beta_{2N} M^1_{2N}\,, \\
\ldots & & & \\
\alpha_{i1} M^K_{i1} + \beta_{i1} M^1_{i1} & \alpha_{i2}. M^K_{i2} + \beta_{i2} M^1_{i2} & \ldots & \alpha_{iN} M^K_{iN} + \beta_{iN} M^1_{iN}\,, \\
\ldots & & & \\
\alpha_{N1} M^K_{N1} + \beta_{N1} M^1_{N1} & \alpha_{N2}. M^K_{N2} + \beta_{N2} M^1_{N2} & \ldots & \alpha_{NN} M^K_{NN} + \beta_{NN} M^1_{NN}\,.
\end{array}\right|$$

Taking in mind that α terms can accept values 0 or 1, while β terms – 0 or 2, it becomes evident that in the matrices of permitted connections R^K for $k =1,..., K$, the terms may get four possible values, namely: 0, 1, 2 or 3.

– zero – no permission;
– one – a permission for message transfer in direction A_i to B_j;
– two – a permission for message transfer in direction B_j to A_i;

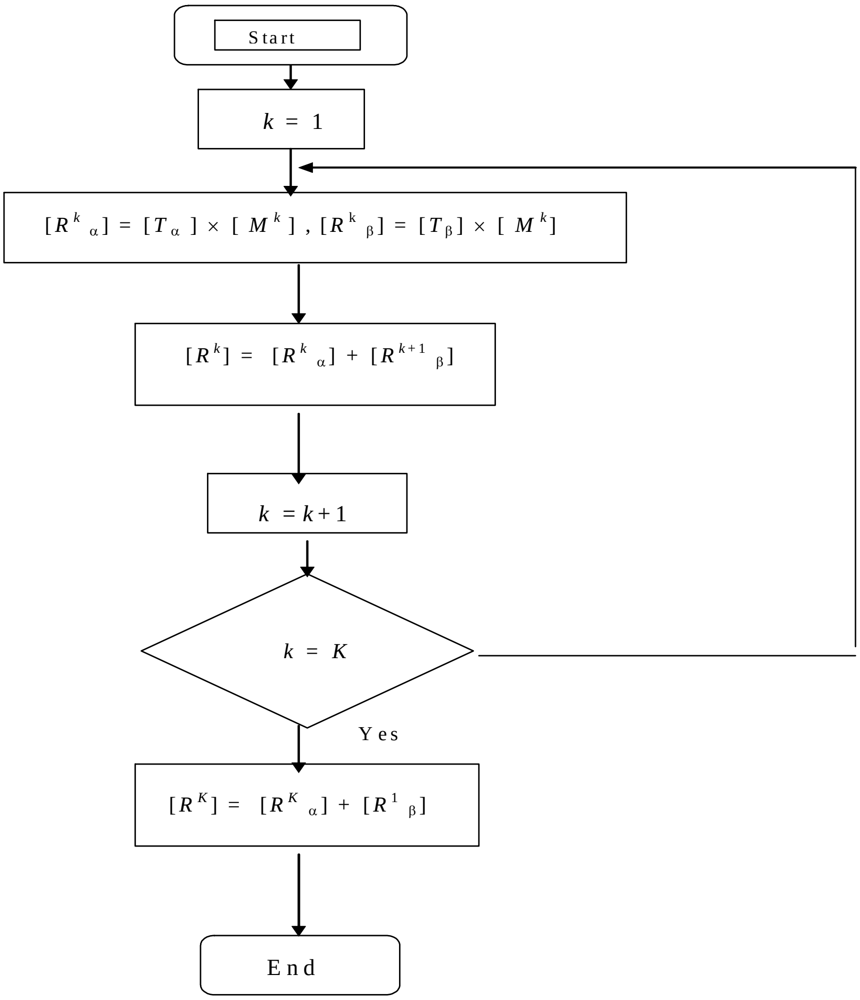

Fig. 2

– three – a permission for message transfer in both directions simultaneously – from A_i to B_j and from B_j to A_i (rf. to Fig. 1)

Comparison between the algorithm for acquiring a non-conflict schedule [1] and this of the optimized non-conflict schedule, accounting the direction of messages transfer

The comparison is done on the basis of algorithms block diagrams. The difference is on one side in the synthesis of the matrices of permitted connections $[R^k]$, and on the other – in their type. The decomposition of the matrix of requested connections in the approach accounting the direction of messages transfer of two matrices $[T_\alpha]$ and $[T_\beta]$, and the synthesis of permitted connections $[R^k] = [R^k_\alpha] + [R^{k+1}_\beta]$, $k = 1, ..., K - 1$, $[R^K] = [R^K_\alpha] + [R^1_\beta]$ enables the non-conflict transfer in both directions. The matrices of permitted connections consist of two parallel to the main diagonal neighbouring diagonals of permitted connections (satisfied requests). In the approach, which does not account the direction of messages transfer [1], there is no possibility to match the requests for bi-directional transmission and reception respectively and this is the reason why in the matrices of permitted connections they are theoretically twice fewer.

Each matrix of permitted connections consists of one diagonal only with satisfied requests [1]:

$$[R^k] = [R^k_\alpha] + [R^{k+1}_\beta],\ k=1,..., K-1,\ [R^K] = [R^K_\alpha] + [R^1_\beta];\ [R^k_\alpha] = [T_\alpha] \times [M^k],$$

$$[R^k_\beta] = [T_\beta] \times [M^k],$$

$$[R^k] = [T] \times [M^k].$$

An example can illustrate this. The matrix of the requested connections of a communication node with dimensions 4×4 is given in Fig. 3, not accounting the directions of messages transfer. Fig. 4 shows the same matrix accounting the directions of messages transmission.

1	1	1	0
1	1	1	1
1	1	1	1
1	0	1	1

Fig. 3

1	2	3	0
2	3	3	1
1	2	3	3
2	0	2	2

Fig. 4

T_α is shown in Fig. 5, and T_β in Fig. 6.

1	0	1	0
0	1	1	1
1	0	1	1
0	0	0	0

Fig. 5

0	2	2	0
2	2	2	0
0	2	2	2
2	0	2	2

Fig. 6

R^4 is shown in Fig. 7, R^7 – in Fig. 8.

1	2		
	1	2	
		1	2
			0

Fig. 7

			0
2			

Fig. 8

Conclusion

Taking into account the direction of messages transfer in a communication node allows the decomposition of the matrix of the requested connections T in two matrices of equal rank T_α and T_β.

The synthesis of two non-crossing sets of permitted connections $R_\alpha{}^k$ and $R_\beta{}^k$, $k = 1,..., K$, enables the construction of a set of matrices with two parallel to the basic diagonal neighbouring diagonals with permitted connections (satisfied requests))

$[R^k] = [R^k_{\alpha}] + [R^{k+}_{\beta}]$, $k = 1, ..., K - 1$, $[R^K] = [R^K_{\alpha}] + [R^1_{\beta}]$.

In comparison with the approach without accounting the directions of messages movement, where the matrices of permitted connections are with one diagonal parallel to the basic one only, under equal other conditions (equal matrix of permitted connections), in the optimized non-conflict schedule the number of permitted connections (satisfied requests) proves to be twice greater.

R e f e r e n c e s

1. K o l c h a k o v, K. One solution of the conflicts problem in the operation of the commutation nodes in distributed information networks. – Working Papers of IIT – BAS, IIT /WP-162B, ISSN1310-652X, October 2003, Sofia.
2. K o l c h a k o v, K. A method for performance improvement of a non-conflict algorithm of a message switching node in distributed information networks. – In: Scientific Series. Vol. 3. Intellectual systems and Technologies. Moscow, Scientific session MIPI-2003, 70-71.
3. K o l c h a k o v, K. An algorithm, guaranteeing non-conflict work of a commutation node in distributed information networks and some procedures for its speeding up. – Working Papers of IIT – BAS IIT-WP-142B, ISSN1310-652X, November 2002, Sofia.
4. K o l c h a k o v, K. Non-conflict algorithm of message switching node in distributed information networks. – Cybernetics and Information Technologies, Vol.1, 2001, No 1, 44-47.
5. C h e n, W., J. M a v o r, P h. D e n y e r, D. R e n s h a w. Traffic routing algorithm for serial superchip system customisation. – IEE Proc., **137**, 1990.

Синтез на оптимизирано безконфликтно разписание с отчитане посоката на придвижване на съобщенията в комуникационния възел

Кирил Колчаков

Институт по информационни технологии, 1113 София

(Р е з ю м е)

В статията се разглеждат проблемите при създаване на безконфликтни разписания в комуникационните възли, като е извършен синтез на оптимизирано безконфликтно разписание. Представена е подробна блоксхема на алгоритъма за синтез на оптимизирано безконфликтно разписание.

Базата за синтез на оптимизирано разписание е в отчитане посоката на придвижване на съобщенията в комуникационния възел и използване на специални матрици-маски. Синтезът на две непресичащи се множества от разрешени връзки позволява съставяне на множество матрици с по два успоредни на главния диагонал съседни диагонала с разрешени връзки (удовлетворени заявки).

Доказва се, че сравнено с подхода без отчитане на посоките на придвижване на съобщенията, където матриците от разрешени връзки са само с един диагонал, успореден на главния, има реална възможност при равни други условия (еднаква матрица на връзките) броят на разрешените връзки (удовлетворени заявки) да бъде два пъти повече при оптимизираното безконфликтно разписание.