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On Global Operator $\boldsymbol{G_{21}}$ Defined over Generalized Nets

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Abstract: *A new global operator defined over the generalized nets is introduced and its basic properties are studied. If two neighbouring transitions in a generalized net have more than one joint place (output for one of the transitions, and input for the other) then operator* $\boldsymbol{G_{21}}$ *merges them in one place. In result we obtain a shorter and more compact generalized net model.*

Keywords: *Generalized net, Operator.*

1. Introduction

The idea for defining operators over Generalized Nets (GNs, see [1, 9]) was introduced in 1982, and up to now essentially developed (see [1-9]). Each operator juxtaposes to a given GN a new GN with specific properties. The operators that can be applied over the generalized nets fall into six categories: global, local, hierarchical, reducing, extending and dynamical [2-9].

Here we shall introduce the definition of a new global operator $\boldsymbol{G_{21}}$ and will discuss its basic properties. All necessary concepts from the theory of the GNs are defined in [9].

2. Definition of the global operator $\boldsymbol{G_{21}}$

Operator $\boldsymbol{G_{21}}$ changes the structure of a given GN, uniting in one place all output places of a given transition, that are simultaneously input places of another transition. The capacity of the new place will be equal to the sum of the capacities of the included places, and the priority of the new place will be equal to the maximum of the priorities of the included places.

In Fig. 1, a GN is shown before (Fig. 1a) and after (Fig. 1b) applying operator $\boldsymbol{G_{21}}$.

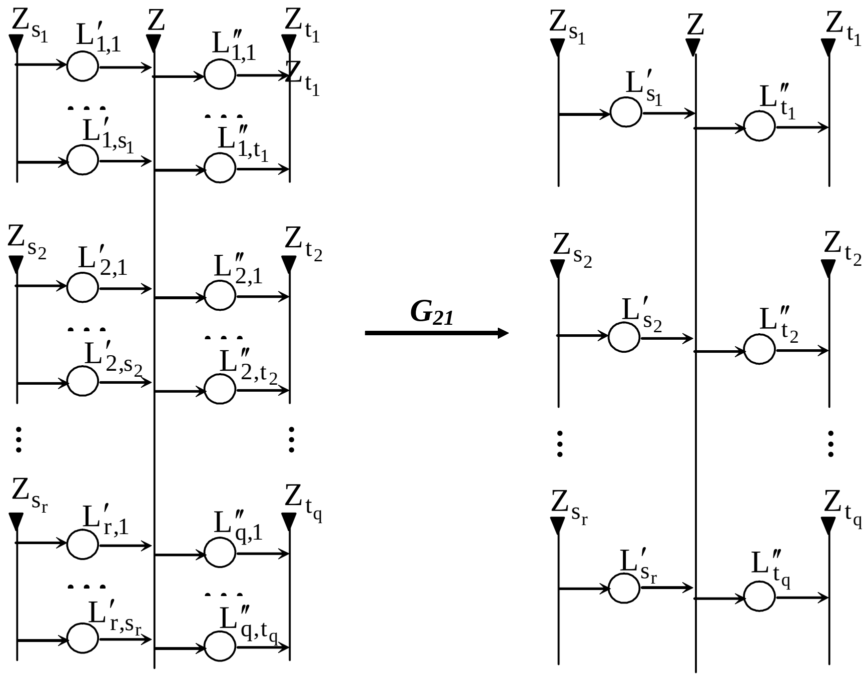

Fig. 1

Places $L'_{i,1}, \ldots, L'_{i,s_i}$, that are outputs of transition Z'_i and inputs for Z in the first GN, in the new GN will be united in place L'_i, that has the following capacity and priority:

$$c(L'_{s_i}) = \sum_{j=1}^{s_i} c(L'_{i,j}),$$

$$\pi_L(L'_{s_i}) = \max_{1 \le j \le s_i}(\pi_L(L'_{i,j})), \quad i = 1, 2, \ldots, r.$$

Analogously, places $L''_{i,1}, L''_{i,2}, \ldots, L''_{i,t_i}$ that are outputs of transition Z and inputs of transition Z''_i, merge in place L''_i, that has the following capacity and priority:

$$c(L''_{t_i}) = \sum_{j=1}^{t_i} c(L''_{i,j}),$$

$$\pi_L(L''_{t_i}) = \max_{1 \le j \le t_i}(\pi_L(L''_{i,j})), \quad i = 1, 2, \ldots, q.$$

3. Relations between operator $\boldsymbol{G_{21}}$ and the other global operators

The global operators change separate GN components or a given GN as whole. In a result a new GN is obtained, which has a previously determined properties.

In some sense, operator $\boldsymbol{G_1}$ “normalizes” the GN-structure.

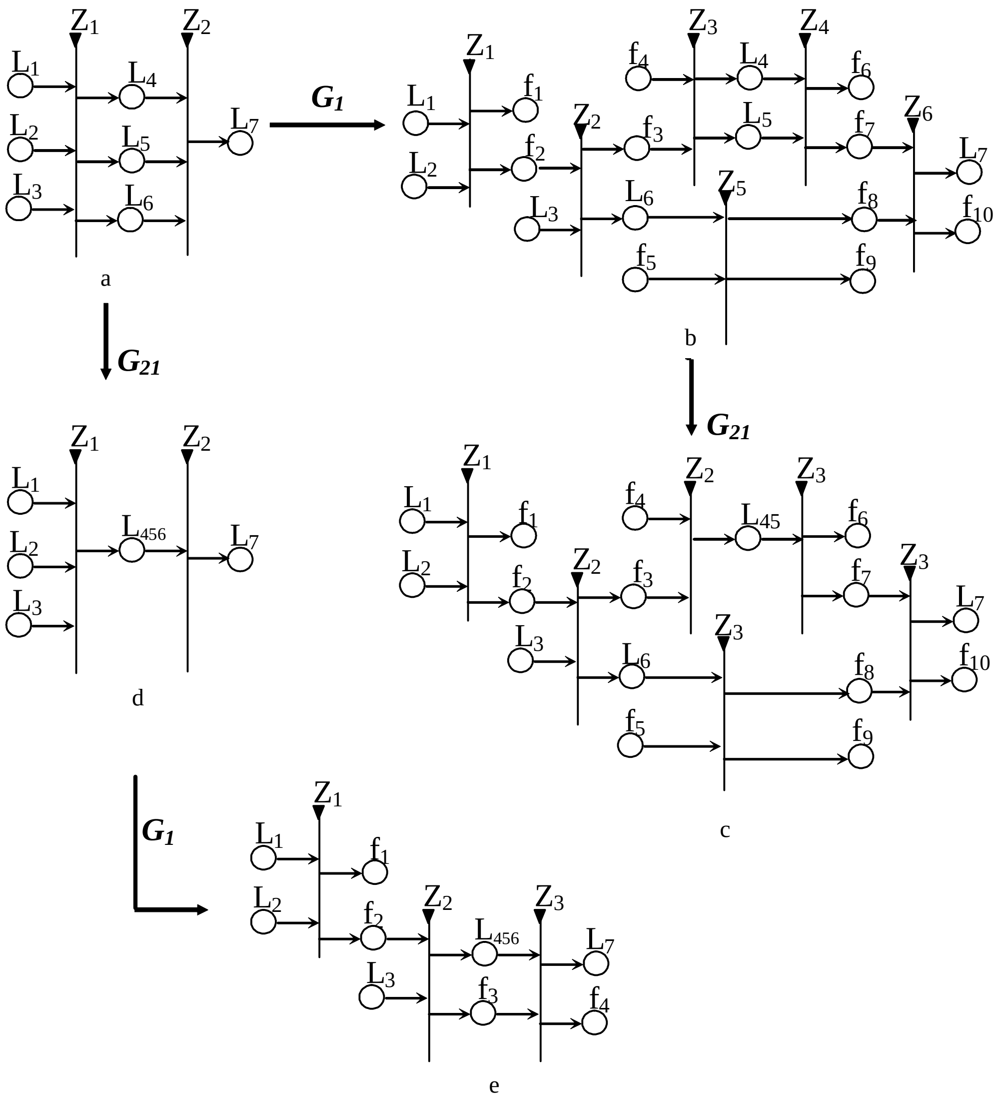

Fig. 2

The GN in Fig. 2a has been constructed to illustrate the relation between operators $\boldsymbol{G_1}$ and $\boldsymbol{G_{21}}$. Over this GN the two operators are applied in different order: in Fig. 2b and 2c – $\boldsymbol{G_1}$, $\boldsymbol{G_{21}}$; in Fig. 2d and 2e – $\boldsymbol{G_{21}}$, $\boldsymbol{G_1}$. Obviously, the obtained GNs (Fig. 2c and 2e) are different. Therefore, the following relation is valid:

$$\boldsymbol{G_1(G_{21}(E)) \neq G_{21}(G_1(E))} \ .$$

Operator $\boldsymbol{G_2}$ “narrows” the GN. The result of its application is a transition with input places – the inputs of the GN, and output places – the outputs of the GN. Since the new operator is not influenced by the input and output GN-places, then the result of application of operators $\boldsymbol{G_2}$ and $\boldsymbol{G_{21}}$ is not related to the order of their application, i.e.,

$$G_2(G_{21}(E)) = G_{21}(G_2(E)).$$

Moreover, for every transition $\boldsymbol{Z}$:

$$G_{21}(Z) = Z,$$

i.e.,

$$G_{21}(G_2(E)) = G_2(E) = G_2(G_{21}(E)).$$

Operator G_3 omits the GN-transitions that have not be fired at least once during the GN-functioning, as well as the GN-places, through which no token has transfered at the same time. Let us assume that in place L_3 of GN from Fig. 3a no token has transferred for the whole time-period of GN-functioning. Then after applying operator G_3, place L_3 will be omitted (see Fig. 3b). After applying operator G_{21} over the result on the first step, the net will obtain the form from Fig. 3c. If we firstly apply operator G_{21} over the given GN, the latest will obtain the form from Fig. **3d**. When over the so-obtained net we apply operator G_3, it cannot change, because place L_3 is now included in the united L_{345}, that cannot be omitted, because it also contains places L_4 and L_5.

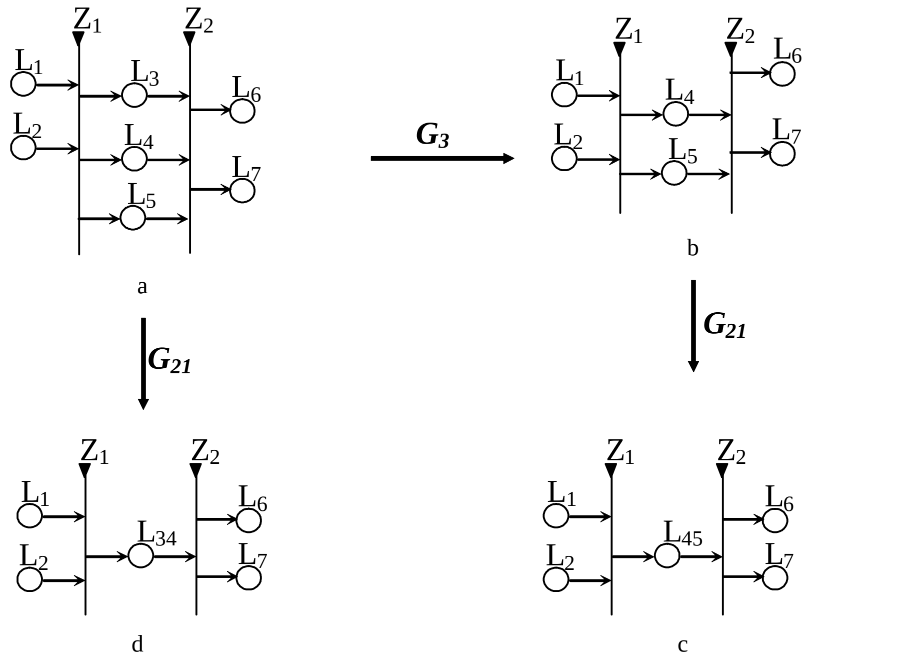

Fig. 3

Therefore, the relation

$$G_3(G_{21}(E)) \neq G_{21}(G_3(E))$$

holds.

Operator G_4 adds new (in some sense, generalized) input and output places to the net. In the new input place will enter all tokens that must enter the net, and through the new output place will pass all tokens that must leave the net. The result of applying the two discussed operators over the net from Fig. 4a is shown in Figs. 4b, 4c, 4d and 4e, and it illustrates why does

$$G_4(G_{21}(E)) \neq G_{21}(G_4(E))$$

hold.

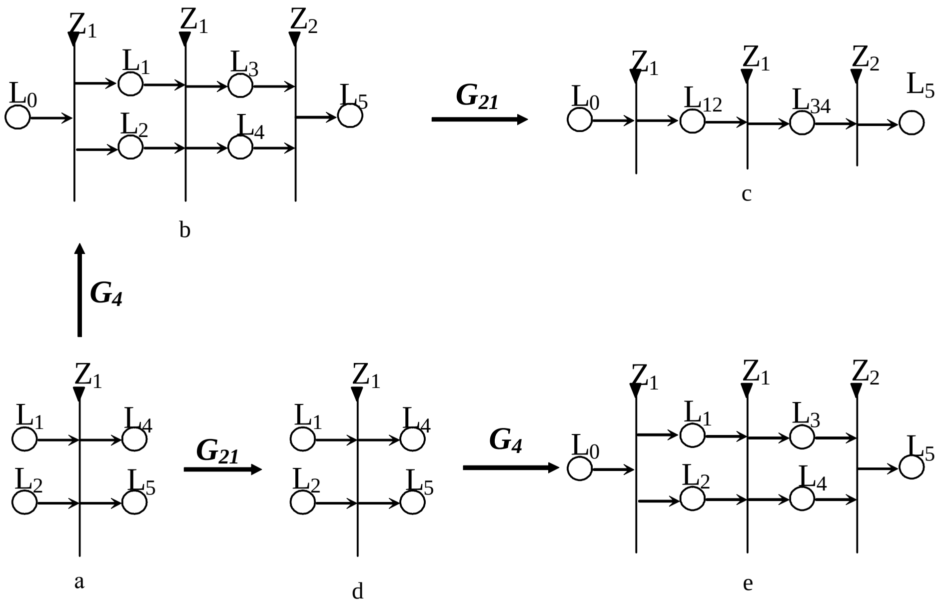

Fig. 4

Operator G_6 “compresses” only a part of a given GN. For example, the net from Fig. **5a** is transformed by this operator to the form from Fig. **5b**. The latest is changed by operator G_{21} to the form from Fig. **5c**. When we apply the operators in opposite order (see Fig. **5d** and **5e**) we obtain the same result. It is easy for us to prove formally, that for each GN E:

$$G_6(G_{21}(E)) = G_{21}(G_6(E)).$$

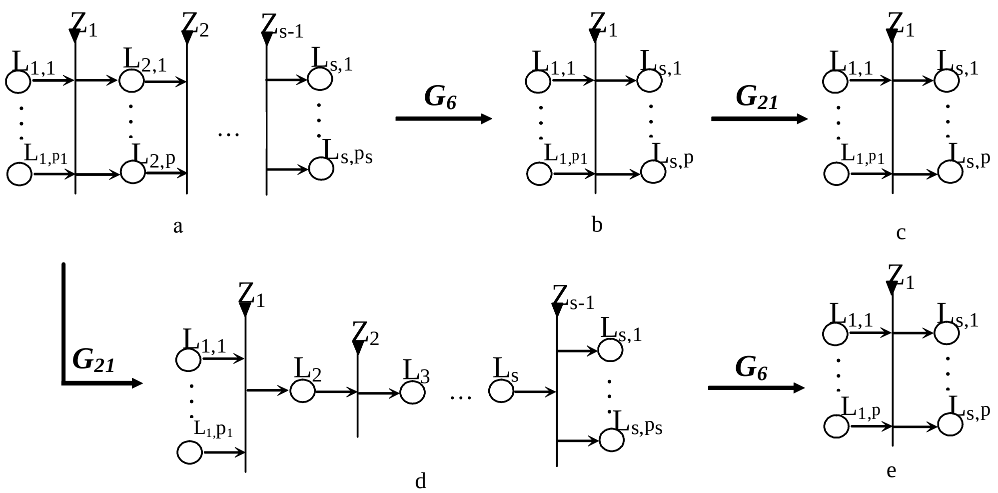

Fig. 5

The rest global operators – temporal (G_7, G_8 and G_9), dynamical (G_{10}, G_{11}, G_{12} and G_{13}) and the operators modifying the auxiliary components (G_5, G_{14}, G_{15}, G_{16}, G_{17}, G_{18}, G_{19} and G_{20}) act over GN-components that have not been influenced by the action of G_{21}. By this reason for i = 5, 7, 8, …, 20 it is valid that

$$G_i(G_{21}(E)) = G_{21}(G_i(E)) \ .$$

4. Relations between operator G_{21} and the local operators and their extensions

The operators from this group are applied over separate components of a given transition. Since the local operators P_1, P_2, P_3, P_4, P_7 and P_{10} and their extensions are related to the changes of transition components, over which G_{21} does not effect, it is easily proved that:

$$P_1 (G_{21} (E, Z, t_1')) = G_{21} (P_1 (E, Z, t_1')),$$
$$P_2 (G_{21} (E, Z, t_2')) = G_{21} (P_2 (E, Z, t_2')),$$
$$P_3 (G_{21} (E, Z, \theta_1')) = G_{21} (P_3 (E, Z, \theta_1')),$$
$$P_4 (G_{21} (E, Z, \theta_2')) = G_{21} (P_4 (E, Z, \theta_2')),$$
$$P_7 (G_{21} (E, Z, \nu')) = G_{21} (P_7 (E, Z, \nu')),$$
$$P_{10} (G_{21} (E, Z, f')) = G_{21} (P_{10} (E, Z, f')).$$

The local operators P_5 and P_6 are related to the changes of the transition structures (the index matrices of transition conditions and of arc capacities, and operators P_8, and P_9 influence the capacities and the characteristic functions of the places, it is obvious, that:

$$P_5 (G_{21} (E, Z, r')) \neq G_{21} (P_5 (E, Z, r')),$$
$$P_6 (G_{21} (E, Z, M')) \neq G_{21} (P_6 (E, Z, M')),$$
$$P_8 (G_{21} (E, Z, k_L')) \neq G_{21} (P_8 (E, Z, k_L')),$$
$$P_9 (G_{21} (E, Z, \Phi_L')) \neq G_{21} (P_9 (E, Z, \Phi_L')).$$

5. Relations between operator G_{21} and the hierarchical operators

The hierarchical operators detailize or enlarge the structure of a given GN.

Hierarchical operator H_1 changes a given place in a GN with a whole net (sub)GN, that the result of its functions is similar to the result of the characteristic function of the place from the GN.

The example from Fig. 6 shows why for operator H_1 the following relation is valid

$$H_1(G_{21}(E)) \neq G_{21}(H_1(E)).$$

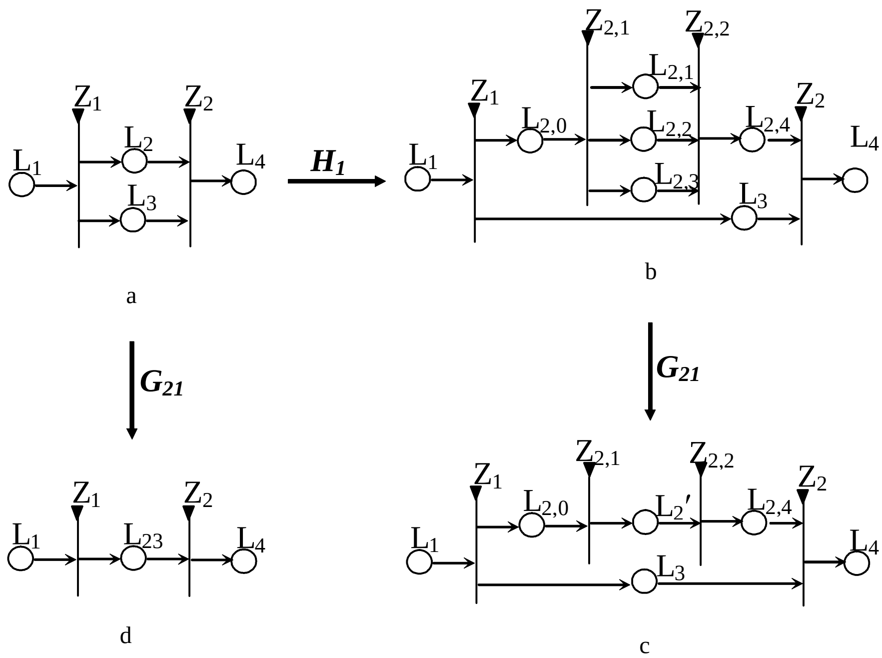

Fig. 6

The same is valid for operators $\boldsymbol{H_2}$, $\boldsymbol{H_3}$, $\boldsymbol{H_4}$, and $\boldsymbol{H_5}$ (see, e.g., Fig. 7 and 8), i.e.,

$$\boldsymbol{H_2(G_{21}(E)) \neq G_{21}(H_2(E))},$$

$$\boldsymbol{H_3(G_{21}(E)) \neq G_{21}(H_3(E))},$$

$$\boldsymbol{H_4(G_{21}(E)) \neq G_{21}(H_4(E))},$$

$$\boldsymbol{H_5(G_{21}(E)) \neq G_{21}(H_5(E))}.$$

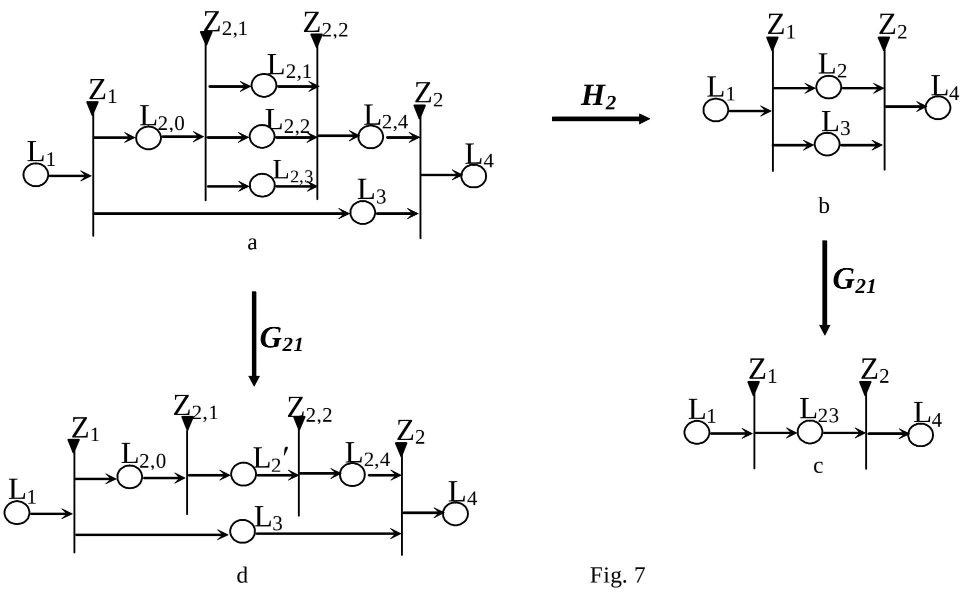

Fig. 7

6. Relations between operator $\boldsymbol{G_{21}}$ and the reducing and extending operators

The reducing operators omit these of the GN-components that are not used in a concrete GN-model. Obviously, operator $\boldsymbol{G_{21}}$ will not give a result, when it is applied over a GN without components that are related to it. Therefore,

$$\boldsymbol{R\ (G_{21}\ (E),Y) = G_{21}\ (R\ (E,\ Y))}$$

for $Y \in \{A_3, A_4, A_5, A_6, A_7, \ \pi_L, \pi_L, c, \theta_1, \theta_2, K, \pi_K, \theta_K, T, t^\circ, t^*, X, \Phi, b\}$.

Operator $\boldsymbol{G_{21}}$ influences only the first two transition components, but applying a reducing operator to them is meaningless. Operator $\boldsymbol{G_{21}}$ is applicable over GN extensions in these cases, when it is applicable over the ordinary GNs. Relation

$$\boldsymbol{O\ (G_{21}\ (E),Y) = G_{21}\ (O\ (E,\ Y))}$$

is not valid only in the case, when O is related to the GN-extension with a complex structure.

7. Relations between operator $\boldsymbol{G_{21}}$ and the dynamical operators

Dynamical operators are related to the change of tokens transfer strategies at the time of GN-functioning or to the change the way of transition condition predicate calculation. They do not incluence the GN-statical structure. Therefore,

$$\boldsymbol{A_{i,j}\ (G_{21}\ (E\ )) = G_{21}\ (A_{i,j}\ (E\ ))},$$

for $i = 1, 2, 3, 4$ and $j = 1, 2, \ldots, 18$.

8. An example of application of operator $\boldsymbol{G_{21}}$

In Fig. 9a there is a model describing the organization of the information exchange in the abstract university Intranet. The L-places stand for the separate network objects (Rector, Deputy rectors, Deans, Heads of Departments, Lecturers, Students and Administrative staff, who exchange information via electronic mail, Web and the Administrative information system). The S-places represent servers, the V-places are related to requests to the Web-server and the AIS, and there is one A-place, which depicts the university's electronic information archive. The different Intranet clients are equal in rights. Receiving and sending information in the so-described model has been realized by models Z_1 and Z_2. Since all L_i places are output places for transition Z_1 and input places for transition Z_2, the above-efined operator $\boldsymbol{G_{21}}$ can be applied. The capacity of the new place L is sum of the capacities of the merged L_i positions, hence equal to the number of clients who exchange information. As a result of the application of operator $\boldsymbol{G_{21}}$, the following conclusions can be drawn:

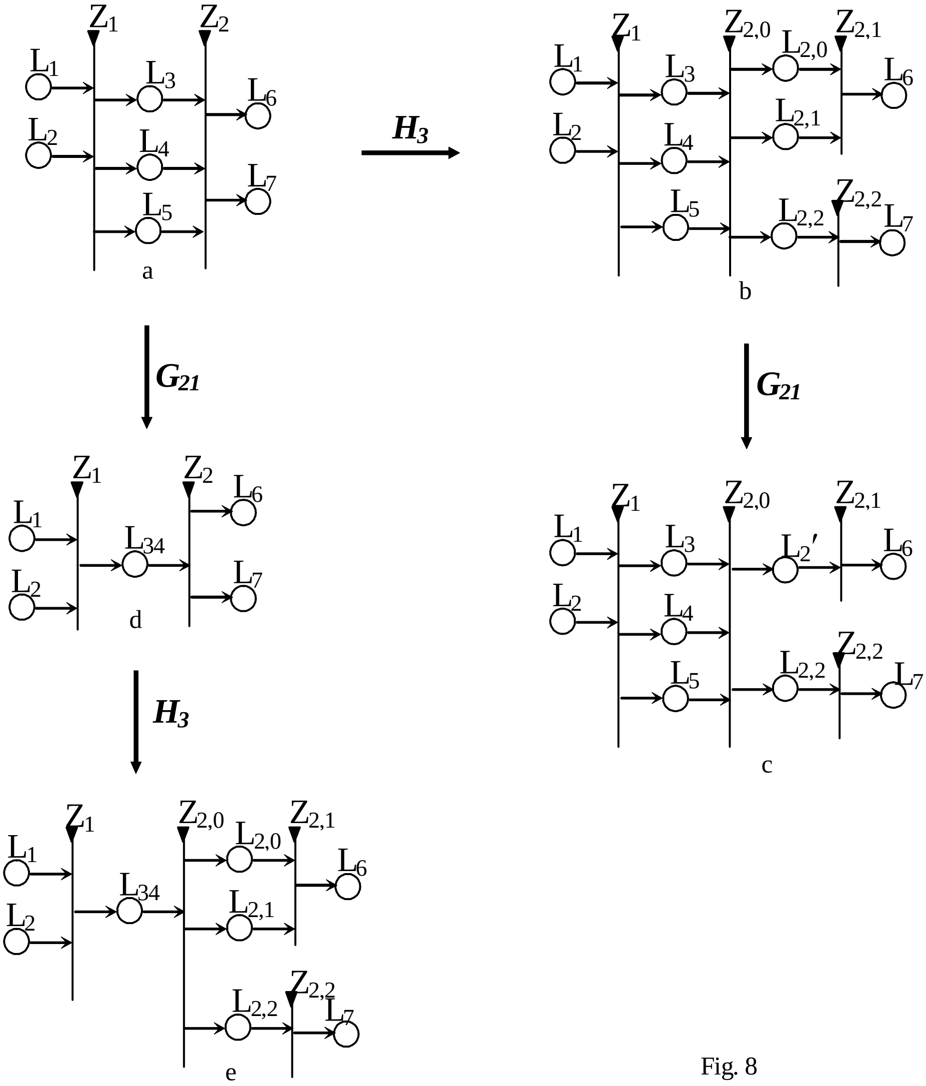

Fig. 8

1. A new generalized net has been obtained, more perspicuous and easier due to its graphical structure

2. The description of the transitions in the new net is shorter and unified, without the redundant repetition of similar information.

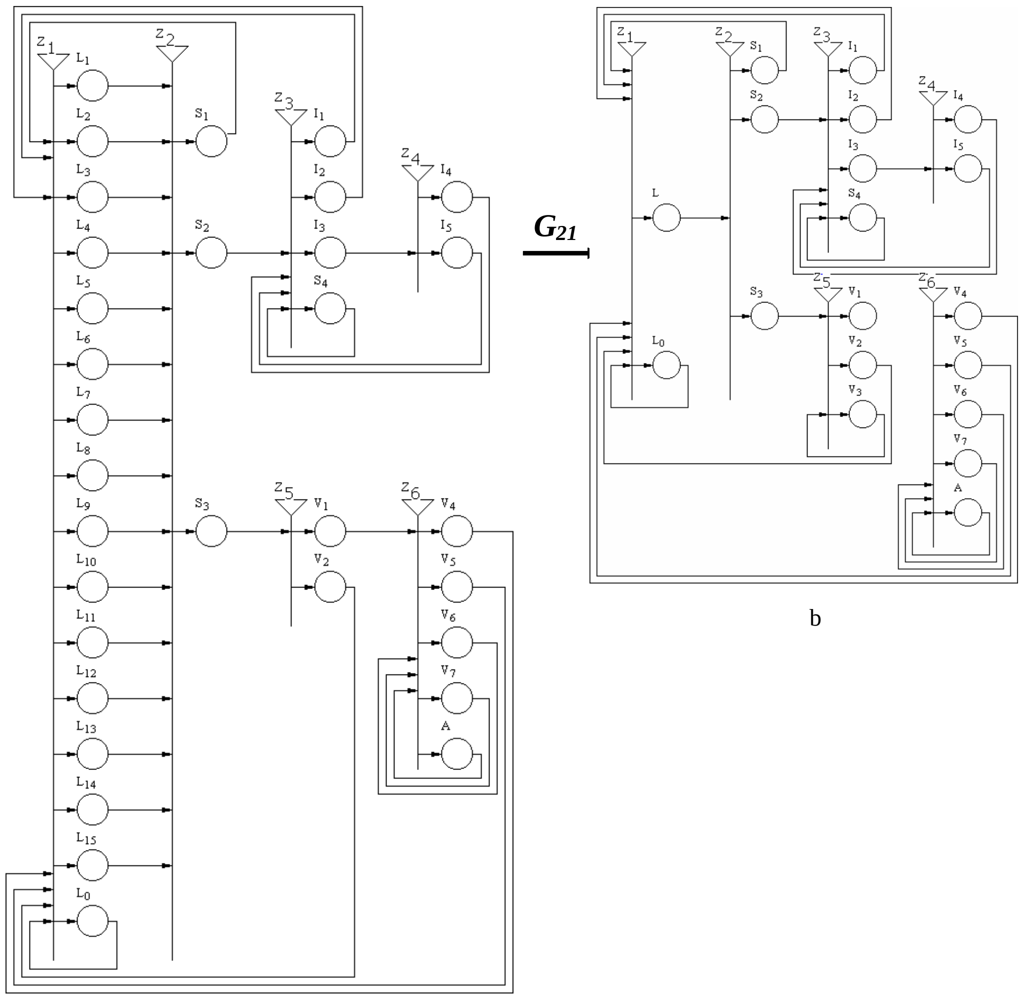

Fig. 9

9. Conclusion

The so-defined global operator $\boldsymbol{G_{21}}$ simplifies the graphical structure of a given GN. It can be used as a tool for constructing of GN-models with suitable simpler form.

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Върху глобалния оператор $\boldsymbol{G}_{21}$, дефиниран над обобщени мрежи

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(Р е з ю м е)

Дефиниран е нов глобален оператор върху обобщени мрежи (ОМ) и са изследвани основните му свойства. Ако два съседни прехода в дадена ОМ имат повече от една обща позиция (изходни за единия преход и входни за другия), оператор $\boldsymbol{G}_{21}$ ги слива в една позиция. В резултат на това се получава по-малък и по-компактен ОМ-модел.