Modeling of the Synthesis of Information Processes with the Help of Generalized Networks

Tasho Tashev, Hristo Radev

Institute of Information Technologies, 1113 Sofia

1. Introduction

The concept of Interacting Open Systems (IOS) [1] of the International Organization of Standardization (ISO) is used by the developers in telecommunication areas as a basis for realization, investigation and development of the information exchange systems. The processes in such systems are characterized by parallelism and asynchronism, and their structure – by high level of complexity.

The Nets of Petri (PN) are applied to these problems due to their possibilities to model the structure of the object investigated and also to model the dynamics of the processes running in it [2]. The Generalized Nets (GN) are their modern extension, suggesting a more detail reflecting of the structural and time connections in parallel processes and a more compact graphical form [3]. But defining which part of the process should get its reflection in the transition structure, and which – in its predicate conditions, is a non-trivial problem in the general case. A possibility to solve it is to use models already developed on the basis of PN and on their ground to build GN-models.

When synthesizing new models of information interactions [4] with the help of PN apparatus, the question should be raised whether the new network obtained enables the achievement of a certain objective. In this situation the synthesis process has to be specified by a more powerful than PN formal tool. It is appropriate this tool to contain PN formalism as a subset. The Generalized Nets apparatus provides such a possibility.

The paper presented discusses a GP model for synthesis of information processes, described by PN tools.

2. Discrete structures synthesis

The initial data for each process of discrete structures synthesis include:

a) formal description of the structure (a static part);

b) initial conditions, from which the processes functioning starts (defining processes development);

c) final status that has to be reached by the system modeled (purpose);

d) criteria (constraints), that have to be always satisfied in the synthesis (for comleteness, correctness):

e) requirements that should be kept in order to complete the synthesis (synthesis operations).

The formal definition of PN structure is given by the following definition [2]:

Definition 1. The four elements below given are called a Petri Net:

$$C = (P, T, I, O),$$

where $P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of positions, $n \geq 0$ and $T = \{t_1, t_2, \ldots, t_m)$ is a finite set of transitions, $m \geq 0$. The set of positions and the set of transitions do not cross, i.e. $P \cap T = \varnothing$, I: $T \rightarrow P^{\infty}$ is an input function – the image of the transitions in complects positions. O: $T \rightarrow P^{\infty}$ is an output function – the image of transitions in complects positions.

Definition 2. The function, representing the set of positions P into a set of non-negative integers N is called marking μ of Petri Net $C = (P, T, I, O)$:

$$\mu: P \rightarrow N.$$

The marking μ can be regarded as n-vector $\mu = (\mu_1, \mu_2, ..., \mu_n)$ where $n = |P|$ and each $\mu_i = N$, $I = 1, \ldots, n$. The vector μ defines the number of cores at a position for each position p_i of PN.

The initial conditions, from which the functioning of one PN starts (sub-point b) are given by a vector of initial marking μ^0.

The status that has to be reached by the system modeled (sub-point c) is expressed by vector μ^p, which will be called "objective (purpose) marking".

On the basis of B. Kulagin's methodology [5], the constraints are given, which have to be always satisfied (sub-point d), and the synthesis operations (sub-point e) are specified [6].

3. GM – a model of the synthesis process

We shall use the GM as a meta tool to describe the complete process of PN synthesis, which is possible since GN are a powerful extension of the classical PN.

Our task is to construct a model of the process of PN synthesis in the form of a GN, with the purpose to obtain objective marking with the help of the PN obtained. The problem solution is graphically shown as a GM model in Fig. 1.

The problem is solved in two stages. At first a generalized common GN model of the synthesis of PN structure is built (z1-z6), after which a model is built of the process of checking for objective marking achievement (decomposition of z6) and an evaluation is made of the properties of the combined model obtained.

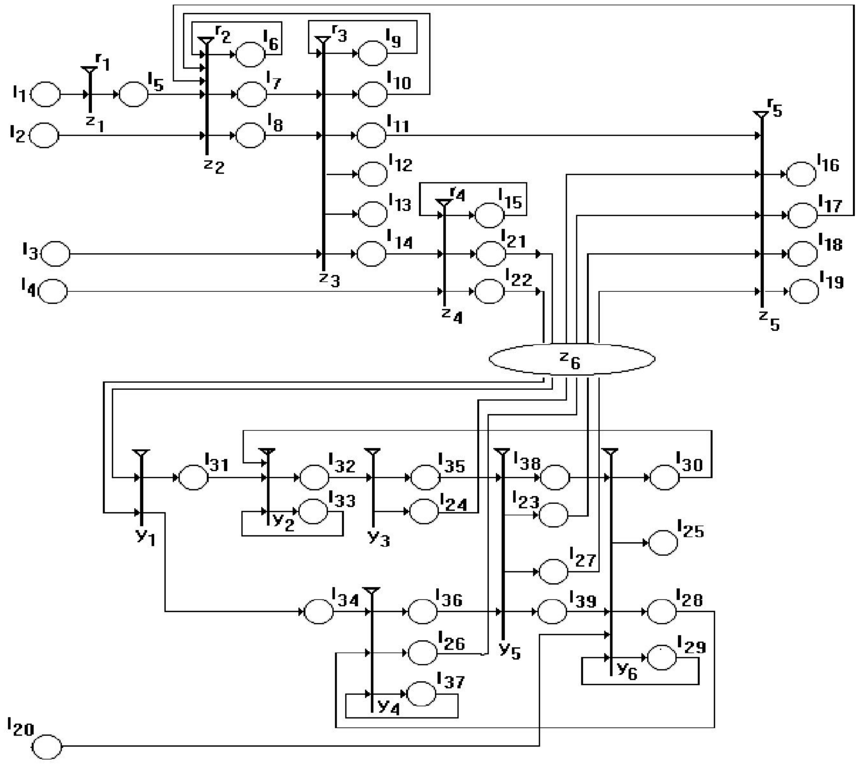

Fig. 1. GN-model of the synthesis process

The core α enters position l_1 with an initial characteristics "output structure description". The core β enters position l_2 with initial characteristics "requirements".The core γ enters position l_3 with initial characteristics "criteria". The core δ enters position l_4 with initial characteristics "initial marking, *Ct*", where *Ct* is a given constant (connected with the maximal number of markings [7]). The core ε enters position l_{19} with initial characteristics "objective". The priority of α is the greatest, and that of ε – the lowest.

The formal description has the form:

$z_1 = \langle \{l_1\}, \{l_5\}$, where $r_1, \wedge l_1 \rangle$, where

$r_1 =$

	l_5
l_1	True

The core α enters position l_1 with initial characteristics x^{α}_0 = "description of the output structure". The characteristic function is Φ_5= "description of the primitive system (PS)", which is a result of the analysis of the output structure according to [5].

$z_2 = \langle \{l_2, l_5, l_6, l_{10}, l_{17}\}, \{l_6, l_7, l_8\}, r_2, \wedge (v\ (l_5, l_6), v\ (l_2, l_{10}, l_{17})) \rangle$, where

$r_2=$	l_6	l_7	l_8
l_2	False	False	True
l_5	True	True	False
l_6	True	True	False
l_{10}	False	False	True
l_{17}	False	False	True

The core β enters position l_2 with initial characteristic x^{β}_0= "list of the positions and transitions that are to be connected". The characteristic function is Φ_5= "description of the connected primitive nets (CPN), as a result of the operations "Connect" [6]. Φ_6 and Φ_8 are "*" (they do not alter the characteristics).

z_3= <{l_3, l_7, l_8, l_9}, {l_9, l_{10}, l_{11}, l_{12}, l_{13}, l_{14}}, r_3, ^(v (l_3, l_9), l_7, l_8))>, where

$r_3=$	l_9	l_{10}	l_{11}	l_{12}	l_{13}	l_{14}
l_3	True	False	False	False	False	False
l_7	False	False	False	False	$\neg W_1$	W_1
l_8	False	$\neg W_1 \wedge W_2$	W_1	$\neg W_1 \wedge \neg W_2$	False	False
l_9	True	False	False	False	False	False

The core γ enters position l_3 with initial characteristic x^{γ}_0= "list of constraints". The characteristic functions Φ_9, Φ_{11} and Φ_{13} are "*"and their meanings are as follows:

Φ_{10}= "modification of the requirements (a list);

Φ_{12}= "no other modification of the requirements is possible";

Φ_{14}= "there is a new PN-structure synthesized", as a result of the matrix transformation [6].

The predicates W_1 and W_2 have the following form:

W_1= "the constraints are satisfied"

W_2 = "the requirements can be modified"

z_4= <{l_4, l_{14}, l_{15}}, {l_{15}, l_{20}, l_{21}}, r_4, ^(v (l_4, l_{15}), l_{14}))>, where

$r_4=$	l_{15}	l_{21}	l_{22}
l_4	True	False	True
l_{14}	False	True	False
l_{15}	True	False	False

The core δ enters position l_4 with initial characteristic x^{δ}_0= "<m_1, m_2, …, m_8>" where the natural numbers m_1, m_2, …, m_8 are elements of the initial marking, result from the analysis of the output PN-model [5]. The characteristic function is Φ_{22}= "initial marking of the PN synthesized", which is a result of the transformation of m_1, m_2, …, m_8 [6]; Φ_{15} and Φ_{21} are "*".

z_5= <{l_{11}, l_{23}, l_{24}, l_{26}, l_{27}}, {l_{16}, l_{17}, l_{18}, l_{19}}, r_5, ^(v (l_{23}, l_{24}), v (l_{26}, l_{27}), l_{11})>, where

$r_5=$	l_{16}	l_{17}	l_{18}	l_{19}
l_{11}	False	W_3	False	$\neg W_3$
l_{23}	True	False	False	False
l_{24}	True	False	False	False
l_{26}	False	False	True	False
l_{27}	False	False	True	False

The characteristic functions Φ_{16}, Φ_{18} and Φ_{19} are "*"and

Φ_{17} = "modification of the requirements (a list)"

The predicate W_3 has the form:

W_3 = "there is a core in position l_{29}"

$z_6 = \langle\{l_{20}, l_{21}, l_{22}, l_{28}, l_{30}\}, \{l_{23}, l_{24}, l_{25}, l_{26}, l_{27}, l_{28}, l_{29}, l_{30}\}, r_6, v\,(\wedge(l_{20}, l_{21}, l_{22}), \wedge(l_{28}, l_{29}, l_{30}))\rangle$, where

$r_6=$	l_{23}	l_{24}	l_{25}	l_{26}	l_{27}	l_{28}	l_{29}	l_{30}
l_{20}	False	False	False	False	False	False	True	False
l_{21}	False	False	False	False	False	False	False	True
l_{22}	False	False	False	False	False	True	False	False
l_{28}	False	False	False	$\neg W_4 \wedge W_5$	W_4	$\neg W_4 \wedge \neg W_5$	False	False
l_{29}	False	False	W_6	False	False	False	$\neg W_6$	False
l_{30}	W_4	$\neg W_4 \wedge W_5$	False	False	False	False	False	$\neg W_4 \wedge \neg W_5$

The core ε enters position l_{20} with initial characteristic x^{α}_{0}= "list of those positions in the synthesized PN, in which there should be a core". The characteristic function Φ_{29} is "*" and

Φ_{23}= "a limit has been reached"

Φ_{24}= "no transitions permitted"

Φ_{25} = "the purpose is achieved"

Φ_{26} = "terminal marking"

Φ_{27} = "the limit for marking is surpassed"

Φ_{28} = "a new marking"

Φ_{30} = "a new transition allowed"

The core α enters position l_{21} with initial characteristic x^{α}_{0}=, and the core δ enters position l_{22} with initial characteristic x^{δ}_{0}=(as above described).

The characteristic function Φ_{31} is already defined and Φ_{34} is *".

$y_2 = \langle\{l_{30}, l_{31}, l_{33}\}, \{l_{32}, l_{33}\}, n_2, \vee(l_{30}, l_{31}, l_{33})\rangle$, where

$n_2=$	l_{32}	l_{33}
l_{30}	True	W_7
l_{31}	True	W_7
l_{33}	True	W_7

This transition could split the core α, and the identifier “with” could be respectively modified for position l_{32}. The characteristic functions are:

$\Phi_{32} \rightarrow$ “the first element of the list of x”

(this characteristic is accepted by the core with an identifier “$x^{\alpha,c}{}_{\text{last}}$”, which comes from positions l_{30}, l_{31} or l_{33}),

$$\Phi_{33} \rightarrow \text{“ } x^{\alpha',c}{}_{\text{last}} - \{x^{\alpha,c}{}_{\text{last}}\}\text{”}$$

The predicate has the following form W_7 = “card $x^{\alpha,c}{}_{\text{last}} > 1$”

$$y_3 = <\{ l_{32}\}, \{l_{24}, l_{35}\}, n_3, \wedge(l_{32})>, \text{ where}$$

$$n_3 = \begin{array}{c|cc} & l_{24} & l_{35} \\ \hline l_{32} & W_5 & \neg W_5 \end{array}$$

The characteristic function Φ_{35} is “*” and Φ_{24} is above described.

The predicate W_5 is modified into W_5 = “card $x^{\alpha,c}{}_{\text{last}} = \varnothing$”.

$$y_4 = <\{l_{28}, l_{34}, l_{37}\}, \{l_{26}, l_{36}, l_{37}\}, n_4, \vee(l_{28}, l_{34}, l_{37})>, \text{ where}$$

$$n_4 = \begin{array}{c|ccc} & l_{26} & l_{36} & l_{37} \\ \hline l_{28} & \text{False} & \text{False} & \text{True} \\ l_{34} & \text{False} & \text{False} & \text{True} \\ l_{37} & W_5 & \neg W_5 & W_8 \end{array}$$

The transition can split the core δ, without modifying its identifier. The characteristic functions Φ_{36} and Φ_{37} are “*”, and Φ_{26} is above defined.

The predicate has the following form : W_8 = “there is a core in position l_{33}”.

$$y_5 = <\{l_{35}, l_{36}\}, \{l_{23}, l_{27}, l_{38}, l_{39}\}, n_5, \wedge(l_{35}, l_{36})>, \text{ where}$$

$$r_5 = \begin{array}{c|cccc} & l_{23} & l_{27} & l_{38} & l_{39} \\ \hline l_{35} & \neg W_4 & \text{False} & \neg W_4 & \text{False} \\ l_{36} & \text{False} & W_4 & \text{False} & \neg W_4 \end{array}$$

This transition modifies the identifier “c” of core δ (for position l_{39}). The characteristic functions Φ_{23} and Φ_{27} are already defined. Φ_{38} accepts the function of Φ_{30} (Φ_{30} and Φ_{28} become “*”), i.e.

$$\Phi_{39} = \begin{cases} m_i + V(t, p_i), \text{ if} & p_i \in t\bullet \text{ and } t = x^{\alpha,c}{}_{\text{last}}, \\ m_i - V(p_i, t), \text{ if} & p_i \in \bullet t \text{ and } t = x^{\alpha,c}{}_{\text{last}}, \\ m_i & \text{otherwise.} \end{cases}$$

The predicate W_4 is modified to: W_4 = “length (c) = Ct”

$$y_6 = <\{l_{20}, l_{29}\, l_{38}, l_{39}\}, \{l_{25}, l_{28}, l_{29}, l_{30}\}, n_6, \wedge(V(l_{20}, l_{29}), l_{38}, l_{39})>, \text{ where}$$

$n_4 =$	l_{25}	l_{28}	l_{29}	l_{30}
l_{20}	False	False	True	False
l_{29}	W6	False	$\rceil W_6$	False
l_{38}	W6	False	False	$\rceil W_6$
l_{39}	W6	$\rceil W_6$	False	False

The characteristic functions Φ_{30} and Φ_{28} are "*", and Φ_{29} is "*" as well. Φ_{25} is above defined, the predicate W_6 also.

The priorities of the split cores α and δ are important for this transition. We assume that in cores with identifiers "α, c", and "α', c" the last has got the higher priority. In cores with identifiers "α, i_1, I_2, ..., i_{q-1}, 1" and "α, i_1, I_2, ..., i_{q-1}, 2" the first one has got the higher priority. The same is valid for cores δ. In this way when permitting a transition (a concession), the movement of core α and core δ with equal identifiers is guaranteed.

4. Properties of the model constructed

The capacity of all the arcs is equal to one. The number of the characteristics of cores α and δ cannot be apriori defined, but the constant *Ct* guarantees the existence of an upper constraint. The selected form of identifiers "α, c" = "α, i_1, i_2,…, i_q" and of predicate W_4 enables the connection $q = Ct$, i.e. *Ct* is the number of hierarchical levels of the tree of achievable markings that will be computed. This is a protection in case of cycles existence in PN. A high value for *Ct* is over-assuring in most of the cases, since maximally quick reaching of the purpose is usually seeked (avoiding the unnecessary cycles). Nevertheless, if possible, the aim will be achieved.

The priorities of the positions are equivalent one to another, and their capacities are different. With exception of positions l_{23}, l_{24}, l_{25} , l_{26} and l_{27}, all the remaining ones have a capacity equal to one. Position l_{25} has got capacity equal to 3. The capacities of …???

The predicates W_4, W_5, W_6 have the following form:

W_4 = "the number of x^{δ}_{last}: |last| = *Ct*" (a limit has been reached),

W_5 = " $x^{\alpha}_{\text{last}} = \varnothing$" W_6 = " $\varepsilon \subseteq x^{\delta}_{\text{last}}$"

The capacities of all the arcs are equal to one. The capacities of all the positions, with the exception of l_{13}, l_{16} and l_{18} are also equal to one. The number of the cores characteristics is not apriori defined but constant *Ct* limits it above for core δ. Priorities of the positions and transitions are not necessary.

This GM does not possess local and global time components. Hence, the GM thus constructed is a reduced form of the common class $\sum$ of all GN. It can describe the synthesis process of other discrete structures also after minimal alterations in the characteristic functions and predicates.

Transition z_6 is a union of transitions y1-y6.

The input positions for transition z_6 are l_{21}, l_{22} and l_{20}.

A core with characteristic x^{α}_0= "structure of the PN synthesized" enters position l_{21}. The PN synthesized does not have loops as a sequence of the synthesis methodology [7, 8]. i.e. x^{α}_0= "P, T, D, V, Ct" , where $P = \{p_1, p_2, ..., p_s\}$ is the set of positions, $T = \{t_1, t_2, ..., t_u\}$ is the set of transitions, $D : (P\times T) \cup (T\times P) \to \{0, 1\}$ is the incidence matrix (representing the input and output functions) , V: $(P\times T) \cup (T\times P) \to N$ is a function of the capacity of PN arcs. Ct is a given constant [9].

A core δ with characteristic x^{δ}_0= "$m_1, m_2, ..., m_s$>" will enter position l_{22}, where the natural numbers $m_1, m_2, ..., m_s$ define the initial marking of the PN synthesized.

A core ε with an initial characteristic "x^{α}_0= "objective marking" will enter position l_{20}. The objective marking is naturally a vector with a dimension of the vector of initial marking of the PN synthesized. The components of this vector are with a value 0 for positions that are not important and natural numbers (greater than 0) in the remaining (significant by default) positions. This is a more general case in comparison with the strict equality, when all the positions are important and it is reflected in the form of predicate W_6. It is a sequence of the often-met situation to allow not "strict determination" of the objective. This flexibility, permitted by the GN is useful, and since GN enables it, we will consider it an advantage of GN.

The core α is of the highest priority, followed by δ, and the core ε is of the lowest priority. The capacity of the positions and the arcs is equal to one.

PN stops its functioning when all the achievable markings are terminal (cores in positions l_{24} and l_{26}). The transition will not be allowed when constant Ct (cores in positions l_{23} and l_{27}) is reached, and also when the objective marking is attained (covered – a core at position l_{25}).

Is there any necessity for a GN model, which will account the specifics of functioning of the classical PN? For this purpose a GN model is used, which represents the functioning of all the modifications of PN [7], introducing some alterations necessary to achieve our aim (including an alteration in some elements like predicates, priorities and capacities).

According to [7] a core in GN can be divided into several cores. When core α is spitted for the first time, the new cores will get identifiers "α, l", ..., "α, q_l" (where $q_l \geq 1$ is the number of the different new cores generated by α). When a core with identifier "α, i" ($l \leq i \leq q_1$)is splitted, the new cores will obtain identifiers "α, i, l", ..., "α, i, q_2" (where $q_2 \geq 1$ is the number of the different new cores generated by the core with identifier "α, i") and so on. The same is valid for core δ.

The core α obtains in positions l_{31} and l_{38} a characteristic (by the characteristic functions Φ_{31} and Φ_{38} that are identical), "$t_{i_1}, t_{i_2}, ..., t_{i_q}$" where $l \leq i_1 < i_2 <...< i_q \leq u$, and for each t_{i_j} ($1\leq j \leq q$) it is true that if $p_v \in \bullet t_{i_j}$, then $m_v \geq V(p_v , t_{i_j})$, i.e. the core obtains as a characteristic a list of the permitted transitions (PN-transitions enabled for activating) where $\bullet t$ is the set of input positions of PN transition t.

Let "c" be the current ranked set of indices having the form above given. At the first step (position l_{31}): $x^{\alpha,c}_{\text{last}} = x^{\alpha,l}$, where $x^{\alpha,c}_{\text{last}}$ is the last characteristic of the core α with identifier "α, c". If $c = i_1, i_2, ..., i_q$ ($q \geq 1$) then let "$c = i_1 , i_2,..., i_{q-1}$ and length (c)= q.

The transitions have a priority in the sequence $y_6, y_5, , y_3, , y_2, , y_1$, the highest priority belonging to y_6.

The description of the transitions in this case is:

$$y_1 = <\{l_{21}, l_{22}\}, \{l_{31}, l_{34}\}, n_1, \wedge (l_{21}, l_{22})>, \text{ where}$$

$$
n_1 = \begin{array}{c|cc} & l_{31} & l_{34} \\ \hline l_{21} & \text{True} & \text{False} \\ l_{22} & \text{False} & \text{True} \end{array}
$$

Positions l_{23}, l_{24}, l_{26} and l_{27} depend on the splitting of cores α and δ. In the general case the capacity of l_{23} and l_{27} will not exceed the number $(u–1)^{(Ct–1)}$. This is the upper limit – in case that an active transition (in the PN synthesized) generates always a list of (u–1) active transitions – for the next hierarchical level of achievable markings. This is seldom met. The same number is an upper limit for the capacity of positions l_{24} and l_{26} and we can say the same – it is an infrequent case to obtain a large number of terminal markings.

The capacity of the positions is a finite number. In this sense the model possesses the necessary property of computability – the process of PN functioning has got an end.

5. Conclusion

The GN model constructed has the potential to develop. Altering the matrix of predicates of the last (z6) transition, problems of achieving some different by a definition purposes can be solved.

The GN-model does not possess local and global time components. The Generalized Net constructed has a reduced form with respect to the common class Σ of all the GN. Its software realization will require less powerful computing resources.

References

1. ISO IS 7498. Information Processing System – Open System Interconnections – Basic Reference Model. 1983.
2. P e t e r s o n, J. L. Petri Net Theory and the Modeling of Systems. Prentice Hall, New York, 1981
3. A t a n a s s o v, K. Generalized Nets. World Scientific, Sing., N.J., London, 1991.
4. T a s h e v, T., H. H r i s t o v. On One Approach for Modification and Expansion of the Information Interaction Models. Problems of Eng. Cybernetics and Robotics, Vol. 49, 2000. Acad. Press “Prof.M.Drinov”, Sofia, Bulgaria. 78-87.
5. К у л а г и н, В. Алгебра сетевых моделей для описания параллельных вычислительных систем. Автоматизация и соврем. технологии. 1993. N2, 25 - 30.
6. H r i s t o v, H., T. T a s h e v. Computer Aided Synthesis of Interacting Processes Models. Problems of Eng. Cybernetics and Robotics, Vol. 51, 2001. Akad. Press “Prof.M.Drinov”, Sofia, Bulgaria. 20-25.
7. A t a n a s s o v, K. The generalized nets which represents all Petri nets. AMSE Review, Vol.12, No3, 1990, 33-37.