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# Application of Monte Carlo Simulation in Pricing of Options[1]

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***Abstract:*** *In this paper we consider a Monte Carlo technique for valuation of derivatives securities. Metropolis algorithm is used to sample probability distribution of histories of underlying stocks. We consider options on portfolios consisting of linear combinations of correlated log-normal assets, including basket and spread options. The present purpose is to examine feasibility and accuracy of the method, so we start with the simplest valuation problem of a European call on a stock with constant volatility and no dividends, where we can easily compare Monte Carlo results with the analytic Black- Scholes solution. In this relation a practical example is discussed in part three.*

***Keywords:*** *Monte Carlo simulation, Metropilis algorithm, assets.*

## I. Application of Monte Carlo simulation

A great number of Monte Carlo simulation models are known and used in practice. The purpose of this paper is not related to the presentation of all the models, but only to some of the major applications of Monte Carlo simulation processes for options evaluation. Accent is placed on the analysis of the results which have been obtained on the basis of practical examples and experiments connected with the application of the simulation models.

### 1. Base models of Monte Carlo method

Monte Carlo simulation is suitable for options, in which tendencies are observed for certain parameters development. This model can also be improved for evaluation of options, whose assessment is influenced by many stochastic processes, accidental variables, etc. Its main disadvantage is that it requires very

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intensive calculations, since a great number of simulations are usually necessary. With the purpose of solving this problem, some techniques have been developed related to variance reduction, control variates, antithetic variates, etc.

Despite that this technique is efficiently used for the European options evaluation, it is not applied on a mass scale for the American derivatives, which are usually evaluated by numerical methods. The problem with the American options is connected with the fact, that the possible dates of exercising the option are several. That is why the option owner has to decide whether to exercise the option or to wait for every of the possible moments. The decision depends on:

a) the amount which the exercising of the option will bring to him (the value of the immediate exercising);

b) the amount which he/she will receive in eventual future exercising (the value of continuation).

Due to this reason the optimal decision will be based on the assessment of the option value on the date of exercising. Recently Longstaff and Schwartz [13] have developed an algorithm where the price of the non-exercised option is evaluated better by means of a regression model with the help of the Least Squares Method (LSM). This method is used along with the intermediate technique of the Least Squares Monte Carlo Simulation (LSMCS) [13]. In this regression model a group of basic functions is used, whose parameters are based on the prices of the assets. The values obtained from these regression models are used as expected values of continuation. The optimal rule for stopping is determined comparing the values of continuation with these of immediate exercising. This procedure is recursively repeated back in time. The price of the American option is found by discounting the cash flows obtained to the initial point. In this relation, in order to understand how the Least Squares Monte Carlo simulation is applied, we have to analyze the so-called risk-neutral evaluation [6]. When using risk-neutral evaluation, in contrast to the European call option, whose equation is given by:

$$E_{\text{call}} = E\left[\exp\left(-\int_0^x r(\overline{w}, s)ds\right)\max\{S(T) - K, 0\}\right],$$

The problem of pricing American call option is to find

$$A_{\text{call}} = \max_r E\left[\exp\left(-\int_0^r r(\overline{w}, s)ds\right)\max\{S(\tau) - K, 0\}\right]$$

for all the moments of discontinuance $\tau \leq T$. In this case $S(\tau)$ is the final asset price at moment $\tau$, $K$ is the price of exercising, $T$ is the date of the option maturity, and $r(\overline{w},s)$ is the possible risk-free interest, related to given tendency of development $w$.

We suppose here that the American option can be exercised only in $L$ discrete moments, so that $0 < t_1 \leq t_2 \leq ... \leq t_{L-1} \leq t_L = T$. In practice the American options can be exercised continuously in time and the option price can be defined taking a sufficiently great number for $L$.

The Least Squares Monte Carlo simulation method consists of two major steps. For the method used $N$ simulations of the stochastic model are necessary. The first of them are related to the assessment of the coefficients of $V(w, t_i)$, where the formula for $V(w, t_i)$ is $V(w, t_i) = a_{i0} + a_{i1} X + a_{i2} X^2$, and of $C(w, s, t_i, T)$ on the basic functions for given tendencies, which are the so-called “in-the-money" options at a fixed moment $t_i$. Based on the function of conditional expectation, the second step is to make a decision for early exercising at moment $t_i$, comparing the value of immediate exercising with the value $V(w, t_i)$ for any possibility of "in-the-money" options. Immediately after taking the decision for option exercising, the option cash flows $C(w, s, t_i, T)$ can be specified. The analysis continues by our returning to the preceding moment $t_{i-1}$ and repeating the procedure until the decisions for the option exercising at any moment for any possibility (tendency of development) are taken. Finally, the price of the American option is given by the following formula:

$$\hat{A}_{\text{catt}} = \frac{1}{N}\sum_{k=1}^{N} \exp\left(-\int_0^{t_i^{(k)}} r(w, s)ds\right) \max\left\{S\left(t_i^{(k)}\right) - K, 0\right\}$$

if the optimal time $t_i^{(k)}$ exists for the pricing $k$, $\max\left\{S\left(t_k^i\right) - K, 0\right\} = 0$.

Let us consider a numerical example, which shows that if we use the Monte Carlo simulation for the least squares, some values are obtained, with which the American option may have a lower price than the corresponding European one.

For example, the American put option is evaluated, under which no dividends are paid. The price of exercising is 1.1 and we have three possible dates of exercising. The compound risk-free interest in the option continuation is equal to 0.05. For the purposes of the analysis we simulate eight tendencies of development of the asset price, as shown in Table 1 [12].

Table 1. The valuation American put option with the Least Squares Monte Carlo Simulation

| Path | $t$=0 | $t$=1 | $t$=2 | $t$=3 | Payoff at $t$=3 |
|---|---|---|---|---|---|
| 1 | 1 | 0.917938 * | 1.272171 | 1.417021 | 0 |
| 2 | 1 | 1.133931 | 1.290983 | 1.669802 | 0 |
| 3 | 1 | 1.162833 | 0.917742* | 1.228432 | 0 |
| 4 | 1 | 1.096706* | 1.081163* | 1.118280 | 0 |
| 5 | 1 | 1.056690* | 0.871784* | 0.818722* | 0.281278 |
| 6 | 1 | 1.416442 | 1.672474 | 1.263264 | 0 |
| 7 | 1 | 0.937138* | 0.945920* | 0.861259* | 0.238741 |
| 8 | 1 | 0.872576* | 0.658605* | 0.475270* | 0.624730 |

The last column of the table shows the realized final yield of the European option. Discounting these yields at moment 0 and finding the average, we can define that the price of the European option is equal to 0.123162. The symbol (*) means that these options are "in-the-money".

For the American put option, the Least Squares Monte Carlo simulation maximizes its value on any date of exercising along the separate trajectories when the options are "in-the-money". For each date we mark by $X$ the price of the reviewed asset and by $Y$ (the discounted) cash flows obtained on future dates, if the option will not be exercised. In this relation, the values obtained are presented in Table 2.

Table 2. The valuation American put option with the Least Squares Monte Carlo Simulation when all the options are "in-the-money".

| Path | $Y$ | $X$ |
|---|---|---|
| 1 | - | - |
| 2 | - | - |
| 3 | $e^{-0.05}\times 0$ | 0.917742 |
| 4 | $e^{-0.05}\times 0$ | 1.081163 |
| 5 | $e^{-0.05}\times 0.281278$ | 0.871784 |
| 6 | - | - |
| 7 | $e^{-0.05}\times 0.238741$ | 0.945920 |
| 8 | $e^{-0.05}\times 0.624730$ | 0.658605 |

The use of multi-asset options as hedging instruments has the advantage that unless the underlying assets are perfectly correlated, the volatility of the multi-asset product or portfolio can be less than that of the individual assets. This should lead in most of the cases to a more cost-efficient hedge as compared to the use of a collection of single-asset options. In this paper we consider put and call options on portfolios consisting of linear combinations of different assets, including basket and spread options.

Numerical integration formulae can provide approximate values for the integrals, with the accuracy of the values improving as the amount of computational effort expended increases. A compromise must be achieved in practice between the amount of the time spent to obtain an option value and the reliability or accuracy of that value. A good method will enable given accuracy to be attained with minimal amount of work, or conversely allow maximal increase in the accuracy for a given increase in the computational time.

Gauss-Hermite integration formulae take the form:

$$\int_R e^{-\frac{1}{2}y^2} f(y)dy \approx \sum_{q=1}^{Q} w_q f(y_q),$$

where $w_q$ are the weights and $y_q$ are the abscissae. Such a formula is exact, if $q$ is a polynomial of a degree less than $2Q$; it will be accurate if $f$ can be closely approximated by such a polynomial, which will be the case if $f$ is smooth. In this paper we do not consider the numerical evidence of convergence: theoretical error estimates and the proof of convergence will be a subject of future work [19].

When the approximation is applied to the recursive formulation of the option pricing formula, we obtain

$$\overline{V}_n\left(\overline{S}, A, \overline{K}\right) \approx \frac{1}{\sqrt{2\pi}} \sum_{q=1}^{Q} w_q \overline{V}_{n-1}\left(\overline{S}_{(2)} e^{A_{(21)} y_q}, A_{(22)}, \overline{K} - \overline{S}_1 e^{A_{11} y_q}\right).$$

The one-dimensional formulae are evaluated using standard routines for the calculation of cumulative normal distributions.

Some simplifications are possible. If the asset weights $\theta_i$ are all positive (so that $\bar{S}_i$ are also positive), $\bar{K}$ is negative. On the other hand, if all the asset weights are negative, and $\bar{K} \le 0$, then the option is worthless. We obtain Algorithm 1 for the evaluation of $\overline{V_n}(\bar{S}, A, \bar{K})$.

The equations can be used to provide a similar algorithm for the calculation of the option deltas. The deltas for the special cases can be found differentiating the corresponding option formulae directly. Incorporating these results gives Algorithm 2 or the calculation of $D_i \overline{V_n}(\bar{S}, A, \bar{K})$.

Once a value for $D_i \overline{V_n}(\bar{S}, A, \bar{K})$ has been found, the true option deltas can be deduced via

$$\frac{\partial V(S)}{\partial S_i} = \theta_i e^{-d_i T - M_{ii}/2} D_i \overline{V_n}(\bar{S}, A, \bar{K}).$$

Notice that Algorithm 2 can be coded in such a way as to provide all the deltas in a single recursive calculation.

Here we consider two option payoffs on two underlying assets. The first one is a call option on a basket and the second one – a call option on a spread. We assume (continuously compounded) interest rate of 10%, and dividend payments of 5% and 0% on the two assets. The volatilities of the two uncorrelated lognormal assets are both 20% and both assets are assumed to have an initial value of $100. The expiry time for each option is 6 months. The option payoffs are:

*Basket*: max $(S_1 + S_2 - 200, 0)$,

*Spread*: max $(S_1 - S_2 - 2, 0)$.

The results of the calculations are shown in Table 3 for the two options. "True" option values were calculated in each case using Algorithm 1 with $N_q = 60$ integration points.

Table3. Multi-assets options

| $N_q$ | Basket | | | | Spread | | | |
|---|---|---|---|---|---|---|---|---|
| | Option value $V = \$\ 8.26120$ | | | | Option value $V = \$\ 6.46868$ | | | |
| | value | error | time | resuts | value | error | time | resuts |
| 1 | 7.25179 | 1.009 | 0.02 | 227 | 5.61559 | 0.85 | 0.02 | 179 |
| 2 | 8.32673 | 0.06553 | 0.02 | 336 | 6.54334 | 0.075 | 0.03 | 359 |
| 3 | 8.25069 | 0.01051 | 0.02 | 785 | 6.46004 | 0.0086 | 0.03 | 760 |
| 4 | 8.26363 | 0.002435 | 0.03 | 1266 | 6.46927 | 0.00059 | 0.03 | 1266 |
| 5 | 8.26046 | 0.0007364 | 0.04 | 2076 | 6.46889 | 0.00021 | 0.04 | 2074 |
| 6 | 8.26146 | 0.0002618 | 0.04 | 3320 | 6.46853 | 0.00015 | 0.04 | 3320 |
| 7 | 8.26110 | 0.0001006 | 0.04 | 4454 | 6.46875 | $6.5e^{-0.5}$ | 0.04 | 4462 |
| 8 | 8.26124 | $3.97e^{-0.5}$ | 0.05 | 6154 | 6.46866 | $2.5e^{-0.5}$ | 0.05 | 6210 |
| 9 | 8.26118 | $1.569e^{-0.5}$ | 0.05 | 8176 | 6.46869 | $9.1e^{-0.6}$ | 0.05 | 8161 |
| 10 | 8.26121 | $6.096e^{-0.5}$ | 0.06 | 11274 | 6.46868 | $3.2e^{-0.6}$ | 0.06 | 11340 |
| 11 | 8.26120 | $2.3e^{-0.6}$ | 0.07 | 14226 | 6.46868 | $1.1e^{-0.6}$ | 0.06 | 14188 |
| 12 | 8.26120 | $8.306e^{-0.7}$ | 0.07 | 18032 | 6.46868 | $3.6e^{-0.7}$ | 0.07 | 18088 |
| 13 | 8.26120 | $2.805e^{-0.7}$ | 0.07 | 21990 | 6.46868 | $1.2e^{-0.7}$ | 0.07 | 22008 |
| 14 | 8.26120 | $8.466e^{-0.8}$ | 0.08 | 26796 | 6.46868 | $3.9e^{-0.8}$ | 0.08 | 26852 |
| 15 | 8.26120 | $2.016e^{-0.8}$ | 0.08 | 33578 | 6.46868 | $1.3e^{-0.8}$ | –0.08 | 33576 |

The results of the calculations are illustrated in Figs. 1, 2 and 3. The figures show the relative errors in the computed option values (i.e., errors relative to the actual option value) for $N_q$ ranging from 1 up to 10. The relative errors are shown in Fig. 1 (for various option expiry times) on a log-log scale as functions of the time required to obtain the corresponding option values. The same number is plotted in Fig. 2 against the number of floating point operations used. In each case the results shown in the four subplots were generated using input data that differed only in the time of expiry for the options being evaluated. Finally we show the results of a similar calculation on a basket of just three assets. Fig. 3 shows the relative errors as a function of time.

## 2. Metropolis algorithm

Before describing the advantages of promoting complete paths to be the fundamental objects of Monte Carlo simulation, we describe Metropolis method for generating probability distribution of the paths, in order to be able to take advantages of importance sampling. Metropolis method constructs a Markov process in the path space, which asymptotically samples the path probability distribution. This process is not related to Markov process that governs the evolution of the state variables. Since it is the formal device to obtain the desired distribution, there is much freedom in constructing this process, which will prove advantageous for variance reduction techniques [5].

The Markov process will be defined by the transition probability $W(\Omega_1 \to \Omega_2)$, which denotes the probability of reaching point $\Omega_2$ starting from $\Omega_1$. There are two restrictions on the choice of the probability $W$. First, the stochastic dynamics defined must be ergodic, i.e., every point in the path space must be accessible. The second requirement is that the transition probability must satisfy the "detailed balance condition"

$$P(\Omega_1)W(\Omega_1 \to \Omega_2) = P(\Omega_2)W(\Omega_2 \to \Omega_1).$$

These two restrictions do not specify uniquely the stochastic dynamics. We will use the transition probability proposed by Metropolis (1953), [8], which is known as the Metropolis algorithm:

$$W(\Omega_1 \to \Omega_2) = \begin{cases} P(\Omega_2)/P(\Omega_1) & \text{if } P(\Omega_1) \geq P(\Omega_2), \\ 1 & \text{if } P(\Omega_1) < P(\Omega_2). \end{cases}$$

We now outline a proof that this Markov chain will asymptotically sample the desired distribution $P(\Omega)$ [1]. If we start from an initial probability distribution $P_0(\Omega)$, then the probability distribution after  steps of the Markov chain will be denoted by $P_n(\Omega)$. Probability distribution at successive Markov steps $n$ and $n+1$ satisfy the following relationship:

$$P_{n+1}(\Omega_{n+1}) = \int D\Omega_n W(\Omega_n \to \Omega_{n+1}) P(\Omega_n).$$

$P(\Omega)$ is the fixed point distribution of this Markov process. Substitution of $P(\Omega)$ for $P_n(\Omega)$ in the above equation, combined with the detailed balance condition, implies that $P_{n+1}(\Omega) = P_n(\Omega) = P(\Omega)$. One has also to show that the distribution converges towards $P(\Omega)$. A simple measure of deviation from the desired distribution is $D_n = \int D\Omega \left| P_n(\Omega) - P(\Omega) \right|$. Deviation decreases as one goes further along the Markov chain:

$$D_{n+1} = \int D\Omega_{n+1} \left| \int D\Omega_n W(\Omega_n \to \Omega_{n+1}) P_n(\Omega_n) - P(\Omega_{n+1}) \right|,$$

$$D_{n+1} = \int D\Omega_{n+1} \left| \int D\Omega_n W(\Omega_n \to \Omega_{n+1}) \left[ P_n(\Omega_n) - P(\Omega_n) \right] \right|,$$

$$D_{n+1} \le \int D\Omega_{n+1} \int D\Omega_n W(\Omega_n \to \Omega_{n+1}) \left| P_n(\Omega_n) - P(\Omega_n) \right| = D_n .$$

Therefore, $P(\Omega)$ is the asymptotic probability distribution of the points generated by this random walk,

$$P(\Omega) = \lim_{n \to \infty} P_n(\Omega).$$

One can view the evolution of the original probability distribution along the Markov chain as a relaxation process towards the "equilibrium distribution", $P(\Omega)$. In practice, one assumes that the relaxation occurs within Markov chain of finite length $R$. The actual number is usually determined by experimenting and depends on both probabilities $P$ and $W$ and the desired accuracy of simulation. Given $P$ and $W$, $R$ has to be chosen large enough so that the systematic error due to deviation from the true distribution is smaller than the statistical error due to the finite size of the sample. In applications with a large number of degrees of freedom, where state vector $\Theta$ may have millions of strongly coupled components, the relaxation process is non-trivial. For the present purpose, the state vector is low-dimensional and relaxation occurs within just a few steps along the Markov chain [14].

The prescription for a **practical algorithm** can now be summarized as follows:

1. Pick an arbitrary initial path.

2. Generate a new trial path.

3. The new path is with probability $W$. Specifically, if $W \geq 1$, the new path is accepted without further tests. If $W < 1$, a random number between 0 and 1 is generated, and the new path is accepted if the random number is smaller than $W$. If the trial path is accepted, it becomes current path $\Omega_v$, otherwise the old path remains as current path $\Omega_v$.

4. If we have enough progressed along the Markov chain so that the relaxation is completed, (i.e. $v \geq R$), the current path is sampled from the desired distribution $P(\Omega)$. We compute the payoff function for the current path $F(\Omega_v)$ and accumulate the result $A = A + F(\Omega_v)$.

5. Perform an estimate of the statistical errors due to Monte Carlo sampling procedure. If the error is above a desired level of accuracy, go to (2), otherwise – go to (6).

6. Compute Monte Carlo estimates of the required integrals. If $L$ denotes the last value of the step index $v$ and $R$ is the number of the relaxation steps, the total number of Monte Carlo measurements is $M_v = L - R$. Monte Carlo estimate of the option price $\langle Q \rangle MC$, given the payoff function $F$, is obtained as:

$$< Q > mc = \frac{A}{M_v} = \frac{1}{M_v} \sum_{v=R+1}^{L} F(\Omega_v).$$

The error estimate requires also the accumulation

$$< Q^2 > mc = \frac{1}{M_v} \sum_{v=R+1}^{L} F^2(\Omega_v).$$

The estimate of the sampling error is obtained as a root of the variance of the Monte Carlo run:

$$\varepsilon. = (< \sigma^2 > mc)^{1/2},$$

$$< \sigma^2 > mc = \frac{1}{M_v} (< Q^2 > mc - < Q >^2 mc).$$

7. Stop.

## II. Application of Monte Carlo simulation models

### 1. Basic conclusions about application of models

Monte Carlo technique for valuation of derivatives securities is a method, which is based on the probability distribution of complete histories of the underlying security process. We used Metropolis algorithm to generate this probability distribution. We showed that this approach is efficient, accurate and allows one to obtain a complete solution of the valuation problem in a single simulation. One can obtain only price in a single simulation using the standard Monte Carlo method. Using path-integral Monte Carlo simulation one can get price sensitivity with respect to all the input parameters and even compute prices for multiple parameter values. Path-integral method can easily incorporate global constraints on the underlying security dynamics, which may prove very useful for applications such as bond option pricing [14].

### 2. Practical examples

Our present goal is to examine the feasibility and accuracy of the method, so we will begin with the simplest valuation problem of a European call on a stock with constant volatility and no dividends, where we can easily compare Monte Carlo results with the analytic Black-Scholes solution. The practical examples are illustrated in Tables 4, 5, 6 and 7.

We show results for a couple of realistic parameter choices in Table 4 and examine their accuracy as the number of Monte Carlo steps is varied. Exact results are always within estimated confidence limits of Monte Carlo results. Statistical errors after 100 000 Monte Carlo steps are less than half percent for all maturities. The error is less than tenth of a percent for $1.6 \times 10^6$ steps. These statistical uncertainties reflect improvements achieved by explicit use of all the symmetries of path probabilities, which enable us accumulate more independent results per path. For example, if a stock prices path is reflected with respect to the deterministic path, its probability is the same, so we can accumulate results for the reflected path as well with negligible computational cost. This can be regarded as a rudimentary variance reduction technique. A promising technique we are experimenting is the inclusion of additional global Monte Carlo updates. Table 4 also shows that statistical errors scale $\varepsilon \propto N^{-1/2}$ with the number of Monte Carlo steps, which is in agreement with the central limit theorem and also shows that successive Monte Carlo steps are not correlated. Correlation between Monte Carlo steps is reduced because at every step we pick with equal probabilities either the current path or any of its reflection symmetry related paths.

In Table 5 we present results for parameter sensitivities using the same parameter choices as in Table 4. They are obtained concurrently with the option price itself. They show even higher level of accuracy than the corresponding option price for a given number of Monte Carlo steps. If these values were obtained by numerical differentiation, it would require at least 3 simulations besides the original one to compute the three partial derivatives. Additional simulation may also be required, depending on the statistical accuracy of Monte Carlo results. If the statistical errors are large, one would need simulation for a few nearby parameter values, combined with a least-squares fit to produce estimates of derivatives. This may lead to unacceptably large errors for higher order derivatives, unless statistical errors for option prices are very small. In the path integral approach there are no additional sources of errors.

The possibility of computing Monte Carlo results for different parameters in a single simulation is illustrated in Table 6, where the option values in a window of about 10% variation of initial stock prices are computed in a single run. Within a few percent difference from the stock price used in the simulation, the results are roughly of the same statistical quality as for the original price. This is a very cheap and efficient way to explore option price variations in a limited parameter range, particularly if there is uncertainty about the input parameter estimates. It is clear from the table that the further one goes from the original simulation parameters, the worse the statistics becomes (larger relative errors) due to inefficient importance sampling. The same trend is apparent for longer periods to maturity, because the differences between the simulation probability distribution and the true ones are amplified for longer time periods. For shorter periods to maturity, there is an apparent asymmetry between the errors, which are much smaller for the initial prices below the simulation price $S_i < S_0$, than for prices above the simulation price, $S_i > S_0$. The reason is that the stock price distribution is skewed towards higher stock prices, so that the overlap between simulation price distribution and actual price distributions is greater for $S_i < S_0$ than for $S_i > S_0$. This effect becomes less and less important for longer time periods to maturity.

As a first step we show results for a jump diffusion model, where jumps are superimposed upon the continuous Wiener process. We will consider the following differential/difference equation corresponding to this process:

$$d \log S = dy = \mu dt + \sigma\, d\xi + dZ,$$

where $dZ$ is the stochastic variable describing the jump process. It is assumed that the number of jumps is Poisson distributed while the jump size is uniformly distributed with average of $dZ = 0$. The finite average value of the jump size will amount to a trivial shift coefficient $\mu$. Merton was able to obtain a series solution

Table 4

| Number of periods to maturity | Stock price variance | $C(1x10^5)$ | $\varepsilon(1x10^5)$ | $C(4x10^5)$ | $\varepsilon(4x10^5)$ | $C(16x10^5)$ | $\varepsilon(16x10^5)$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.001875 | 1.9792 | 0.0064 | 1.9769 | 0.0032 | 1.9750 | 0.0012 |
| 2 | 0.001875 | 2.9508 | 0.0106 | 2.9456 | 0.0053 | 2.9430 | 0.0026 |
| 3 | 0.001875 | 3.7522 | 0.0139 | 3.7492 | 0.0069 | 3.7469 | 0.0034 |
| 4 | 0.001875 | 4.4710 | 0.0168 | 4.4678 | 0.0083 | 4.4673 | 0.0041 |
| 5 | 0.001875 | 5.1446 | 0.0193 | 5.1324 | 0.0096 | 5.1326 | 0.0048 |
| 6 | 0.001875 | 5.7822 | 0.0216 | 5.7610 | 0.0108 | 5.7608 | 0.0054 |
| 7 | 0.001875 | 6.3823 | 0.0238 | 6.3599 | 0.0118 | 6.3595 | 0.0059 |
| 8 | 0.001875 | 6.9456 | 0.0258 | 6.9309 | 0.0129 | 6.9337 | 0.0064 |
| 9 | 0.001875 | 7.4969 | 0.0276 | 7.4881 | 0.0138 | 7.4875 | 0.0069 |
| 10 | 0.001875 | 8.0335 | 0.0294 | 8.0259 | 0.0147 | 8.0235 | 0.0073 |
| 11 | 0.001875 | 8.5593 | 0.0312 | 8.5511 | 0.0156 | 8.5493 | 0.0078 |
| 12 | 0.001875 | 9.0780 | 0.0338 | 9.0663 | 0.0169 | 9.0660 | 0.0084 |
| 1 | 0.002500 | 2.2467 | 0.0075 | 2.2436 | 0.0037 | 2.2415 | 0.0018 |
| 2 | 0.002500 | 3.3269 | 0.0123 | 3.3198 | 0.0061 | 3.3168 | 0.0030 |
| 3 | 0.002500 | 4.2089 | 0.0161 | 4.2024 | 0.0080 | 4.2004 | 0.0040 |
| 4 | 0.002500 | 4.9935 | 0.0195 | 4.9873 | 0.0097 | 4.9864 | 0.0048 |
| 5 | 0.002500 | 5.7262 | 0.0224 | 5.7086 | 0.0112 | 5.7085 | 0.0056 |
| 6 | 0.002500 | 6.4168 | 0.0252 | 6.3873 | 0.0125 | 6.3855 | 0.0062 |
| 7 | 0.002500 | 7.0602 | 0.0277 | 7.0302 | 0.0138 | 7.029 | 0.0069 |
| 8 | 0.002500 | 7.6629 | 0.0301 | 7.6415 | 0.0150 | 7.6434 | 0.0075 |
| 9 | 0.002500 | 8.2483 | 0.0322 | 8.2351 | 0.0161 | 8.2348 | 0.0080 |
| 10 | 0.002500 | 8.8169 | 0.0343 | 8.8066 | 0.0172 | 8.8041 | 0.0086 |
| 11 | 0.002500 | 9.3733 | 0.0364 | 9.3634 | 0.0182 | 9.3594 | 0.0091 |
| 12 | 0.002500 | 9.9226 | 0.0395 | 9.9076 | 0.0197 | 9.9028 | 0.0099 |

Table 5

| Number of periods to maturity | r(f) | Stock price variance | $\delta$ (MC) | $\varepsilon$ | $\delta$ | $\kappa$ (MC) | $\varepsilon$ | $\kappa$ | $\rho$ (MC) | $\varepsilon$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00483 | 0.001875 | 0.5530 | 0.00039 | 0.5532 | 0.1143 | 0.0005 | 0.1141 | 0.04443 | 0.00005 |
| 2 | 0.00483 | 0.001875 | 0.5745 | 0.00045 | 0.5750 | 0.1610 | 0.0006 | 0.1600 | 0.09083 | 0.00011 |
| 3 | 0.00483 | 0.001875 | 0.5914 | 0.00049 | 0.5916 | 0.1949 | 0.0008 | 0.1942 | 0.13848 | 0.00019 |
| 4 | 0.00483 | 0.001875 | 0.6060 | 0.00051 | 0.6054 | 0.2219 | 0.0010 | 0.2222 | 0.18710 | 0.00027 |
| 5 | 0.00483 | 0.001875 | 0.6167 | 0.00053 | 0.6175 | 0.2469 | 0.0012 | 0.2463 | 0.23555 | 0.00035 |
| 6 | 0.00483 | 0.001875 | 0.6276 | 0.00054 | 0.6283 | 0.2687 | 0.0013 | 0.2673 | 0.28485 | 0.00043 |
| 7 | 0.00483 | 0.001875 | 0.6386 | 0.00055 | 0.6382 | 0.2867 | 0.0014 | 0.2862 | 0.33526 | 0.00052 |
| 8 | 0.00483 | 0.001875 | 0.6474 | 0.00056 | 0.6473 | 0.3034 | 0.0016 | 0.3032 | 0.38522 | 0.00061 |
| 9 | 0.00483 | 0.001875 | 0.6564 | 0.00057 | 0.6558 | 0.3179 | 0.0017 | 0.3188 | 0.43615 | 0.00071 |
| 10 | 0.00483 | 0.001875 | 0.6635 | 0.00057 | 0.6638 | 0.3323 | 0.0018 | 0.3330 | 0.48614 | 0.00080 |
| 11 | 0.00483 | 0.001875 | 0.6707 | 0.00057 | 0.6713 | 0.3451 | 0.0019 | 0.3461 | 0.53639 | 0.00089 |
| 12 | 0.00483 | 0.001875 | 0.6772 | 0.00058 | 0.6784 | 0.3577 | 0.0021 | 0.3583 | 0.58630 | 0.00099 |
| 1 | 0.00483 | 0.002500 | 0.5485 | 0.00035 | 0.5486 | 0.1146 | 0.0004 | 0.1143 | 0.043845 | 0.000045 |
| 2 | 0.00483 | 0.002500 | 0.5682 | 0.00040 | 0.5685 | 0.1617 | 0.0006 | 0.1605 | 0.089164 | 0.000106 |
| 3 | 0.00483 | 0.002500 | 0.5838 | 0.00044 | 0.5837 | 0.1960 | 0.0009 | 0.1950 | 0.135424 | 0.000174 |
| 4 | 0.00483 | 0.002500 | 0.5966 | 0.00046 | 0.5964 | 0.2236 | 0.0010 | 0.2235 | 0.182223 | 0.000248 |
| 5 | 0.00483 | 0.002500 | 0.6074 | 0.00048 | 0.6075 | 0.2490 | 0.0012 | 0.2481 | 0.229169 | 0.000324 |
| 6 | 0.00483 | 0.002500 | 0.6167 | 0.00049 | 0.6175 | 0.2715 | 0.0013 | 0.2697 | 0.276238 | 0.000402 |
| 7 | 0.00483 | 0.002500 | 0.6270 | 0.00050 | 0.6266 | 0.2901 | 0.0015 | 0.2892 | 0.324568 | 0.000488 |
| 8 | 0.00483 | 0.002500 | 0.6354 | 0.00051 | 0.6350 | 0.3072 | 0.0016 | 0.3068 | 0.372574 | 0.000572 |
| 9 | 0.00483 | 0.002500 | 0.6433 | 0.00052 | 0.6428 | 0.3226 | 0.0017 | 0.3230 | 0.420538 | 0.000658 |
| 10 | 0.00483 | 0.002500 | 0.6506 | 0.00052 | 0.6502 | 0.3373 | 0.0019 | 0.3380 | 0.468612 | 0.000745 |
| 11 | 0.00483 | 0.002500 | 0.6569 | 0.00053 | 0.6572 | 0.3508 | 0.0020 | 0.3519 | 0.516353 | 0.000833 |
| 12 | 0.00483 | 0.002500 | 0.6629 | 0.00053 | 0.6637 | 0.3641 | 0.0021 | 0.3648 | 0.563794 | 0.000929 |

for the option price only under the assumption that the jump size distribution is normal [3]. This restriction can be lifted in a Monte Carlo simulation, so we chose an uniform distribution for experimentation purposes, since it is computationally cheap and there is no analytic solution. The results for a European call on an asset following this process are shown in Table 7. The prices and sensitivities are concurrently obtained and the accuracy is comparable to the one achieved in Black-Scholes problem. Relative errors for $1\times10^5$ steps are below one percent for option price and below two tenths of a percent for some $\delta$ and $\rho$ sensitivities. As for Black-Scholes model, price sensitivities are more accurately determined than the option price itself. The relative quality of the estimators depends on the form of the corresponding function, which is integrated with respect to the path probability measure. If one computes the sensitivities using numerical differentiation, errors would be at least as large as the price error.

## Examples

Example No 1 is shown in Table 4 – Comparison of Monte Carlo estimates and exact results for European call values. This table shows accuracy, which can be achieved as the number of Monte Carlo steps ranges from $1\times10^5$ up to $16\times10^5$. Risk-free rate per period is set to $r_j$ = 0.004853; $N_j$ is the number of periods to maturity, $\sigma^2$ is the stock price variance per period. European call values Monte Carlo estimates after $N$ Monte Carlo steps and $\varepsilon(N)$ is the error estimate after $N$ Monte Carlo steps. Initial stock price is $S$ = 100, the strike price is $X$ = 100 for all data in the table (see Appendix No 1).

Example No 2 is shown in Table 5 – option price sensitivities to input parameters. This table lists some of the price sensitivities, which are computed along the option price in a path-integral simulation. The initial stock value is set to $S$ = 100 and the strike price is $X$ = 100. The number of Monte Carlo steps is $1\times10^5$. Each parameter sensitivity estimate (MC) is immediately its error estimate $\varepsilon$ and the exact value obtained by differentiation of Black-Scholes formula. $\delta$ is the stock pricing sensitivity ($\delta = \partial C/\partial S$), $\kappa$ is the volatility sensitivity ($\kappa = \partial C/\partial S$), $\rho$ is the interest rate sensitivity ($\rho = \partial C/\partial r_j$) (see Appendix No 2).

Example No 3 is shown in Table 6 – Computation of option prices for multiple parameters in a single simulation. This table shows the level, which can be obtained if multiple option prices are computed in a single simulation. The number of Monte Carlo steps is $1\times10^5$. The initial stock price is $S_0$ = 100 and the strike price is $X$=100, volatility per period is $\sigma^2$ = 0.0025 and risk less interest rate $r_f$ = 0.004853 per period. Each option price estimate $C(S_i)$ for initial stock price $S_i$ is followed by its error estimate exact value from Black-Scholes formula $C$; $N_i$ denotes the number of time periods to maturity (see Appendix No 3).

Example No 4 is shown in Table 7 – Call option price and sensitivities for a jump diffusion process. This table lists results for the option, its input parameter sensitivities when a jump is superimposed on the continuous process of Black-Scholes formula. The initial stock value is set to $S$ = 100 and the strike price is $X$ =100. The number of Monte Carlo steps is $1\times10^5$. The Monte Carlo results are

Table 6

| Number of periods to maturity | $C(95)$ | ε | $C$ | $C(99)$ | ε | $C$ | $C(101)$ | ε | $C$ | $C(105)$ | ε |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.4632 | 0.0006 | 0.4629 | 1.7364 | 0.0047 | 1.7325 | 2.8358 | 0.0117 | 2.8287 | 5.8448 | 0.0572 |
| 2 | 1.1827 | 0.0051 | 1.1790 | 2.7907 | 0.0080 | 2.7757 | 3.9362 | 0.0154 | 3.9121 | 6.8103 | 0.0639 |
| 3 | 1.8600 | 0.0098 | 1.8613 | 3.6494 | 0.0107 | 3.6390 | 4.8268 | 0.0183 | 4.8059 | 7.6560 | 0.0714 |
| 4 | 2.5060 | 0.0145 | 2.5049 | 4.4087 | 0.0130 | 4.4080 | 5.6044 | 0.0207 | 5.6004 | 8.4129 | 0.0774 |
| 5 | 3.1353 | 0.0192 | 3.1165 | 5.1283 | 0.0151 | 5.1164 | 6.3470 | 0.0230 | 6.3310 | 9.1655 | 0.0826 |
| 6 | 3.7185 | 0.0232 | 3.7024 | 5.8012 | 0.0170 | 5.7813 | 7.0462 | 0.0251 | 7.0160 | 9.8878 | 0.0877 |
| 7 | 4.2810 | 0.0269 | 4.2671 | 6.4267 | 0.0188 | 6.4133 | 7.6846 | 0.0271 | 7.6662 | 10.5269 | 0.0926 |
| 8 | 4.8267 | 0.0312 | 4.8140 | 7.0248 | 0.0205 | 7.0191 | 8.2981 | 0.0288 | 8.2888 | 11.1462 | 0.0968 |
| 9 | 5.3438 | 0.0345 | 5.3457 | 7.5966 | 0.0220 | 7.6032 | 8.8850 | 0.0305 | 8.8887 | 11.7508 | 0.1010 |
| 10 | 5.8616 | 0.0380 | 5.8641 | 8.1616 | 0.0235 | 8.1691 | 9.4637 | 0.0321 | 9.4693 | 12.3372 | 0.1052 |
| 11 | 6.3681 | 0.0415 | 6.3710 | 8.7055 | 0.0249 | 8.7194 | 0.0198 | 0.0336 | 10.0335 | 12.9117 | 0.1086 |
| 12 | 6.8597 | 0.0450 | 6.8676 | 9.2435 | 0.0264 | 9.2561 | 0.5745 | 0.0352 | 10.5834 | 13.5007 | 0.1139 |

Table 7

| Number of periods to maturity | Δ | $C$ | ε | δ | ε | κ | ε | ρ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.02 | 2.12671 | 0.01505 | 0.55233 | 0.00121 | 0.12487 | 0.00108 | 0.04426 |
| 2 | 0.02 | 3.16259 | 0.02458 | 0.57517 | 0.00141 | 0.17626 | 0.00159 | 0.09056 |
| 3 | 0.02 | 3.97857 | 0.03243 | 0.59134 | 0.00151 | 0.21151 | 0.00202 | 0.13789 |
| 4 | 0.02 | 4.77069 | 0.03896 | 0.60583 | 0.00159 | 0.24485 | 0.00244 | 0.18604 |
| 5 | 0.02 | 5.47055 | 0.04516 | 0.61457 | 0.00161 | 0.27364 | 0.00281 | 0.23328 |
| 6 | 0.02 | 6.19997 | 0.05057 | 0.62528 | 0.00165 | 0.30257 | 0.00316 | 0.28164 |
| 7 | 0.02 | 6.85696 | 0.05560 | 0.63553 | 0.00168 | 0.32708 | 0.00349 | 0.33073 |
| 8 | 0.02 | 7.50037 | 0.06024 | 0.64449 | 0.00170 | 0.34980 | 0.00382 | 0.37966 |
| 9 | 0.02 | 8.10141 | 0.06485 | 0.65231 | 0.00172 | 0.36877 | 0.00412 | 0.42847 |
| 10 | 0.02 | 8.68738 | 0.06891 | 0.66075 | 0.00173 | 0.39014 | 0.00443 | 0.47823 |
| 11 | 0.02 | 9.23218 | 0.07319 | 0.66640 | 0.00174 | 0.40439 | 0.00472 | 0.52623 |
| 12 | 0.02 | 9.78240 | 0.07923 | 0.67459 | 0.00175 | 0.42227 | 0.00503 | 0.57677 |
| 1 | 0.05 | 0.02526 | 0.02526 | 0.55565 | 0.00110 | 0.19220 | 0.00192 | 0.04376 |
| 2 | 0.05 | 0.04117 | 0.04117 | 0.57536 | 0.00120 | 0.28671 | 0.00285 | 0.08821 |
| 3 | 0.05 | 0.05421 | 0.05421 | 0.59300 | 0.00131 | 0.35139 | 0.00363 | 0.13372 |
| 4 | 0.05 | 0.06491 | 0.06491 | 0.60581 | 0.00136 | 0.41691 | 0.00432 | 0.17878 |
| 5 | 0.05 | 0.07494 | 0.07494 | 0.61854 | 0.00141 | 0.47293 | 0.00502 | 0.22451 |
| 6 | 0.05 | 0.08419 | 0.08419 | 0.62963 | 0.00144 | 0.53323 | 0.00569 | 0.26977 |
| 7 | 0.05 | 0.09320 | 0.09320 | 0.63949 | 0.00146 | 0.58649 | 0.00627 | 0.31485 |
| 8 | 0.05 | 0.10119 | 0.10119 | 0.64944 | 0.00149 | 0.63476 | 0.00693 | 0.36049 |
| 9 | 0.05 | 0.10962 | 0.10962 | 0.65942 | 0.00153 | 0.67822 | 0.00754 | 0.40639 |
| 10 | 0.05 | 0.11694 | 0.11694 | 0.66944 | 0.00156 | 0.73089 | 0.00826 | 0.45278 |
| 11 | 0.05 | 0.12495 | 0.12495 | 0.67883 | 0.00159 | 0.76241 | 0.00883 | 0.49956 |
| 12 | 0.05 | 0.13614 | 0.13614 | 0.68505 | 0.00160 | 0.81260 | 0.00942 | 0.54295 |

immediately followed by its error estimate ε. The jump rate per period is set to $k_p = 0.1$. The risk-free rate per period is set to $r_f = 0.004853$ and the variance per period is $\sigma^2 = 0.001875$. The jump sizes are uniformly distributed in the interval $(-\Delta, +\Delta)$. δ is the stock pricing sensitivity ($\delta = \partial C/\partial S$), $\kappa$ is the volatility sensitivity ($\kappa = \partial C/\partial S$), $\rho$ is the interest rate sensitivity ($\rho = \partial C/\partial r_j$) (see Appendix No 4).

## III. Problems with Monte Carlo simulation

### 1. Basic problems with Monte Carlo simulation

Monte Carlo simulation has enjoyed resurgence in financial literature in recent years. This paper explores the reasons why implementing Monte Carlo simulation is very difficult at best and can lead to incorrect decisions at worst. The problem is that the typical assumption set used in Monte Carlo simulation assumes normal distributions and correlation coefficients of zero, neither of which are typical in the world of financial markets. It is important for planners to realize that these assumptions can lead to problems connected with their analysis.

In 1981 Rubinstein has developed a set of criteria to be used in deciding whether it is appropriate to use Monte Carlo simulation [15]. Monte Carlo simulation is appropriate when

- It is impossible or too expensive to obtain data;
- The system observed is too complex;
- The analytical solution is difficult to be obtained;
- It is impossible or too costly to validate the mathematical experiment.

Monte Carlo is just one type of simulation used to generate values for the exogenous variables. Exogenous variables are changed to reflect certain courses of action. This is the famous "what if" simulation technique. What if we implement this action? What if that happens? This type of simulation generally relies on a model built on historical data and sometimes may be called historical simulation. However, it can be used with other types of models including Monte Carlo simulation. The key is to set up a model of how the world works and then test different policies or decisions on the model to see what works.

In finance, exploratory simulation is generally the most useful one. It does not create a large computational burden and is relatively easy to implement. Monte Carlo variables assume that the processes being studied are independent on each other and that each value is a random draw from a distribution, or serially independent. Proponents of Monte Carlo simulation point out that the available computer programs can handle dependent relationships between exogenous variables. However, the problem is that the inter-relationships between two or more variables are generally quite complex and it is difficult to determine the correct relationships and distributions. The portfolio performance generated by Monte Carlo simulation to model the portfolio is lackluster. It does reduce the risk compared to an all-stock portfolio but with a lower return. The lower return results in poor risk-reward performance as evidenced in a lower reward to semivariability (*R*/*SV*) ratio.

Since the investment literature is fond of pointing out legal reasons, past performance is not a guarantee of future returns [18]. This counsel applies equally to both exploratory simulation and Monte Carlo simulation. Picking historic periods for an exploratory simulation that are equivalent to the current situation is problematic. However, still more problematic is picking the distribution to use in a Monte Carlo simulation. It should be noted that we are not forecasting per se when we use exploratory simulation. We are trying different policies to find out which one will best meet our needs. Monte Carlo simulation homogenizes away the factors that drive stock returns. Can you forecast stock returns without bringing forward a forecast for any financial variable? Monte Carlo simulation lets you do this by simply specifying the distribution for stock returns. The probability results from Monte Carlo simulation may appear impressive to a client. However, if that number is derived from assumptions that are not realistic, there is no value to the number.

When should a financial planner use Monte Carlo simulation? Whenever a variable in the problem cannot be estimated or is not available. Appropriate variables might include a person's life span or irregular cash flow needs for a retirement problem. This seems to be an appropriate application of Monte Carlo simulation, but only for those variables where the data is not available. Monte Carlo simulation gives information through its assumption set whenever variables with readily available data are used [17].

## 2. Other Problems with Monte Carlo Simulation

Other problems with Monte Carlo simulation are concern the presentation of a picture of Monte Carlo simulation that is, at best incomplete, especially in the contexts of long range financial planning. It is important to know how simulation methods can be used as the "gold standard" to evaluate simplified models suitable for use in everyday financial planning practice.

There are important differences between the application programs that answer financial planning questions and the methods used to compute the outcome of a financial plan. The computer application programs are set up to give answers to certain questions. Different methods are used to compute the outcome of a financial plan. The Monte Carlo simulation is such a method. Historical data for asset returns and inflation are summarized in statistical quantities (e.g., mean, standard deviations) and these quantities are inserted into assumed probability distributions. Then, samples are drawn from these assumed distributions for each year in the planning horizon to arrive to an outcome. This is replicated many times (from 1000 up to 10 000), and these outcomes are summarized into results such as "probability of meeting retirement goal", "mean terminal wealth", etc.

In the financial world Monte Carlo simulation is preferred instead of using a formula to compute returns. The formula is derived under the assumptions that the returns have a lognormal distribution and no serial correlation. Then, after citing references which describe investment returns as being other than log-normal, the authors state that the same log-normal assumptions are bad assumptions which "… can lead to incorrect decisions and that the implementation of Monte Carlo simulation is going to take a great deal of care." [4]

Another problem is to display the nonlinear relationship between the S&P 500 from one day to the next (daily data) from 1970 to 2000. This is quite informative because the linear correlation, the usual measure of dependence, is very small, but there is a strong non-linear relationship.

As an example, a two-asset portfolio consisting of the S&P 500 and timber is studied. The example has been chosen to show the advantages of historical simulation over Monte Carlo simulation. The example intends to show that the annual return of a 50-50 portfolio using historical simulation is 10.4% and only 8.5% when using Monte Carlo simulation. This difference is ascribed to the inter-relationships between the two data series that are not modeled by Monte Carlo simulation.[17]

Table 8 below shows the essential simulation results.

Table 8. Computation of arithmetic means from geometric means (Data historical simulation and Monte Carlo simulation)

| Simulation method: 50-50 portfolio | Annual return (geometric mean), % | Semi-annual geometric mean, % | Semi-annual standard deviation, % | Semi-annual variance | Semi-annual arithmetic mean, % | Inferred annual arithmetic mean, % |
|---|---|---|---|---|---|---|
| Monte Carlo | 8.55 | 4.19 | 9.98 | 0.0100 | 4.66 | 9.54 |
| Historical | 10.40 | 5.07 | 9.00 | 0.0081 | 5.45 | 11.21 |

## Basic conclusions

Monte Carlo simulation is useful for those cases where data and analytic models simply are not available. Otherwise, it requires more work and does not result in a demonstrably better answer than other analytic techniques. The benefit/cost ratio just is not there.

The problem with Monte Carlo simulation is connected with the assumptions that are made in the model in order to easily deploy Monte Carlo simulation. Since few planners have formal training on operations research, they will tend to make these assumptions without understanding their implications. Other forms of simulation, exploratory and tactical, do not make these assumptions and are easier to deploy. Monte Carlo simulation implies that we are operating under conditions of risk and know the underlying distributions. However, the financial markets are really operating under conditions of uncertainty where we do not know the distribution. Under these conditions the best policy is the one that adapts to uncertain conditions. It is important to stress-test policies to see which have proved most adaptable under severe conditions. This is the role of exploratory simulation with historic data.

Finally, the proponents of Monte Carlo simulation have to demonstrate the additional benefits of Monte Carlo simulation before its recommendation for wider use within the profession. Its benefits do not lie in the area of analyzing aggregate market returns. However, it could prove useful in other areas of financial planning practice, where data is not readily available. The proponents of Monte Carlo method should explore those areas so that it becomes valuable addition to financial planning.

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## Използване на Монте Карло-симулация при оценка на опции

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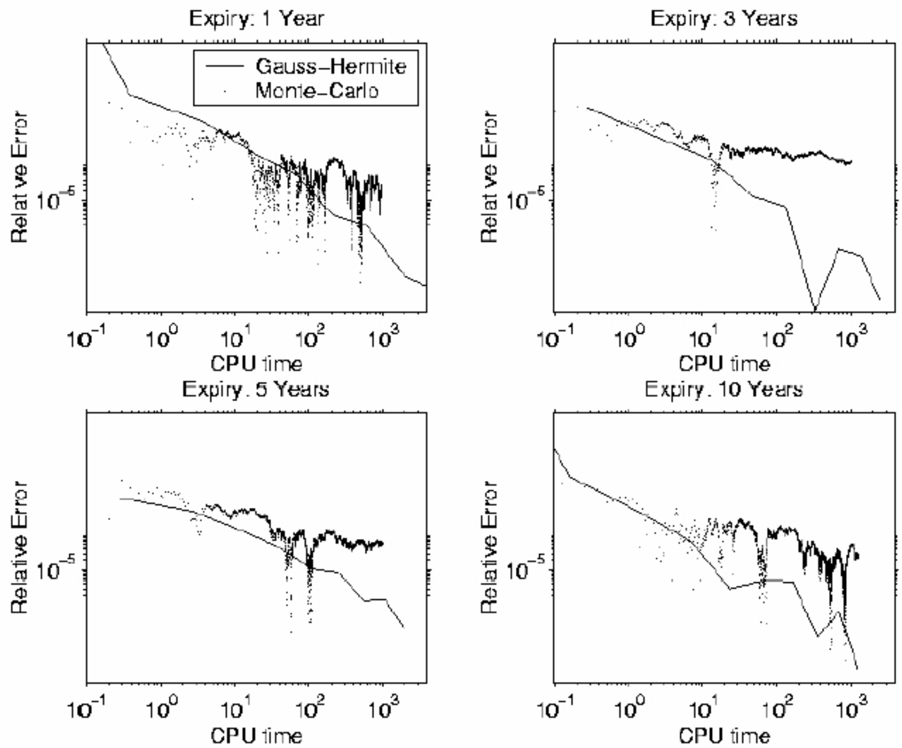
(Р е з ю м е)

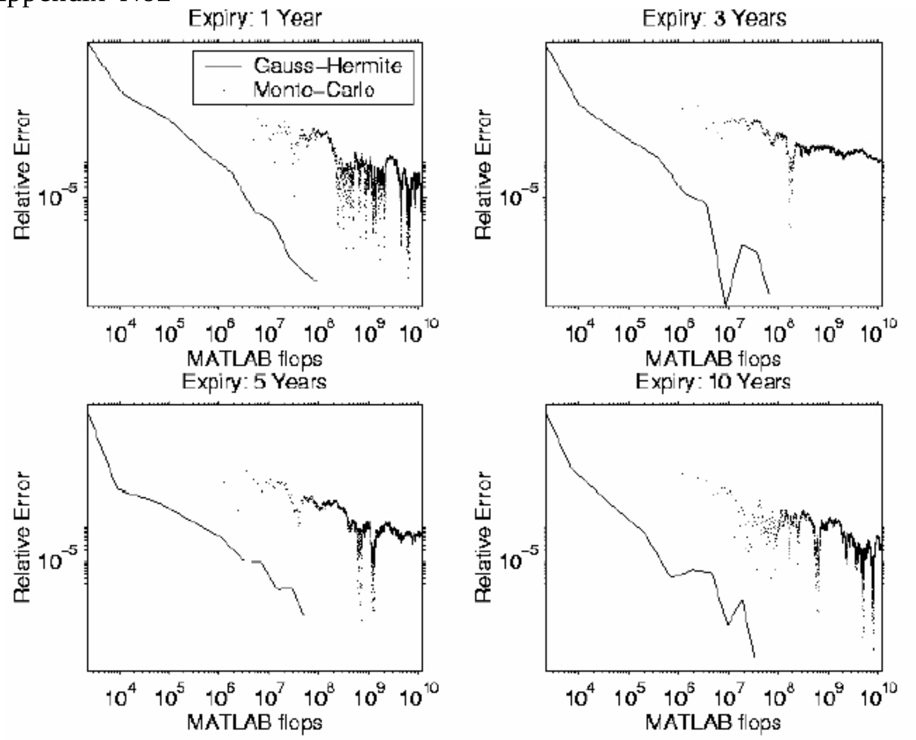
В настоящата статия е разгледана една техника на процеса Монте Карло-симулация. Алгоритъмът "Метрополис" е използван, за да покаже вероятностното разпределение на определени исторически параметри на базовите акции. Разгледани са портфейли от опции, съдържащи линейна комбинация от корелирани активи, включващи опции от типа "кошница" и "спред".

Целта е да се разгледа приложимостта и точността на метода, като се започне от най-простия проблем за оценка на Европейска кол опция, която е създадена на базата на акция с постоянно ниво на променливост и без дивиденти по нея. Така се дава възможност да са направи лесно сравнение на резултатите, получени чрез процеса Монте Карло-симулация, с резултатите, получени по модела на Блек-Скулс. В тази връзка са разгледаните практически примери в част трета.

Appendix No1



Appendix No2




Appendix No3