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A Partition-Based Interactive Method to Solve Discrete Multicriteria Choice Problems

Subhash C. Narula[*], Vassil Vassilev[**], Krassimira Genova[**], Mariana Vassileva[**]

* *School of Business, Virginia Commonwealth University, Richmond, VA 23284-4000, USA*
** *Institute of Information Technologies, 1113 Sofia*

Abstract: *A learning-oriented interactive method is proposed intended to solve discrete multicriteria choice problems with a large number of discrete alternatives and a few quantitative criteria. The method suggested is inspired by the partition-based methods designed to solve multiple objective mathematical programming problems. At each iteration, the DM may choose the current preferred alternative from one ranked set or from two ranked sets of alternatives. The first ranked set of alternatives is obtained by solving a discrete optimization scalarizing problem based on the preference information given by the DM about the desired changes of the values, desired directions of changes and desired intervals of changes for some or for all of the criteria in relation to their values in the current preferred alternative. The second ranked set is obtained using AHP or an outranking procedure when the DM is willing (able) to provide additional preference information, e.g., pairwise comparisons of the criteria or inter- and intra-criteria information. The DM can successively and systematically screen the set of non-dominated alternatives by the proposed method. The method is illustrated with the help of an example.*

***Keywords*:** *multicriteria decision analysis problem, discrete multicriteria choice problem, multiobjective optimization problem, AHP method, outranking methods.*

1. Introduction

Multiple criteria decision-making problems can be divided, V i n c k e [30], into two classes according to their formal statement: in the first one a finite number of explicitly stated constraints implicitly determine an infinite number of feasible alternatives, whereas in the second a finite number of alternatives are stated explicitly. The first class of problems is called multiple objective mathematical programming (MOMP) problems or multiple criteria choice problems with continuous alternatives, Y u, H u a n g and M a s u d [8], C h a n k o n g and H a i m e s [4], S t e u e r [26] and M i e t t i n e n [18]. The multiple criteria decision analysis (MCDA) problems, which are also called discrete multiple criteria or multi-attribute analysis problems, belong to the second category, H u a n g and Y o o n [9] and V i n c k e [30].

Interactive methods, B e n a y o u n, de M o n t g o l f i e r, T e r g n y, and L a r i c h e v [2], W i e r z b i c k i [31], K o r h o n e n and L a a k so [15], N a k a y a m a [20], B u c h a n a n [3], M i e t t i n e n and M a k e l a [19], and V a s s i l e v, N a r u l a and G o u l j a s h k i [28], are widely used to solve the MOMP problems. In these methods, the decision and computational phases are executed alternately. In the computational phase, a scalarizing problem is solved to generate one or several (weak) non-dominated solutions that satisfy the local preferences of the decision-maker (DM) to the greatest extent. In the decision phase, the DM chooses the best local (the preferred) solution. If this solution also satisfies his/her global preferences, it becomes the best global (the most preferred) solution. Otherwise, the DM provides additional information about his/her local preferences that is used in the next computational phase to search for new solutions. In these methods it is assumed that the DM optimizes an implicit value (utility) function or that by learning during the search process he/she tries to satisfy his/her aspirations concerning the values of the criteria (the aspiration levels of the criteria) to the highest degree. Convergence of the solution process is presumed in both types of methods, G a r d i n e r and V a n d e r p o o t e n [7]. Mathematical convergence of the computational process is ensured in the "search-oriented methods", whereas in the "learning-oriented methods", the DM ensures behavioural or intuitive convergence of the solution process. In both types of methods it is assumed that the DM can compare two solutions, that is, decide whether to prefer one of them or the two are equivalent to him/her.

The problems of MCDA can be classified into three main groups, H w a n g and Y o o n [9]. In the first group of problems, the discrete multicriteria choice problem, DMCCP, the objective is to search for the best-preferred non-dominated alternative. In the second group of problems, the ranking problem, the non-dominated alternatives are ranked in a descending order, i.e., starting from the best to the worst alternative. In the third group of problems, the sorting problem, the set of alternatives is partitioned into separate groups.

Most of the MCDA methods use two types of preference models or DM preference structures, V i n c k e [30], for comparing two nondominated alternatives. The first type of DM preference model does not allow the existence of incomparable alternatives, and preference information obtained by the DM is sufficient to determine whether one of the alternatives is to be preferred or the two alternatives are equivalent to the DM. This type of DM preference model is used in the multiattribute utility theory methods, F i s h b u r n [6], K e e n e y and R a i f f a [11], F a r q u h a r [5], and the analytical hierarchy process methods (AHP), Saaty [11]. The second type of DM preference model allows the existence of incomparable alternatives, and the preference information obtained by the DM may be insufficient to determine whether one of the alternatives is to be preferred or whether the two alternatives are equivalent to the DM. This type of DM preference model is used by the outranking methods, R o u b e n s [22], B r a n s and M a r e s c h a l [1], and R o y [23, 24]. In the outranking methods, the DM provides inter- and intra-criteria information, whereas in AHP method he/she has to provide pairwise information on the criteria.

To solve multicriteria choice problems with a large number of alternatives and a small number of quantitative criteria, which can be regarded as close to MOMP problems, the "optimizationally motivated" interactive methods, inspired by MOMP methods, have been suggested (see K o r h o n e n [13], S u n and S t e u e r [27], L o t f i, S t e w a r t and Z i o n t s [16], and J a s z k i e w i c z and S l o w i n s k i [10]. The first two methods use the first type of DM preference model and the DM provides in every iteration only the desired values of the criteria. The last two methods use the second DM preference model where the DM provides at each iteration not only the desired values of the criteria but also inter- and intra-criteria information.

We propose a learning-oriented, optimizationally motivated, interactive method designed to solve multicriteria choice problems with a large number of alternatives and a small number of quantitative criteria that is inspired by the MOMP method described in V a s s i l e v, N a r u l a, V a s s i l e v a and G e n o v a [29]. This method uses the first type of DM preference model. Besides the desired values of the criteria, the DM may provide the desired intervals and desired directions of changes. In addition, if the DM wants, he/she may also provide information such as pairwise comparisons of the criteria, or inter- and intra-criteria information.

The rest of the paper is organized as follows: we give some notations and definitions in the next section. We describe the proposed method in section 3 and state the algorithmic scheme of the method in section 4. We provide an illustrative example in section 5. We conclude the paper with a few remarks in section 6.

2. Preliminary considerations and definitions

The discrete multiple criteria decision analysis problem is defined as follows: Given a set I of n ($n > 1$) deterministic alternatives and a set J of k ($k \geq 2$) quantitative criteria, we define a $n \times k$ decision matrix A. The element a_{ij} of the matrix A denotes the evaluation of the alternatives $i \in I$ with respect to criterion $j \in J$. The vector $(a_{i1}, a_{i2}, ..., a_{ik})$ shows the evaluation of alternative $i \in I$ with respect to all the criteria in the set J. The column vector $(a_{1j}, a_{2j}, ..., a_{nj})^{\mathrm{T}}$ gives the assessment of all the alternatives in set I for criterion $j \in J$. The objective is to search a non-dominated alternative that satisfies the DM mostly with respect to all the criteria simultaneously, where more is better than less for each criterion. We assume that all the alternatives are known in advance.

Definition 1. The alternative $i \in I$ is called non-dominated if there is no other alternative $s \in I$, for which $a_{sj} \geq a_{ij}$ for all $j \in J$ and $a_{sj} > a_{ij}$ for at least one $j \in J$.

Definition 2. A current preferred alternative is a non-dominated alternative chosen by the DM at the current iteration.

Definition 3. The most preferred alternative is a preferred alternative that satisfies the DM to the greatest degree.

Definition 4. The desired changes in the criteria values are the amounts by which the DM wishes to improve or agrees to worsen the criteria compared to their values in the current preferred alternative.

Definition 5. The desired directions of change of the criteria are the directions in which the DM wishes to change (to improve or agrees to worsen) the criteria with respect to their values at the current preferred alternative.

Definition 6. The desired intervals of changes in the values of the criteria specified by the DM denote the limits within which the values of the criteria may lie.

Definition 7. A current ranked sample of alternatives is a subset of l non-dominated alternatives (l being set by the DM) that are obtained after a discrete optimization-scalarizing problem is solved.

Definition 8. A current re-ranked sample of alternative is obtained from the current ranked sample of alternatives based on the pairwise comparison of the criteria or inter- and intra-criteria information provided by the DM using AHP or outranking methods.

Since it is comparatively simple to identify dominated alternatives, (S u n and S t e u e r [27]), we shall assume in the rest of the paper that matrix A contains only non-dominated alternatives.

3. Method description

To solve multicriteria discrete choice problems with a large number of alternatives and a small number of quantitative criteria, we propose a partition-based interactive method that is inspired by MOMP method described in V a s s i l e v at al. [29]. At every iteration of the proposed method, the DM has the possibility to choose from the current ranked set or the current re-ranked set of the alternatives to improve the current preferred alternative.

To obtain the current ranked set, we use a Tchebychev type optimization scalarization problem that is a discrete analog of the scalarization problem described in Vassilev et al. [29]. This scalarizing problem is based on the information given by the DM for the desired changes of the values, desired directions of changes and the desired intervals of changes for some or for all the criteria in relation to their values in the current preferred alternative. Using this scalarizing problem, the alternatives are ranked in an increasing order by the value of the objective function of the scalarizing problem. The smaller the value of the objective function of a given alternative is, the more preferable (desirable) the alternative is. The first l alternatives in this ranking order establish the current set of the alternatives that are shown to the DM for evaluation and choice of the current preferred alternative, where $l << n$ is specified by the decision maker. It is possible that there may not be l alternatives which satisfy the requirements of the DM.

If the DM is willing (able) to provide additional preference information, e.g., pairwise comparisons of the criteria, S a a t y [25] or inter- and intra-criteria information – B r a n s and M a r e s c h a l [1], then it is possible to re-arrange the current ranked set of alternatives. This ranking can be accomplished by AHP method, S a a t y [25], or an outranking method such as PROMETHEE II method, B r a n s and M a r e s c h a l [1]. The DM can choose the current preferred alternative from the current ranked set or the current re-ranked set. Note that the DM has two rankings of the same set of alternatives based on different information.

3.1. Discrete partition-based scalarizing problem

To improve the current preferred alternative, we propose that the DM uses a discrete analog of the continuous partition-based scalarizing problem described by V a s s i l e v et al. [29]. The scalarizing problems available in the literature for discrete choice problems until recently are discrete analogs of the continuous aspiration level scalarizing problem proposed by W i e r z b i c k i [31]. In the partition-based scalarizing problem, the DM can express his/her desires not only in the form of aspiration levels, but also as aspiration directions and aspiration intervals of changes in the values of the criteria. In this way the DM can express his/her desires with more flexibility and precision. This flexibility is more important in the discrete choice problems where the number of alternatives is finite (although large) than in MOMP problems where the number of choices is infinite.

Before presenting the discrete partition-based scalarizing problem, we introduce the following notations.

Let h denotes the index of the current preferred alternative.

I' is the set of alternatives without alternative with index h; $I' = I \setminus h$;

a_{hj} – the value of a criterion with an index $j \in J$ in the current preferred alternative.

$K_h^{\geq} \cup K_h^{>}$ is the set of indices $j \in J$ of the criteria, for which the DM wishes to increase their values compared to their values in the current preferred alternative, where:

$K_h^{\geq}$ is the set of indices of the criteria $j \in J$ that the DM wants to improve by desired (aspiration) values Δ_{hj};

$K_h^{>}$ – the set of indices of the criteria $j \in J$ that the DM wants to improve but for which he/she is able to provide only the direction.

$K_h^{\leq} \cup K_h^{<}$ is the set of indices $j \in J$ of the criteria for which the DM agrees to worsen their values compared to their values in the current preferred alternative, where:

$K_h^{\leq}$ is the set of indices of the criteria $j \in J$ that the DM agrees the values of the criteria to be worsened by no more than δ_{hj};

$K_h^{<}$ – the set of indices of the criteria $j \in J$ that the DM agrees to worsen but for which he/she is able to provide only the direction.

$K^{><}$ is the set of indices of the criteria $j \in J$ for which the DM wants the criteria values to lie within an interval, $(a_{hj} - t^-_{hj} \leq a_{hj} \leq a_{hj} + t^+_{hj})$ around the current value a_{hj};

$K^{=}$ – the set of indices of the criteria $j \in J$ for which the DM wants to either preserve or improve the current value of the criteria;

K^0 – the set of indices of the criteria $j \in J$ about which the DM is indifferent about the value of these criteria and as such may be altered freely;

$\bar{a}_{hj}$ – the desired (aspiration) value of the criterion with an index $j \in K_h^{\geq}$ and

$$\bar{a}_{hj} = a_{hj} + \Delta_{hj}, \ j \in K_h^{\geq};$$

Λ'_j – the difference between the maximal and minimal value for the criterion with an index $j \in J$ and

$$\Lambda'_j = \max_{i \in I} a_{ij} - \min_{i \in I} a_{ij} .$$

Now we can formulate the discrete partition-based scalarizing problem (S) as follows:

$$\text{(S): } \min_{i \in I} S(i) = \min_{i \in I}\{\max[\max_{j \in K_h^{\geq}} (\bar{a}_{hj} - a_{ij}) / \Lambda_j , \max_{j \in K_h^{<} \cup K_h^{\leq}} ((a_{hj} - a_{ij})/\Lambda_j] + \max_{j \in K_h^{>}} (a_{hj} - a_{ij})/\Lambda_j\}$$

subject to:

$$a_{ij} \geq a_{hj}, \quad i \in I', \quad j \in K_h^{>} \cup K_h^{=} ;$$

$$a_{ij} \geq a_{hj} - \delta_{hj} , \quad i \in I', \quad j \in K_h^{\leq} ;$$

$$a_{ij} \geq a_{hj} - t_{hj}^{-} , \quad i \in I', \quad j \in K_h^{><} ;$$

$$a_{ij} \leq a_{hj} + t_{hj}^{+} , \quad i \in I', \quad j \in K_h^{><} ,$$

where

$$\Lambda_i = \begin{cases} \varepsilon & \text{if } \Lambda'_j \leq \varepsilon , \\ \Lambda'_j & \text{if } \Lambda'_j > \varepsilon \end{cases}$$

and ε is a small positive number.

When solving a discrete optimization problem S, the value of $S(i)$ is computed for each alternative i that satisfies the constraints of this problem. The objective function $S(i)$ denotes the distance to the "modified" Tchebychev metric for each feasible alternative from the virtual alternative as defined by DM's wishes.

3.2. The ranked and re-ranked current sets of alternatives

By solving the discrete partition-based scalarizing problem S, the alternatives are ranked in an ascending order of the value of $S(i)$, i.e., from the smallest to the largest value. The first l ranked alternatives are included in the current ranked subset M_1, where l denotes the number of the alternatives specified by the DM. These alternatives satisfy to the greatest extent the preferences of the DM in relation to the current preferred alternative with an index h.

If the DM wishes, it is possible and may be preferable, to re-rank the alternatives that belong to the set M_1 by a formal procedure based on his/her pairwise comparison of the criteria, or inter- and intra-criteria preference information. The formal procedure depends on the type of information provided by the DM and may be accomplished by AHP, PROMETHEE II or any other procedure. This re-ranked set of alternatives is the set M_2.

4. Algorithmic scheme

Based on the discrete partition-based scalarizing problem and in order to obtain the ranked and re-ranked sets M_1 and M_2, we develop an algorithmic scheme to solve discrete multicriteria choice problems with a large set of alternatives and a small number of quantitative criteria.

The main steps of the algorithmic scheme are:

Step 1. Reject (see Remark 1) all dominated alternatives in the decision matrix A. The new decision matrix A consists of non-dominated alternatives only. Assign index h to the first initial preferred alternative (see Remark 2). Set $iter = 1$, and LIST $= \varnothing$, where LIST is a set of stored preferred alternatives.

Step 2. If the DM wants to store the current preferred alternative h, check if it has been saved before. If "not" – add h to LIST.

Step 3. Ask the DM to define the desired changes, direction of changes and the intervals of changes for the criteria values in relation to the current preferred alternative h. Ask the DM to specify a parameter l – the number of alternatives he/she would like to see in M_1.

Step 4. Solve the discrete partition-based scalarizing problem S and determine the current ranked set of alternatives M_1.

Step 5. Present set M_1 to the DM for evaluation. If the DM would like to re-rank the alternatives in set M_1 to obtain set M_2, ask him/her to provide additional preference information (pairwise comparison of criteria, or inter- and intra-criteria). If the DM does not provide the required information, ask him/her to choose the preferred alternative from set M_1. If the DM provides the required additional information, determine set M_2. Ask the DM to choose the preferred alternative from sets M_1 and M_2.

Step 6. If the DM selects the preferred alternative chosen in Step 5 as the most preferred alternative – *Stop*.

Step 7. If the DM selects the preferred alternative chosen in Step 5 as the current preferred alternative – assign index h to it, update $iter = iter + 1$, and go to Step 2. Otherwise, ask him/her to choose the current preferred alternative or most preferred alternative from the stored list of alternatives, LIST. In the first case, assign index h to this alternative, update $iter = iter + 1$ and go to Step 3. In the second case – *Stop*.

Remark 1. The dominated alternatives are rejected only once in the initial phase of the algorithm, S u n and S t e u e r [27]. The computational complexity of this operation is of order $O(kn^2)$.

Remark 2. Any alternative can be selected as an initial preferred alternative, or the DM may specify an alternative as an initial preferred alternative.

5. An illustrative example

We illustrate the method proposed by a part of a real problem in Bulgarian "Melinvest" fund. Their interest was to acquire state-owned companies in "Wine Industry". The four characteristic ratios most often used as criteria for evaluation of these companies are: assets turnover ratio, liquidity ratio, profitability on net sales, and gearing ratio. When choosing an enterprise, it is desirable to maximize the values of the first three criteria and to minimize the value of the fourth criterion. Twenty non-dominated enterprises with respect to the values of these criteria are included in the analysis.

In order to solve this discrete multiobjective choice problem, we used the experimental decision support system DSS for solving multicriteria analysis problems. The system consists of a control and an interface module, an editing module, and the modules that include the AHP method, the ELECTRE methods, the PROMETEE methods and the proposed partition-based interactive method. Our main purpose is to illustrate the more important features of the interactive method, but not to satisfy the preferences of a real DM.

The decision matrix A for the multiple criteria choice problem is shown in Table 1. The ratios have been scaled for ease of computations. Since the proposed algorithm searches for maximum values of each criterion, whereas the minimal value for the fourth criterion (gearing ratio) is desirable, the last column of the decision matrix has been multiplied by (–1).

Table 1. The decision matrix A

i \ j	1	2	3	4
1	13.5	9.8	1.0	–23.7
2	7.7	11.8	14.6	–42.5
3	12.1	21.8	1.6	–16.0
4	5.1	27.5	1.9	–67.2
5	15.4	16.9	2.1	–35.8
6	5.8	6.6	3.4	–23.8
7	5.0	5.9	15.0	–36.0
8	3.8	25.5	10.5	–23.8
9	7.8	30.1	2.8	–12.6
10	7.3	21.3	5.1	–35.2
11	5.5	9.5	7.2	–22.0
12	6.3	7.1	14.4	–35.5
13	11.5	37.2	6.5	–53.1
14	6.9	21.1	9.0	–57.0
15	4.3	14.0	5.5	–34.7
16	10.3	15.7	5.3	–37.7
17	8.8	19.5	3.4	–32.7
18	6.1	7.5	14.1	–23.8
19	8.4	8.2	5.1	–15.8
20	12.7	13.1	2.9	–18.8
Max	15.4	37.2	15.0	–12.6
Min	3.8	5.9	1.0	–67.2
Λ_j	11.6	31.3	14.0	54.6

Let us assume that the fifth alternative is selected as the initial preferred alternative based on the maximal value of the first criterion. Set $-h=5$, $iter=1$ and LIST={5}.

Suppose that with respect to the fifth alternative, the DM would like to improve the value of the third criterion by Δ_{53} = 7.9 and $\bar{a}_{53}$ = 10. For the second and fourth criteria the DM expresses his/her wish to improve their values and agrees that, if necessary, he/she is willing to worsen the value of the first criterion. The DM wants to examine a set of three alternatives (l = 3).

The sets $K_5^{\geq} = \{3\}$, $K_5^{>} = \{2, 4\}$ and $K_5^{<} = \{1\}$ are formed. For each alternative $i \in I'$, that satisfies the conditions $a_{i2} \geq a_{52}$ and $a_{i4} \geq a_{54}$, the values of the scalarizing function $S(i)$ are computed:

i	3	8	9	10	17
$S(i)$	0.6000	0.7802	0.5143	0.6873	0.5123
rank	3	5	2	4	1

The set M_1 = {17, 9, 3}consists of the first three alternatives with the smallest values for $S(i)$. The set M_1 is presented to the DM for evaluation. The DM chooses alternative 17 at this iteration as the current preferred alternative.Then h = 17, *iter* = 2 and LIST={5, 17}.

At the second iteration the DM sets his/her local preferences relative to the values of the criteria for the current preferred alternative h = 17 as follows: the value of the first criterion is to be preserved within the interval $t^-_{171} = 3$ and $t^+_{171} = 3$; $-K_{17}^{><} = \{1\}$, if necessary, he/she agrees to worsen the value of the second criterion $-K_{17}^{<} = \{2\}$; he/she wishes to improve the value of the third criterion $-K_{17}^{>} = \{3\}$; and for the fourth criterion, he/she sets an aspiration level $\bar{a}_{174} = -22.7$, $-K^{\geq} = \{4\}$. At this iteration the DM also changes the number of alternatives he/she wants to see in the set M_1, assign l as l = 4.

The feasible set of alternatives of scalarizing problem (S) is defined by the constraints:

$$a_{i1} \geq 5.8,$$
$$a_{i1} \leq 11.8,$$
$$a_{i3} \geq 3.4.$$

The value of function $S(i)$ is computed for each alternative in the feasible set:

The first four alternatives with the smallest value of $S(i)$ are included in the set M_1 = {18, 10, 12, 19}. In order to make his/her choice, the DM decides to re-rank the

i	2	6	10	12	13	14	16	18	19
$S(i)$	0.3626	0.4121	0.2289	0.2344	0.5568	0.6282	0.2747	0.0202	0.2395
rank	6	7	2	3	8	9	5	1	4

set M_1 providing weights, and indifference and strict preference thresholds for each criterion.

A re-ranked set M_2 = {19, 18, 12, 10} is obtained on the basis of this information

j	**1**	**2**	**3**	**4**
w_j	0.2	0.4	0.1	0.3
q_j	0.5	1	0.5	1
p_j	5	10	5	10

using outranking procedure PROMETHEE II in the DSS. The sets M_1 and M_2 are presented to the DM for selection of a preferred alternative. The DM selects alternative 18 as a current preferred alternative – h = 18, *iter* = 3 and decides to save it in LIST – and LIST ={5, 17, 18}.

The DM sets his/her local preferences at the third iteration as follows: to improve the values of the first and the second criterion $-K_{18}^{>}=\{1, 2\}$; if necessary, he/she is willing to worsen the values of the third and fourth criterion, with deterioration of the fourth criterion not greater than $\delta_{18\,4}= 14.2$, $-K_{18}^{<}=\{3\}$, $K_{18}^{\leq}=\{4\}$.

The constraints of scalarizing problem (S), formed at this iteration are:

$$a_{i1} \geq 6.1,$$
$$a_{i2} \geq 7.5,$$
$$a_{i3} \geq -38.$$

The values of the scalarizing problem $S(i)$ are computed for the alternatives satisfying the preceding inequalities. The set M_1 = {16, 3, 17, 10} is presented for evaluation to the DM and he/she chooses alternative 16 as the most preferred alternative.

i	1	3	5	9	10	16	17	19	20
$S(i)$	0.8622	0.4360	0.5568	0.6606	0.5394	0.3666	0.5315	0.6205	0.6211
rank	9	2	5	8	4	1	3	6	7

6. Conclusion

We have proposed a learning-oriented interactive method for solving discrete multicriteria choice problems with a large number of alternatives and a small number of quantitative criteria. The method assists the DM in learning about the problem and in evaluating systematically the set of alternatives. At every iteration the DM can provide not only aspiration levels, as it is in most of the known interactive methods, but also aspiration directions and aspiration intervals of changes in the values of the criteria. In this way the DM can express his/her wishes with more flexibility and precision. If the DM wants, he/she may provide additional preference information such as pairwise comparison of the criteria or inter- and intra- criteria information. On the basis of this information, the method proposed enables the use of discrete optimization scalarizing problems, weighting and outranking procedures, with the help of which the DM has the possibility for a more systematic and successful screening of the set of alternatives.

The software for the proposed partition-based interactive method is included in the experimental decision support system for solving multicriteria analysis problems.

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Интерактивен метод, използващ разделяне на критериите за решаване на дискретни задачи за многокритериален избор

Субхаш С. Нарула[*], *Васил Василев*[**], *Красимира Генова*[**], *Мариана Василева*[**]

** School of Business, Virginia Commonwealth University, Richmond, VA 23284-4000, USA*
*** Институт по информационни технологии, 1113 София*

(Р е з ю м е)

Предложен е интерактивен метод, използващ разделяне на критериите, за решаване на дискретни задачи за многокритериален анализ с голям брой алтернативи и малък брой критерии. Методът е инспириран от интерактивните методи на многокритериалната оптимизация. На всяка итерация лицето, вземащо решение (ЛВР), може да избере текущата предпочитана алтернатива от едно или от две подредени множества алтернативи. Първото подредено множество алтернативи се получава, като се решава дискретна оптимизационна задача, основана на информация за предпочитанията, предоставена от ЛВР за желаните или допустимите промени в стойностите, желаните или допустимите посоки на промяна и желаните или допустимите интервали на промяна в стойностите на някои (или на всички) критерии по отношение на техните стойности в текущата предпочитана алтернатива. Второто подредено множество се получава, използвайки AHP или ранкиращата процедура, когато ЛВР желае (може) да даде допълнителна информация за предпочитанията, например сравнение по двойки на критериите, интер- и интракритериална информация. ЛВР може последователно и систематично да анализира множеството от недоминирани алтернативи чрез предложения метод. Методът е илюстриран с пример.