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A Method for Nadir Point Estimation in MOLP Problems

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Abstract: *It is proposed to use two points of support when estimating the nadir vector components in Multiobjective Linear Programming (MOLP) problems: to consider the frontier of the feasible set S and to use the reference point method. The proposed approach gives upper bounds of the estimated values.*

Keywords*: multiobjective linear programming, nadir vector, feasible set, payoff table.*

1. Introduction

The importance of mathematical models for decision making under a set of criteria is steadily increasing. These models find application in different theoretical and practical investigations. This is confirmed by the immense amount of journal papers and books concerning the multiobjective optimization problems.

The simplest model of this kind (with continuous variables) is the Multiobjective Linear Programming (MOLP) problem. It can be presented as follows

$$\max\ f_1(x)$$
$$\max\ f_2(x)$$
(1) $$\dots$$
$$\max\ f_m(x)$$

s.t.

$$x \in S \subseteq R^n$$

The symbol $f_i(x)$, $i = 1, 2, \dots, m$, denotes a linear fuction, defined on R^n. The functions $f_i(x)$ are the optimization criteria in MOLP problem (1). The vector $f(x) = (f_1(x), f_2(x),\dots, f_m(x))$ is called criterion vector. The set $S \subseteq R^n$ is called decision space or feasible set in R^n. In MOLP problems the set S is defined in the following way:

$$S = \{x \in R^n \mid c_i(x) \leq 0,\ i = 1, 2, \ldots, k\}.$$

All $c_i(x)$ are linear functions too. We will consider problems, where S is a bounded set. In general the set of constraints describing S includes the constraints $x_j \geq 0$, $j = 1, 2, \ldots, n$. But in the algorithm presented in this paper we take notice of these constraints especially. The set

$$Z = \{z \in R^m \mid z = f(x),\ x \in S\}$$

is called objective space or criterion space.

The point $z^1 = f(x^1) \in Z$, $x^1 \in S$, is called a nondominated (Pareto) point if there does not exist a point $x^2 \in S$, $x^2 \neq x^1$, such that

$$f_i(x^2) \geq f_i(x^1),\ \forall i,$$

and

$$f_l(x^2) > f_l(x^1) \text{ for one } l \text{ at least.}$$

If the point $z^1 = f(x^1)$ is nondominated, the point x^1 is called efficient point. The set P of all nondominated points in Z is called nondominated (Pareto) set. The set E of all efficient points in S is called efficient set. The point $x^1 \in S$ is called weak efficient if there does not exist another point $x^2 \in S$, $x^2 \neq x^1$, such that $f_i(x^2) > f_i(x^1)$, $\forall i$. The set of all weak efficient points in S is denoted by E_w, $E \subseteq E_w$. If $x^1 \in E_w$, then $f(x^1)$ is called weak nondominated point in Z. The set P_w of all weak nondominated points in Z is called weak Pareto set $P \subseteq P_w \subseteq Z$.

It is accepted in the most part of the studies on multiobjective optimization problems that the needed solution $x^s \in E$ and $f(x^s) \in P$ respectively. Then the following question arises: for a chosen criterion $f_j(x)$ what are the minimal and the maximal values of $f_j(x)$ on the set E? It is clear that

$$f_j(x) \leq \max_{x\in E} f_j(x) = \max_{x\in S} f_j(x) = U_j,\ j = 1, 2, \ldots, m.$$

On the other hand

$$f_j(x) \leq \min_{x\in E} f_j(x) = L_j,\ j = 1, 2, \ldots, m.$$

The vector $L = (L_1, L_2, \ldots, L_j, \ldots, L_m)$ is called nadir vector. The computing of the vector L is not easy even in the case of MOLP problem (1) because the set E is not a convex set. In this paper a method for estimating the numbers L_j, $\forall j$, is proposed .

2. A short survey of the literature

In most textbooks on multiobjective optimization it is noted that the problem of determining the vector L is a difficult one. In Steuer's book [12] for such purposes an optimization problem is proposed, where a linear function must be minimized under a set of nonlinear constraints. The author points out that this problem is hardly usable because it is large. The problem for determining the vector L is noted in the book of V i n c k e [14], too. K. M i e t t i n e n [9] refers to three methods for finding the vector L but all they are attended with great computational difficulties .

For estimation of the vector L the payoff table is used very often. Having in mind problem (1) the corresponding payoff table is a $m\times m$ matrix . The first row of this matrix consists of the vector $f(x^1)$, where x^1 is determined as a solution of the problem

$$\max_{x\in S} f_1(x)$$

under the additional assumption that $x^1 \in E$. The second row consists of the vector $f(x^2)$, where x^2 is determined as a solution of

$$\max_{x\in S} f_2(x)$$

under the assumption $x^2 \in E$. Following this rule the whole payoff table is fulfilled by analogy. The numbers in the first column are the values of the first criterion $f_1(x)$ when each one of the rest part of criteria is maximized (plus the maximal value of $f_1(x)$, of course). The second column contains the values of the criterion $f_2(x)$ when each one of the rest part of criteria is maximized, and so on. The minimal value in the first column is an estimate of the number L_1, the minimal value in the second column is an estimate of the number L_2, and so on. Many authors point out that the L_i estimates obtained from the payoff table may be very different from the exact values.

An approach using the reference direction idea is proposed in the paper of K o r h o n e n, S a l o, S t e u e r [7]. The idea of reference direction is a well known extension of the notion of reference point (W i e r z b i c k i [17, 18]) and is used with the purpose to obtain a sequence of Pareto points on the base of corresponding reference points. In the obtained Pareto points sequence the values of chosen criterion decrease. It is a heuristic method and the computational results show that the obtained results are much better than the estimates obtained from the payoff table. The same paper cites several other papers (B e n s o n [1, 2, 3], D a u e r [4], E c k e r and S o n g [5]), describing procedures "which are theoretically able to calculate the nadir criterion components but which present formidable computerization challenges". With the intention to avoid similar complications the paper [7] proposes a technique which uses simultaneously an extended LP software (ADBASE) and a realization of the VIG method (K o r h o n e n [8]).

It is clear that determining of the numbers L_j needs a method for maximizing (minimizing) a function over the set E. Many papers concerning this problem can be found in the literature of the last years. Some of the first results are based on the idea to organize a movement in the set of efficient extreme points only [1, 5]. In the next years the stream of ideas has development. In one of the recent papers Y a m a m o- t o [20] proposes a classification of the existing algorithms for optimization over the efficient set. This classification contains six classes:

– adjacent vertex search algorithms;
– nonadjacent vertex search algorithms;
– branch and bound search algorithms;
– lagrangean relaxation based algorithms;
– dual approach;
– bisection algorithms.

In Yamamoto's paper each class is presented by one typical algorithm and these algorithms are compared from computational point of view.

H o r s t and T h o a i [6] proposed to use utility function. They described some conditions allowing to use such function. The proposed method gives ε-approximate solutions.

D. J. W h i t e [19] proposed to use penalty function. The paper contains information about the computational aspects, ε-efficiency, the closeness to the original problem, some nonlinear extensions.

T h o a i [13] proposed to use a special quasiconvex function of the criteria f_i and considers maximization of this function. The proposed algorithm gives satisfactory results in the case of small number of criteria and relatively large dimensionality of decision space.

A branch and bound type algorithm is proposed in the paper of Y a m a d a, T a- n i n o and I n u i g u c h i [21] for maximization of convex function over the set E_w of a multiobjective optimization problem.

Even this short review shows that the problems concerning optimization over the efficient set can be too complex and the authors try to cover this complexity as good as possible. The purpose of this paper is to propose a simple method for estimating the nadir vector in the case of MOLP problems.

3. An auxiliary LP problem

The following LP problem is used in the rest part of this paper. This problem is formulated on the base of the reference point method theory (W i e r z b i c k i [17, 18]) and under the assumption that problem (1) is a MOLP problem and the used LP software computes nonnegative variables only. Having in mind problem (1), the auxiliary LP problem is

(2) $$\min (D_1 - D_2)$$

s.t.

(a1) $$f_i(x) - FA_i + FB_i = 0, \; i = 1, 2, \ldots, m,$$

(a2) $$c_j(x) \le 0, \; j = 1, 2, \ldots, k,$$

(a3) $$\sum_i (FA_i - FB_i) - FA_s + FB_s = 0,$$

(a4) $$FA_i - FB_i + 0.01(FA_s - FB_s) + D_1 - D_2 \ge r_i, \; i = 1, 2, \ldots, m.$$

We need to minimize the quantity D, but D is nonrestricted in sign. So we minimize $D_1 - D_2$, where D_1 and D_2 are nonnegative. Following the same idea constraints (a1) introduce the pairs (FA_i , FB_i), $i = 1, 2, \ldots, m$, for each corresponding criterion $f_i(x)$ (FA_i and FB_i are nonnegative for all i.) Constraints (a2) describe the feasible set in decision space of problem (1). The pair (FA_s, FB_s) is introduced through constraint (a3) and is used in constraints (a4) giving the guaranty for obtaining efficient (or Pareto) points. Constraints (a4) contain the reference point components r_i, $i = 1, 2, \ldots, m$. The reference point theory has the following result: for an arbitrary reference point $r \in R^m$ the obtained solution of problem (2) determines an efficient point in decision space of problem (1) (a Pareto point in the criterion space of the same problem).

4. The proposed algorithm

We will use the notion of "a wall of the set S". Let us consider the sets W_j

$$W_j = \{x \in S \mid c_j = 0\}, \quad j = 1, 2, \ldots, k.$$

All these sets W_j are walls of the set S. In addition there are constraints $x_j \geq 0$, $j = 1, 2, \ldots, n$. Suppose that the constraints $x_j \geq 0$, $j = 1, 2, \ldots t$, $t \leq n$, are not redundant. Then the rest part of walls of the set S are

$$W_{k+s} = \{x \in S \mid x_s = 0\}, \; s = 1, 2, \ldots, t; \; t \leq n,$$

where the constraints $x_s = 0$, $s = 1, 2, ..., t$, are not redundant. Then the set of all walls of S consists of

$$W_1, W_2, \ldots, W_k, W_{k+1}, \ldots, W_{k+t}, \; t \leq n.$$

We will use also the finite sequence Q:

$$Q = \{ S, W_1, W_2, \ldots, W_k, W_{k+1}, \ldots, W_{k+t}\}.$$

An arbitrary element of Q will be denoted by U.

The algorithm is based on the following reason. We suppose that each efficient point of problem (1) is lying of some wall W_j. Therefore we will not inspect the inner part of S. Trying to estimate the component L_i of the nadir vector L we will consider each wall W_j, we will solve the problem

$$\min_{x \in W_j} f_i(x).$$

So we will obtain the solution x^j and next using the reference point method we will find the point $x_e^j \in E$. We obtain the estimate

$$\min_{x \in E} f_i(x) = L_i \leq \min_j f_i(x_e^j).$$

Algorithm "W + RP"(Wall + Reference Point)

Suppose that we are looking for estimate of L_1.

1. Let $U = S$, $l := 1$
2. Solve the LP problem $\min_{x \in U} f_1(x)$.

As result we obtain the solution $x^u \in U$ and the corresponding values

$$f_i(x^u) = FA_i - FB_i, \; i = 1, \ldots, m.$$

3. Set $r_i = f_i(x^u)$ for all i in LP problem (2), solve this problem and denote its solution as $x^l \in E$. The values $f_i(x^l)$ are the components of the corresponding Pareto point. This point may or may not be a point of U. Set $q_1^l = f_1(x^l)$.

4. If $l < k + t + 1$, set $l := l + 1$, the next element of the sequence Q is denoted as U, go to 2.

5. If $l = k + t + 1$, stop. The obtained result is

$$L_1 = \min_{x \in E} f_1 \leq \min_l q_1^l, \; l = 1, 2, \ldots, k + t + 1.$$

6. End of the algorithm.

For the rest part of criteria $f_i(x)$ $(i > 1)$, this algorithm is executed by analogy.

5. An illustrative example

We will consider the following example from the book of S t e u e r [12], ex.14, p. 267. The formulation of the example is

	x_1	x_2	x_3	x_4	x_5	x_6	
f_1:	3	–2			–5		max
f_2:		–2	–2		–2	1	max
f_3:	–2	2		2		2	max
f_4:	1			–2	1	–2	max ,

s.t.

	x_1	x_2	x_3	x_4	x_5	x_6	
c_1:		2				1	$\le$ 10
c_2	2		1	5			$\le$ 9
c_3:	3		4	5	4	2	$\le$ 8
c_4:	3	1	4		4	1	$\le$ 10
c_5:	3	4				3	$\le$ 10
c_6:			3	5	4		$\le$ 5
c_7:	1						$\ge$ 0
c_8:		1					$\ge$ 0
c_9:			1				$\ge$ 0
c_{10}:				1			$\ge$ 0
c_{11}:					1		$\ge$ 0
c_{12}:						1	$\ge$ 0,

where the constraints $c_1, c_2,\ldots, c_{12}$ describe the feasible set S.

Note. The constraints $c_7, c_8, \ldots, c_{12}$ are taken into account automatically by the software when solving standard LP programs . However they are written here because Algorithm “W+RP” can use them taken explicitly as equalities (see below).

The set Q is: $Q = \{S, W_1, W_2, W_3,\ldots, W_{12}\}$.

The payoff table for this example is:

f_1	f_2	f_3	f_4
8.00	0.00	–5.333333	2.666667
0.0	3.333333	6.666667	–6.666667
–2.75	–1.25	7.75	–5.0
7.0	–1.0	–4.333333	2.666667.

The maximal values of the criteria on the whole S are

f_1	f_2	f_3	f_4
8.00	3.333333	7.75	2.666667.

The criteria minimal values taken from the payoff table are

f_1	f_2	f_3	f_4
–2.75	–1.25	–5.333333	–6.666667.

The criteria minimal values on the whole S are

f_1	f_2	f_3	f_4
–7.5	–8.333333	–5.333333	–7.2.

Criterion f_3. Because each row of the payoff table is a Pareto point we see that

$$\min_{x \in E} f_3 = \min_{x \in S} f_3 = -5.333333.$$

This is the **exact** value of the third component L_3 of the nadir point.

Criterion f_1. We execute the first step of Algorithm "W + RP". Here $U = S$ and

$$\min_{x \in S} f_1 = -7.5.$$

The corresponding point in the objective space is

f_1	f_2	f_3	f_4.
–7.5	–7.5	5.0	1.25

This is Pareto point (problem (2) was used); then

$$\min_{x \in E} f_1 = \min_{x \in S} f_1 = -7.5,$$

and this is the **exact** value of L_1.

We can note that here $f_2 = -7.5$, but we will obtain this value later in the general way.

Criterion f_4. The first step of Algotithm "W + RP" ($U = S$) gives :

$$\min_{x \in S} f_4 = -7.2,$$

The corresponding point in the objective space is

f_1	f_2	f_3	f_4
0.0	3.333333	7.2	–7.2

This is a Pareto point (problem (2) was used again); therefore

$$\min_{x \in E} f_4 = \min_{x \in S} f_4 = -7.2,$$

and, as we see, this is the **exact** value of L_4.

Criterion f_2. Solving $\min_{x \in S} f_2$, we obtain the criterion vector (–5, –8.33333, 5, 0), it is not Pareto vector and the corresponding Pareto vector obtained in problem (2) is (–5, –5, 5, 0). After this we need to apply the full power of the algorithm. Firstly we describe the sets W_j

$$W_1 = \{x \in S \mid c_1 = 10\},$$
$$W_2 = \{x \in S \mid c_2 = 9\},$$
$$W_3 = \{x \in S \mid c_3 = 8\},$$
$$W_4 = \{x \in S \mid c_4 = 10\},$$
$$W_5 = \{x \in S \mid c_5 = 10\},$$
$$W_6 = \{x \in S \mid c_6 = 5\},$$
$$W_7 = \{x \in S \mid x_1 = 0\},$$
$$W_8 = \{x \in S \mid x_2 = 0\},$$
$$W_9 = \{x \in S \mid x_3 = 0\},$$
$$W_{10} = \{x \in S \mid x_4 = 0\},$$
$$W_{11} = \{x \in S \mid x_5 = 0\},$$
$$W_{12} = \{x \in S \mid x_6 = 0\}.$$

We have made the computations prescribed by the algorithm and the obtained results are presented in the next table. The first column shows the corresponding set W_j. The second column contains the objective vector, obtained when solving the problem min $f_2(x)$, $x \in W_j$.

The third column shows whether this objective vector is a Pareto point or not. The next column contains the corresponding Pareto point obtained as solution of problem (2)

W_1	(–5, –5, 5, 0)	Pp (–5, –5, 5, 0)
W_2	(3, 0.0, 0.0 –1)	NPp (3.2353, 3.2353, 3.2353, –0.7647)
W_3	(–2.9231, –7.5385, 3.4462, 0.4)	NPp (–2.9231, –4.3077, 3.4462, 0.4)
W_4	(–3.3333, –7.7777, 3.7037, 0.3703)	NPp (–3.3333, –4.4444, 3.7037, 0.3703)
W_5	(–5, –8.333333, 5, 0)	NPp (–5, –5, 5, 0)
W_6	(–5, –8.333333, 5, 0)	NPp (–5, –5, 5, 0)
W_7	(–5, –8.333333, 5, 0)	NPp (–5, –5, 5, 0)
W_8	(0.0, –3.3333, 0.0, 0.0)	NPp (0.7843, –2.5490, 0.7843, 0.7843)
W_9	(–7.5, –7.5, 5, 1.25)	Pp (–7.5, –7.5, 5, 1.25)
W_{10}	(–5, –8.33333, 5, 0)	NPp (–5, –5, 5, 0)
W_{11}	(–5, –8.333333, 5, 0)	NPp (–5, –5, 5, 0)
W_{12}	(–5, –8.333333, 5, 0)	NPp (–5, –5, 5, 0).

Now we can consider the last column, containing the obtained Pareto points. The best value of the second components of these vectors is –7.5. On the other hand S t e u e r [12] confirms that this is the exact value of L_2. Thus we see that for this example the proposed method for estimating the nadir vector gives the **exact** values of its components.

It is clear that in general the proposed method guarantees obtaining of upper bounds T_i^2 only for the values L_i. If we would like to have lower bounds T_i^1 in order to have the inequalities

$$T_i^1 \leq L_i \leq T_i^2,$$

we can consider the set

$$\Lambda_i = \{x \in S \mid f_i(x) = T_i^1\}.$$

If this set does not contain efficient points, then $T_i^1 \leq L_i$. For the considered example we have obtained $L_2 > -7.6$.

Several examples of S t e u e r [12, chapt.9] as well as all examples from the paper [7] have been also analyzed. The obtained estimates of the values L_i are much better than the estimates obtained from the payoff tables.

6. Discussion

The main idea of the proposed algorithm can be expressed as follows. We suppose that all efficient points of problem (1) are points from the frontier of S. The union of all walls W_j covers the frontier. Therefore each efficient point belongs to one or more walls. This allows to see a simple way for searching good efficient points (with low value of f_i): take a wall W_j, find the best point x^j (through min f_i), and the corresponding point $f(x^j)$ in criterion space (with minimal value of f_i on the wall W_j) and then find a good nondominated point using the reference point method; repeat this series of operations with all walls and finally take the point that is the best from the obtained.

The usage of the reference point method does not guarantee that the obtained solution determines an extreme point of S. But this condition must be satisfied because in MOLP problems we handle linear functions. This means that the proposed algorithm could be extended with an addition, that guarantees obtaining of an extreme point of S.

Of course it can be recommended to construct a version of the algorithm that does not check all walls. But in this moment it is not clear – will the profit be sufficiently high? On the other hand changing the parameters of problem (2) we keep the possibilities to influence on some properties of the obtained Pareto points.

The proposed approach does not need any special extensions of LP software. Standard (even old!) versions are sufficient. There is no need of some special optimization techniques with corresponding program realizations. A small knowledge about the reference point method is sufficient. In addition, the proposed method is not based on the theory of vector optimization and (as consequence) does not use the notion of maximally efficient facet and any methods for determining such facets.

It must be added also that the proposed method does not use the earlier developed methods (see for example [1, 5]) for realizing a movement in the set of extreme efficient points .

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Метод за оценка на надир-вектора в задачи на многокритериалното линейно програмиране

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(Р е з ю м е)

Предлага се основата на подхода за оценяване компонентите на надир-вектора в задачата на многокритериално линейно програмиране да съдържа два главни елемента: да се разглежда границата на допустимото множество S и да се използва методът на еталонната точка. Получават се горни граници за оценяваните величини .