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Recurrent Kalman Procedure for ISAR Image Reconstruction from Barcer's Phase Code Modulated Trajectory Signals

Andon Lazarov

Institute of Information Technologies, 1113 Sofia

Abstract: *This work presents an original approximate recurrent approach for image reconstruction from inverse synthetic aperture radar (ISAR) data, obtained by illuminating the target with Barcer's phase code modulated transmitted signal. Geometrical model of ISAR scenario and mathematical expressions of quadrature components of ISAR signal with Barcer's phase code modulation, reflected by object with a complex geometry, are derived. Approximation matrix functions are constructed and used for modeling deterministic ISAR signals reflected by point scatterers, located at nodes of a uniform grid (model) that is depicted in the object's coordinate system. The computational equations of Kalman filtering procedure for target feature extraction from Barcer's phase code modulated ISAR signal are described. To demonstrate the validity and correctness of the developed recurrent Kalman image extraction procedure, numerical experiment is performed. The computational results disclose the capability of the Kalman procedure to obtain high resolution images by short inverse synthetic aperture length, unambiguous and convergent estimates of the point scatterers' intensities of a target from simulated ISAR data.*

Key words*: Inverse Synthetic Aperture Radar, Recurrent Least Mean Square Method, Kalman Recursive Procedure, ISAR Image reconstruction method, Barcer's Phase Code Modulation in ISAR application*

1. Introduction and background

The inverse aperture synthesis is a process of recording the complex amplitude of the trajectory signal reflected from a moving target. The object image referred to as a spatial distribution of its reflectivity function can be retrieved from the complex trajectory signal by applying correlation and spectral procedures, and high resolution time-frequency transforms, such as the short-time Fourier transform and bilinear transform, such as the Wigner-Ville distribution [1, 2]. Algorithms, referred to as parametric and semiparametric spectral estimation procedures, for target feature extraction and complex image formation via synthetic aperture radar are analyzed in [10-13]. The parametric spectral estimation nonlinear least square (NLS) methods for feature extraction, based on certain parametric data sinusoidal models to model ideal point scatterers, are devised and fast Fourier transform (FFT) methods to the simulated phase history data to forming ISAR images are applied. Applying of this approach requires to be ignored the quadratic term of the ISAR signal that means processing should be accomplished over date only obtained by for far-field ISAR measurements.

The traditional tools for target imaging from ISAR data are the correlation and fast Fourier transform. Application of Fourier transform requires that, the scatterers must remain in their range cells, and their Doppler frequency shifts must be constant during the ISAR time interval. Furthermore, the FFT method is known to yield poor resolution if the synthetic aperture spanned by the collected data is not large enough. As known, FFT methods can be employed in the far-field Fraunhofer approximation zone and in cases of rectilinear movement and real or apparent rotation round the object's geometric center. When the target exhibits complicated maneuvers combining translation and rotation motions, these methods demand motion compensation procedures. In case the correlation ISAR image restoration methods are employed, development of certain reference functions is required.

It is known that in ISAR applications the large inverse synthetic aperture data set (large time-interval or large spatial-interval data set) often contains large trajectory distortions caused by the moving target. The spatial Doppler spectrum becomes smeared and image reconstruction is blurred. A small inverse synthetic aperture length (small ISAR data set) has to be used to avoid motion-based distortions in the image reconstruction. Given a small inverse synthetic aperture data set, the correlation and FFT methods cannot yield high resolution. It is worth to note that in case the ISAR data are obtained through illuminating by Barcer's phase code modulated pulses correlation range compression procedure can only be exploited whereas FFT technique is not applicable.

In this work a new technique for ISAR image reconstruction from Barcer's phase code modulated signals, referred to as recurrent Kalman method is presented. This can cope with a small inverse synthetic aperture data set and yield a high resolution image. Approximate methods can be assumed as an alternative to correlation and FFT methods, and NLS parametric estimation techniques. The Kalman method can be related to the adaptive filtering theory and assumed as an approach that solves the inverse problem of image reconstruction. The computation of the Kalman methods is also efficient, compared to other available high resolution methods [3, 4, 5, 6]. In that context, this paper deals with the approximate Kalman method that can be utilized for image restoration from ISAR data, obtained from the rectilinearly moving target, which is illuminated by Barcer's phase code modulated transmitted signal. The application of recurrent procedures requires building of approximation functions. The main purpose of the present research is to reveal the composition of approximation functions in linear approximation and to develop ISAR image reconstruction methods based on recurrent quasi-linear estimation of invariant geometric parameters in the complex amplitude of the ISAR trajectory signal with Barcer's phase code modulation.

2. ISAR geometry

The object presented as assembly of point scatterers is detected in a 2-D object's space in a form of a grid of reference points, which is described in own 2-D Cartesian coordinate system *O'XY* (Fig. 1). The object's space is moving rectilinearly with a constant vector velocity V in the 2-D Cartesian coordinate system *Oxy*. The object and ISAR are placed in separate coordinate systems. Point *O'*(0) is the location of the origin of the coordinate system *O'XY* at the moment $p = N/2$ (*N* is the full number of emitted pulses), which corresponds to the middle of the ISAR length; point *O'*(*p*) is the location of the origin of the coordinate system *O'XY* at the *p*-th moment.

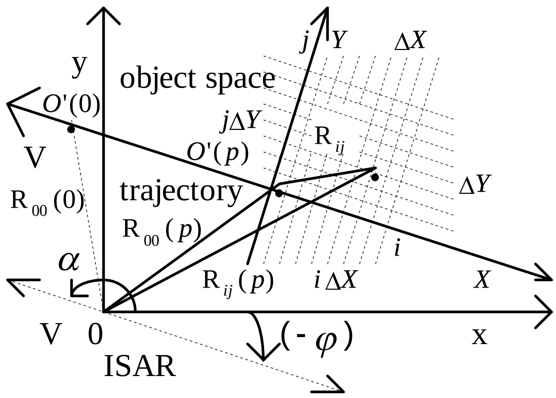

Fig. 1. ISAR geometry

The range vector $\mathbf{R}_{ij}(p) = [x_{ij}(p), y_{ij}(p)]^{\mathrm{T}}$ from ISAR placed in the origin of the coordinate system *Oxy* to the *ij*-th reference point of the object at the *p*-th moment is defined by the vector equation [7]

$$\mathbf{R}_{ij}(p) = \mathbf{R}_{00}(0) + \mathbf{V}\left(\frac{N}{2} - p\right)T_p + \mathbf{A}\mathbf{R}_{ij}, \tag{1}$$

where $\mathbf{R}_{ij} = [X_{ij}, Y_{ij}]^{\mathrm{T}}$ is the geometric distance vector of the *ij*-th reference point in coordinate system *O'XY*; $\mathrm{V} = [V_x, V_y]^{\mathrm{T}}$ is the vector velocity of the object; $\mathbf{R}_{00}(0) = [x_{00}(0), y_{00}(0)]^{\mathrm{T}}$ is the range vector to the object's geometric center, point *O'*(0), defined in the coordinate system *Oxy*; $\mathbf{A} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$ is the transformation matrix; φ is the angle between the coordinate axes *Ox* and *O'X*; $X_{ij} = i(\Delta X)$ and $Y_{ij} = j(\Delta Y)$ are the discrete coordinates of *ij*-th reference point; ΔX and ΔY are the dimensions of the 2-D grid cell on the coordinate axes *O'X* and *O'Y* respectively, $i = \overline{1, I}$ is the number of the reference point on the axis *O'X*; $j = \overline{1, J}$ is the number of the reference point on the axis *O'Y*, *I* is the full number of the reference points placed on the axis *O'X* in the object space, *J* is the full number of the reference points placed on the axis *O'Y* in the object space.

3. ISAR transmitted barcer's phase code modulated pulses and modeling deterministic components of ISAR signal

ISAR transmitted Barker's phase code modulated pulses can be described by the equation

(2) $$S(p) = \sum_{p=1}^{N} a(t - pT_p)\exp\left\{- j\left[\omega\left(t - pT_p\right) + \pi b\left(t - pT_p - \bar{k}T_k\right)\right]\right\}$$

where $a(t - pT_p)$ is the amplitude of the transmitted pulses, $\omega = 2\pi c/\lambda$ is the signal angular frequency, φ_0 is the initial phase of the Barcer's phase code modulated (BPCM) pulse; $t = (k - 1)\Delta T$ is the discrete current time, $\bar{k} = \overline{1,\bar{K}}$, is the number of the segment of the BPCM signal, $\bar{K} = \frac{T}{T_k} = 13$ is the full number of segments of the BPCM signal, T_k is the time duration of a segment of the BPCM pulse, $k = \overline{1,K}$ is the sample number of the transmitted BPCM pulse; $\Delta T = \frac{T_k}{2}$ is the time duration of the sample of the BPCM signal, $K = \frac{T}{\Delta T}$ is the number of samples of the BPCM pulse, T is the time duration of the full BPCM pulse, $\Delta R = cT_k/2$ is the dimension of the range resolution cell. For each $p = \overline{1,N}$ phase sign parameter is defined by

(3) $$b\left(t - pT_p - \bar{k}T_k\right) = \begin{cases} 0, t = \bar{k}T_k, \bar{k} = \overline{1,5}, \\ 1, t = \bar{k}T_k, \bar{k} = \overline{6,7}, \\ 0, t = \bar{k}T_k, \bar{k} = \overline{8,9}, \\ 1, t = \bar{k}T_k, \bar{k} = 10, \\ 0, t = \bar{k}T_k, \bar{k} = 11, \\ 1, t = \bar{k}T_k, \bar{k} = 12. \\ 0, t = \bar{k}T_k, \bar{k} = 13 \end{cases}$$

The deterministic components of the ISAR signal return for every p-th pulse are derived by applying the physical optics principle of Huggens-Fresnel, i.e.

(4) $$S(p,k) = \sum_{i=1}^{I}\sum_{j=1}^{J} a_{ij}\exp\left\{- j\left[\omega\left(t - t_{ij}\right) + \pi b\left(t - t_{ij} - \bar{k}T_k\right)\right]\right\},$$

where a_{ij} is the reflection coefficient (intensity) of the point scatterer of the object space.

The time dwell t of the ISAR signal return for each transmitted puls p can be written as $t = t_{ij,\min}(p) + (k - 1)\Delta T$, where $k = \overline{1, K + L}$ is the number of the ISAR signal discrete, $t_{ij}(p) = \frac{2R_{ij}(p)}{c}$ is the time delay of the ISAR signal, reflected from ij-th point scatterer, $L = \mathrm{int}\left[\frac{t_{ij,\max}(p) - t_{ij,\min}(p)}{\Delta T}\right]$ is the relative time dimension of the target, $t_{ij,\min}(p) = \frac{2R_{ij,\min}(p)}{c}$ is the minimum time delay of the ISAR signal, reflected from the target, $t_{ij,\max}(p) = \frac{2R_{ij,\max}(p)}{c}$ is the maximum time delay of the ISAR signal, reflected from the target, $R_{ij}(p)$ is the module of the range distance vector to the point scatterer, defined by the expression

(5) $$R_{ij}(p) = \left[x_{ij}^2(p) + y_{ij}^2(p)\right]^{\frac{1}{2}},$$

where

(6) $$x_{ij}(p) = x_{00}(0) + V_x T_p\left(\frac{N}{2} - p\right) + X_{ij}\cos\varphi - Y_{ij}\sin\varphi,$$

(7) $$y_{ij}(p) = y_{00}(0) + V_y T_p\left(\frac{N}{2} - p\right) + X_{ij}\sin\varphi + Y_{ij}\cos\varphi,$$

$V_x = V\cos\alpha$, $V_y = V\sin\alpha$ are the components of the target vector velocity.

The Barcer's phase code parameter $b\left(t - t_{ij} - \bar{k}T_k\right)$ of the ISAR signal is defined as follows:

(8) $$b\left(t - t_{ij} - \bar{k}T_k\right) = \begin{cases} 0, t = t_{ij} + \bar{k}T_k, \bar{k} = \overline{1,5}, \\ 1, t = t_{ij} + \bar{k}T_k, \bar{k} = \overline{6,7}, \\ 0, t = t_{ij} + \bar{k}T_k, \bar{k} = \overline{8,9}, \\ 1, t = t_{ij} + \bar{k}T_k, \bar{k} = 10, \\ 0, t = t_{ij} + \bar{k}T_k, \bar{k} = 11, \\ 1, t = t_{ij} + \bar{k}T_k, \bar{k} = 12, \\ 0, t = t_{ij} + \bar{k}T_k, \bar{k} = 13. \end{cases}$$

The ISAR signal return $\xi(p, k)$ is an additive sum of a deterministic component $S(p, k)$ and zero mean complex Gaussian noise $n(p)$, i.e.

(9) $$x(p, k) = S(p, k) + n(p, k).$$

The expressions (4)–(9) can be applied for modeling Barcer's phase code modulated ISAR signal return in case the object is moving on a rectilinear trajectory in 2-D coordinate system.

4. Recurrent Kalman image reconstruction procedure estimating the invariant geometric parameters of Barcer's phase code isar signal quadrature components

4. A. Vector measurement equation and state equation

As known iterative minimum mean square error (MMSE) image reconstruction procedures [9] relay on priory knowledge of the moment values of the ISAR signal. This prohibits evaluation of the invariant parameters in real time. This limitation can be overcome through a recurrent Kalman filtering procedure. Kalman filtering is MMSE estimator and has two distinct characteristics. The first of these characteristics is that Kalman filtering is grounded on the state-space concept that allows processing the system as a whole unit. The second of these characteristics is that Kalman filtering is recursive procedure. The update of the estimate of the state is computed from the current estimate and the current input measurement data. This property makes Kalman filter more computationally efficient than MMSE iterative methods. This recursive capability also eliminates the necessity to store all previous measurement data and previous state estimates [14]. Kalman filtering as an image reconstruction and future extraction procedure is exploited in [9, 15]

The vector measurement equation and vector state equation describing the vector law of parameter variations at the discrete moment p are

$$\begin{aligned} \xi(p,k) &= \mathbf{S}[p, k, \mathbf{a}(p)] + \mathbf{n}(p, k), \\ \mathbf{a}(p) &= \mathbf{g}[p, k, \mathbf{a}(p-1)] + \mathbf{n}_0(p, k), \end{aligned} \tag{10}$$

where $\xi(p, k)$ is a column vector with dimension $[2(K + L); 1]$; $\mathbf{S}[p, k, \mathbf{a}(p)]$ is the deterministic process, defined in the field of discrete vector arguments $\mathbf{a}(p)$, and yields a column-vector with dimensions $[2(K + L)$; $\mathbf{g}[p, k, \mathbf{a}(p - 1)]$ is a column-vector function that describes the law of variation of the vector arguments at discrete time moments yielding dimensions of $[I \times J; 1]$; $\mathbf{n}(p, k)$, $\mathbf{n}_0(p, k)$ are sequences of random vector values with zero expected value and covariance matrices $\psi(p, k)$ with dimensions $[2(K + L); 2(K + L)]$ and $\mathbf{V}(p)$ with dimensions $[I \times J; I \times J]$, respectively. The vector of arguments, $\mathbf{a}(p)$, which has dimensions $[I \times J; 1]$ accounts for a vector estimates of intensities of point scatterers $\hat{a}_{ij}$.

For modeling the quadrature components of the ISAR trajectory signal, it is supposed that in the Cartesian coordinate space Oxy an object moves with a rectilinear trajectory at a constant speed V. The object is situated in a coordinate network whose origin may coincide with the geometric centre of the object. The shape of the object is described by the intensities (reflection coefficients), a_{ij} of point scatterers distributed in accordance with its geometry.

The distance variation to the ij-th point scatterer $\mathbf{R}_{ij}(p)$ of the object is determined by expressions (5), (6), (7). The quadrature components of the ISAR signal (return) reflected by point scatterers of the object space, are additive mixtures of deterministic and random components presented by (8). The expression (9) can be exploited to model the deterministic quadrature components of the ISAR signal.

4. B. Approximation functions

In the most general case, the functions **S**[*p*, *k*, **a**(*p*)] and **g**[*p*, *k*, **a**(*p* – 1)] in (10) would be non-linear. This circumstance would lead to ambiguity in invariant parameter definition. One of the main purposes of the present work is to reveal the composition of **S**[*p*, *k*, **a**(*p*)] and **g**[*p*, *k*, **a**(*p* – 1)] under linear approximation and to develop an algorithm for a quasi-linear Kalman filtration of the invariant vector parameters in the complex amplitude of the ISAR trajectory signal.

The approximation function **S**[*p*, *k*, **a**(*p*)] is defined by the quadrature components of a complex signal, reflected by the point scatterers, placed on the nods of a uniform grid, i.e.

$$\mathbf{S}[p,k,\mathbf{a}(p)] = \mathbf{S_c}[p,k,\mathbf{a}(p)] + j\mathbf{S_s}[p,k,\mathbf{a}(p)]. \tag{11}$$

The Taylor's expansion after ignoring the higher order terms results in the following linear equations:

$$\begin{aligned} \mathbf{S}_c\left[p,k,\mathbf{a}(p)\right] &= \mathbf{S}_c\left[p,k,\dot{\mathbf{a}}(p)\right] + \sum_{j=1}^{J}\sum_{i=1}^{I}\frac{\partial \mathbf{S}_c\left[p,k,\dot{\mathbf{a}}(p)\right]}{\partial a_{ij}}\left(a_{ij}-\dot{a}_{ij}\right), \\ \mathbf{S}_s\left[p,k,\mathbf{a}(p)\right] &= \mathbf{S}_s\left[p,k,\dot{\mathbf{a}}(p)\right] + \sum_{j=1}^{J}\sum_{i=1}^{I}\frac{\partial \mathbf{S}_s\left[p,k,\dot{\mathbf{a}}(p)\right]}{\partial a_{ij}}\left(a_{ij}-\dot{a}_{ij}\right), \end{aligned} \tag{12}$$

where $\dot{\mathbf{a}}(p) = \left[\dot{a}_{11}...\dot{a}_{1J}, \dot{a}_{21}...\dot{a}_{ij}...a_{IJ}\right]^T$ is the vector-estimates of the invariant geometrical parameters with dimensions [$I \times J$; 1], the superscript "T" denotes matrix transpose, the product $I \times J$ denotes the full number of estimates of isotropic point scatterers of the grid with intensity $\dot{a}_{ij}$. The constant coefficients of the Taylor expansion is defined by

$$\begin{aligned} \mathbf{S}_c\left[p,k,\dot{\mathbf{a}}(p)\right] &= \sum_{j=1}^{J}\sum_{i=1}^{I}\dot{a}_{ij}\cos\left[\omega\left(t-t_{ij}\right)+\pi b\left(t-t_{ij}-\bar{k}T_k\right)\right], \\ \mathbf{S}_s\left[p,k,\dot{\mathbf{a}}(p)\right] &= \sum_{j=1}^{J}\sum_{i=1}^{I}\dot{a}_{ij}\sin\left[\omega\left(t-t_{ij}\right)+\pi b\left(t-t_{ij}-\bar{k}T_k\right)\right]. \end{aligned} \tag{13}$$

The coefficients of the linear terms of the Taylor's expansion (12) defined by the expressions

$$\frac{\partial \mathbf{S}_c\left[p,k,\dot{\mathbf{a}}(p)\right]}{\partial a_{ij}} = \cos\left[\omega\left(t-t_{ij}\right)+\pi b\left(t-t_{ij}-\bar{k}T_k\right)\right] \tag{14}$$

$$\frac{\partial \mathbf{S}_s\left[p,k,\dot{\mathbf{a}}(p)\right]}{\partial a_{ij}} = \sin\left[\omega\left(t-t_{ij}\right)+\pi b\left(t-t_{ij}-\bar{k}T_k\right)\right]. \tag{15}$$

If the vector-estimated parameters are Gaussian and Markov, the state transition matrix function **g**[*p*, *k*, **a**(*p* – 1)], linking the vector estimates of invariant parameters in two consecutive moments in linear approximation, is given by the expression [9]:

$$\mathbf{g}[p,k,\mathbf{a}(p-1)] = \mathbf{g}[p,k].\ \dot{\mathbf{a}}(p-1)], \tag{16}$$

where $\mathbf{g}(p,k) = \mathrm{diag}\left[\exp\left(\frac{NT_p}{\tau_{ij}}\right)\right]$ is the diagonal matrix with dimensions [$I\times J$; $I\times J$],

τ_{ij} is the correlation time of the parameter a_{ij}.

If the observation time NT_p, is considerably less than the correlation time, τ_{ij}, then the state transition matrix $\mathbf{g}(p, k)$ becomes approximately an identity matrix, i.e., the estimated geometrical parameters are invariant in the ISAR observation time interval.

4. C. Kalman procedure

The modified recurrent Kalman procedure for quasilinear estimation of invariant geometric parameters can be defined as follows [9]:

(17) $$\dot{\mathbf{a}}(p) = \mathbf{g}(p,k).\dot{\mathbf{a}}(p-1) + \mathbf{K}(p,k)\left\{\boldsymbol{\xi}(p,k) - \mathbf{S}\left[p,k,\dot{\mathbf{a}}(p-1)\right]\right\},$$

where

(18) $$\xi(p,k) = \left[\xi_c(p,k),\ \xi_s(p,k)\right]^{\mathrm{T}}$$

is the new measurement vector with dimensions $[2(K + L;\ 1]$;

(19) $$\mathbf{S}\left[p,k,\mathbf{a}(p-1)\right] = \begin{Bmatrix} \mathbf{S}_c\left[p,k,\mathbf{a}(p-1)\right] \\ \mathbf{S}_s\left[p,k,\mathbf{a}(p-1)\right] \end{Bmatrix}$$

is the measurement prediction vector with dimensions $[2(K + L;\ 1]$;

(20) $$\mathbf{K}(p,k) = \mathbf{R}(p,k)\mathbf{H}^{\mathrm{T}}(p,k)\psi^{-1}(p,k)$$

is the Kalman filter gain-matrix with dimensions $[I\times J;\ 2(K + L]$; $\mathbf{R}(p, k)$ is the update state error covariance matrix with dimensions $[I\times J;\ I\times J]$, determined by

(21) $$\begin{aligned}\mathbf{R}^{-1}(p,k) = \left[\mathbf{g}^{\mathrm{T}}(p,k)\mathbf{R}(p-1,k)\mathbf{g}(p,k) + \mathbf{V}^{-1}(p,k)\right]^{-1} + \\ + \mathbf{H}^{\mathrm{T}}(p,k)\psi^{-1}(p,k)\mathbf{H}(p,k);\end{aligned}$$

(22) $$\mathbf{H}(p,k) = \begin{bmatrix} \dfrac{\partial \mathbf{S}_c\left[p,k=1,\dot{\mathbf{a}}(p-1)\right]}{\partial a_{11}} \cdots\cdot \dfrac{\partial \mathbf{S}_c\left[p,k=1,\dot{\mathbf{a}}(p-1)\right]}{\partial a_{IJ}} \\ \cdots\cdots\cdots\cdots \qquad \cdots\cdots\cdots\cdots \\ \dfrac{\partial \mathbf{S}_c\left[p,k=K,\dot{\mathbf{a}}(p-1)\right]}{\partial a_{ij}} \cdots\cdot \dfrac{\partial \mathbf{S}_c\left[p,k=K,\dot{\mathbf{a}}(p-1)\right]}{\partial a_{IJ}} \\ \cdots\cdots\cdots\cdots \qquad \cdots\cdots\cdots\cdots \\ \dfrac{\partial \mathbf{S}_s\left[p,k=1,\dot{\mathbf{a}}(p-1)\right]}{\partial a_{11}} \cdots\cdot \dfrac{\partial \mathbf{S}_s\left[p,k=1,\dot{\mathbf{a}}(p-1)\right]}{\partial a_{IJ}} \\ \cdots\cdots\cdots\cdots \qquad \cdots\cdots\cdots\cdots \\ \dfrac{\partial \mathbf{S}_s\left[p,k=K,\dot{\mathbf{a}}(p-1)\right]}{\partial a_{11}} \cdots\cdot \dfrac{\partial \mathbf{S}_s\left[p,k=K,\dot{\mathbf{a}}(p-1)\right]}{\partial a_{IJ}} \end{bmatrix}$$

is the state-to-measurement transition matrix with dimensions $[2(K + L;\ I\times J]$.

The elements of the matrix $\mathbf{H}(p, k)$ for each $p = \overline{1,N}$ can be generally described by expressions

$$h^{c}_{k,(i-1)J+j} = \frac{\partial \mathbf{S}_{\mathrm{c}}\left[p,k,\dot{\mathbf{a}}(p-1)\right]}{\partial a_{ij}} = \cos\left[\omega\left(t-t_{ij}\right)+\pi b\left(t-t_{ij}-\bar{k}T_{k}\right)\right], \tag{23}$$

$$h^{s}_{k,(i-1)J+j} = \frac{\partial \mathbf{S}_{\mathrm{s}}\left[p,k,\dot{\mathbf{a}}(p-1)\right]}{\partial a_{ij}} = \sin\left[\omega\left(t-t_{ij}\right)+\pi b\left(t-t_{ij}-\bar{k}T_{k}\right)\right]. \tag{24}$$

The matrix $\mathbf{R}(p-1, k)$ is the predicted state error covariance matrix with dimensions $[I\times J; I\times J]$. In the beginning of the procedure $p = 1$, the initial predicted state error covariance matrix $\mathbf{R}(0, k)$ is an identity matrix. The process noise covariance matrix $\mathbf{V}(p, k)$ and the measurement covariance matrix $\psi(p, k)$ are diagonal with elements $\mathbf{N}_0/2T_p$, where $\mathbf{N}_0$ is the spectral density of the Gaussian noise.

5. Numerical experiment

To substantiate the properties of the proposed 2-D model of ISAR signal with Barcer's phase code modulation and to verify the correctness of the developed Kalman image reconstruction procedure a numerical experiment was carried out. It is assumed that the target is moving rectilinearly in a 2-D observation Cartesian coordinate system *Oxy* and is detected in 2-D coordinate system *O'XY*. The trajectory parameters of the target are as follows: the module of the vector velocity $V = 600$ m/s; the guiding angle of the vector velocity $\alpha = \pi$; the angle between coordinate axes, $\varphi = 0$; the coordinates of the mass-center at the moment $p = N/2$: the initial coordinates of the target geometric center: $x_{00}(0) = 0$ m, $y_{00}(0) = 5.10^{4}$ m. The ISAR transmitted pulse is characterized be following parameters. The wavelength is $\lambda = 3.10^{-2}$ m.

The ISAR transmitted pulse is characterized be next parameters: the wavelength is $\lambda = 3\times10^{-2}$ m; the carry frequency is $f = 10^{10}$ Hz; the repetition period for signal registration (aperture synthesis) is $T_p = 2.5\times10^{-2}$ s; the time duration of a segment of the BPCM pulse is $T_k = 3.3\times10^{-9}$ s; the number of the segment of the BPCM signal is $\bar{k} = \overline{1,13}$; the full number of segments of the BPCM signal is $\bar{K} = 13$; the time duration of the sample of the BPCM signal is $\Delta T = 1.65\times10^{-9}$ s; the time duration of the transmitted BPCM pulse is $T = 42.9\times10^{-9}$ s; the dimension of the range resolution cell is $\Delta R = 0.5$ m; the number of samples of BPCM transmitted signal is $K = 26$; the sample number of the transmitted BPCM pulse is $k = \overline{1, 26}$; the number of transmitted pulses during inverse aperture synthesis is $N = 100$.

Modeling ISAR signal requires that the whole geometry of the target be enclosed in a 2-D regular rectangular grid, described in the coordinate system *O'XY* (Fig. 2). The dimensions of the grid's cell are $\Delta X = \Delta Y = 0.5$ m. The number of the reference points of the grid on the axes X is $I = 32$ and on the axis Y is $J = 32$. Equivalent point scatterers are placed at each node of the regular grid. The intensities of the point scatterers placed on the target are $a_{ij} = 0.1$. The intensities of the point scatterers placed out of the target are $a_{ij} = 0.001$. The approximation functions account for the quadrature components of Barcer's phase code modulated ISAR signal reflected by a regular grid with the same dimensions as in case of modeling, and consisting of isotropic point scatterers with an equal initial intensity $\dot{a}_{ij} = 0.001$.

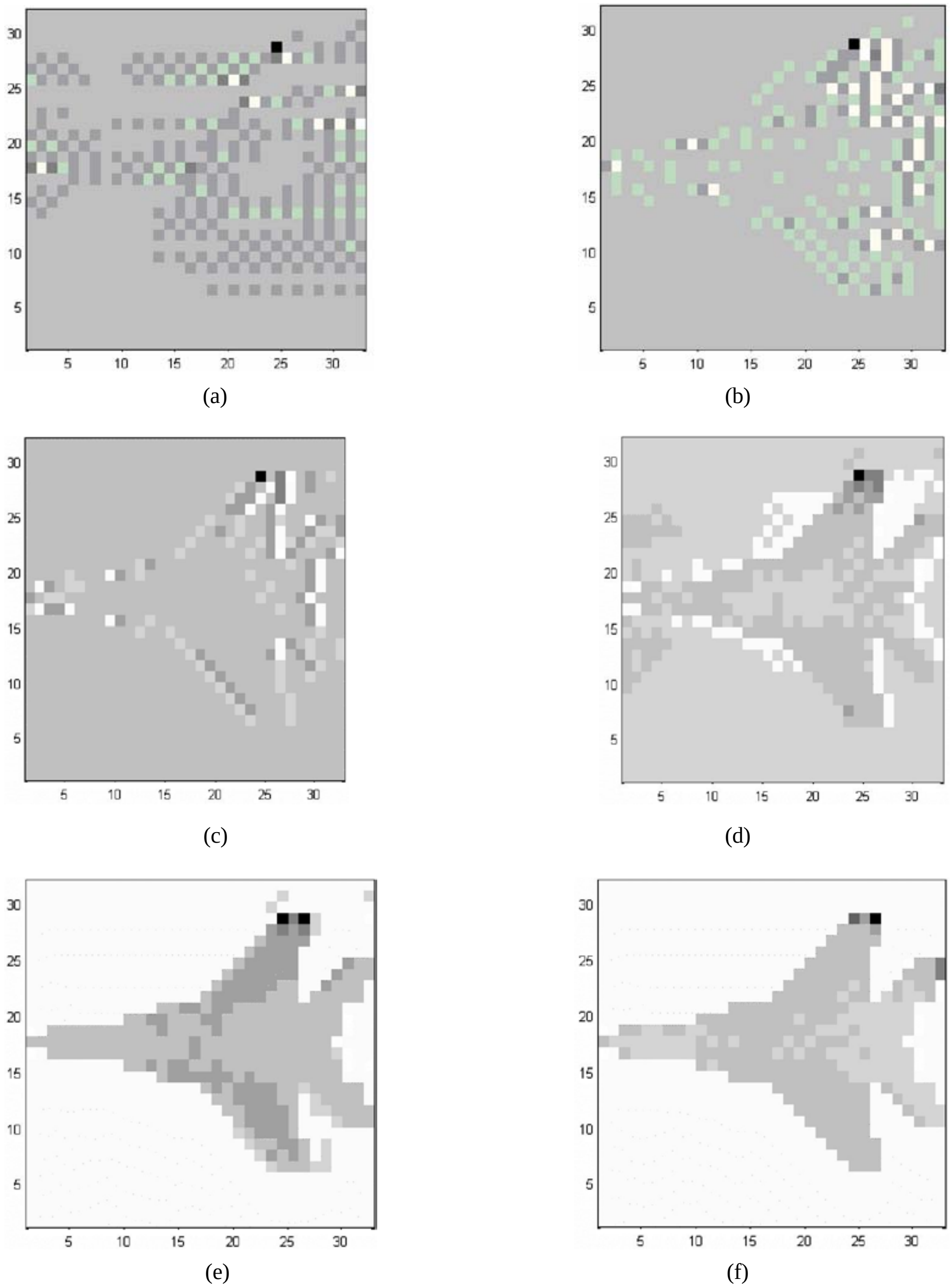

Fig. 2. Six phases by $p = 1$ (a), $p = 14$ (b), $p = 30$ (c), $p = 47$ (d), $p = 65$ (e) and $p = 100$ (f) of the Kalman image reconstruction procedure of the aircraft MIG-29

The results of the numerical experiment are presented in Fig. 2, a, b, c, d, e, f, where six phases (extrapolations) of the image reconstruction processes of the aircraft MIG -29 are depicted. The evolution of the quality of the picture by each step of the procedure demonstrates the correctness of the geometrical model and mathematical expressions of the quadrature components of ISAR signals and Kalman equations as well. The developed Kalman procedure is featured by high calculation speed, steady and quick convergence of computations. Therefore the recurrent Kalman procedure is an exceptional tool for image reconstruction with high quality and could be exploited successfully for extracting invariant geometric parameters from ISAR data obtained by Barcer's phase code modulated transmitted pulses.

6. Comparative analysis

The extreme azimuth resolution capability of the approximate recurrent image reconstruction method does not directly depend on the inverse synthetic aperture duration, unlike the correlation and FFT azimuth compression methods. It is known that if the correlation and FFT algorithms are utilized, the azimuth resolution is of better quality when the inverse synthetic aperture length is larger.

By applying the approximate image reconstruction procedure the number of the ISAR measurements is determined by the number of the estimated point scatterers, placed at appropriate nodes of the grid model. A minimum number of measurements, i.e. minimum inverse synthetic aperture relative length, correspond to the least number of the estimated parameters.

Analysis of the factors influencing azimuth resolution properties of the image reconstruction procedures discloses that in order to achieve certain azimuth resolution the required inverse synthetic aperture length is much less if the recurrent method is applied than in the case of correlation and FFT azimuth compression algorithms. In the small length inverse synthetic aperture, the following advantage can be derived. The trajectory of the object can be approximated with a straight line. The restrictions related to the motion distortions can be eliminated. The approximation functions of the intensities applied in the recurrent procedure are unambiguous and smoothly varying. The basic geometrical parameters – intensities of the point scatterers can be assumed to be invariant

The computational results illustrate that the recurrent image reconstruction algorithm that extracts the invariant geometrical parameters from the simulation ISAR data is steady when the intensities of point scatterers are estimated. Only 100 extrapolations are needed to achieve a satisfying resolution by applying the Kalman procedure. This is the main reason to use proposed ISAR image reconstruction procedures in practical applications.

7. Conclusions

The goal of this work is to examine the properties of the approximate recurrent Kalman techniques for image reconstruction from Barcer's phase code modulated ISAR data. The major contribution of this investigation is to construct approximation functions and to develop the recurrent algorithm for quasilinear estimation of invariant vector geometric parameters in ISAR complex amplitude, obtained through illuminating the target by Barcer's phase code modulated signal. The properties and capabilities of the correlation and FFT azimuth compression methods for ISAR image retrieval to compare with recursive Kalman feature extraction method are revealed. Recurrent Kalman image reconstruction procedure to replace correlation and FFT azimuth compression techniques is proposed. The Kalman procedure retrieves the target image by extracting the estimates of the invariant geometric parameters from quadrature components of ISAR signal with Barcer's phase code modulation.

Simulation experiments are presented to illustrate the capability of the approximate recurrent method for ISAR imaging of rectilinear moving targets. The results and comparative analysis of the image reconstruction approaches clearly demonstrate that the approximation techniques can be used to replace correlation and Fourier transforms as a means of ISAR image reconstruction without suffering image blurring. Therefore, the restriction of the correlation and Fourier transform can be overcome. The recurrent Kalman procedure demonstrates high effectiveness in the target image reconstruction using simulated ISAR data.

The results from the mathematical modeling of the ISAR image reconstruction process as well as recurrent Kalman algorithms for extracting invariant geometrical parameters from the quadrature components of the trajectory complex signal can be used to develop a microprocessor system for the object's image restoration from ISAR data, obtained by Barcer's phase code modulated pulses.

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Рекурентна Калманова процедура за възстановяване на ISAR-изображения от фазово модулирани по код на Баркер траекторни сигнали

Андон Лазаров

Институт по информационни технологии, 1113 София

(Р е з ю м е)

Настоящата разработка представя апроксимационен рекурентен подход за възстановяване образи от измервания на радиолокационни системи със синтезирана апертура (ISAR), получени посредством облъчване на целта с радиоимпулси, фазово модулирани с код на Баркер. Предложена е геометрия на ISAR-сценария и са изведени математически изрази за определяне на квадратурните компоненти на ISAR-сигнали с фазова кодова модулация на Баркер, отразени от обект със сложна геометрия. Конструирани са апроксимиращи матрични функции, които са използвани за моделиране на детерминираните ISAR-сигнали, отразени от точкови излъчватели, разположени във възлите на еднородна решетка (модел), която е изобразена в координатната система на обекта. Дефинирани са изчислителните уравнения на филтриращата процедура на Калман за извличане на геометричните характеристики на целта от ISAR-сигнали с фазова кодова модулация на Баркер. Реализиран е числен експеримент за доказване на достоверността и коректността на разработената Калманова процедура за извличане на образ. Изчислителните резултати разкриват възможностите на процедурата на Калман за получаване на образи с висока разделителна способност посредством синтезирана апертура с малка дължина на вълната и на еднозначни и сходими оценки на интензивностите на точковите излъчватели на целта от симулираните ISAR-измервания.