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Fuzzy Multicriteria Decision Making[1]

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Abstract: *The purpose of this investigation is directed towards summarizing and researching of models for decision making support in multicriteria problems under uncertainties from fuzzy type. The models simulate (approximate) human decision making by means of applying of the fuzzy sets theory. Multicriteria fuzzy decision making problems will be considered in cases of fuzziness present in initial information and the stages of problem's solutions, as well.*

Keywords: *Multicriteria decision making, fuzzy sets theory, fuzzy relations, fuzzy numbers, aggregation operators.*

I. Introduction

Alternatives in decision making problems are usually evaluated from different points of view, which corresponds to particular criteria. In real-life situations, evaluations are neither certain nor precise. There are three main sources of uncertainty [50]:

– imprecision, because of the difficulty of determining the scores of alternatives on particular criteria;

– interdetermination, since the method of evaluation results from a relatively arbitrary choice from several possible definitions;

– uncertainty, since the values involved vary in time.

Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling and controlling uncertain systems. They are facilitators for approximate reasoning in decision making in the absence of complete and precise information. Their role is significant

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when applied to complex phenomena, which are not easily described by traditional mathematics.

Let $A = \{a_1, a_2, \ldots, a_n\}$ be a finite set of alternatives, K_j – a criterion, c_j – the weight of the criterion j $(j = 1, \ldots, m)$. The purpose of the decision making can be the choice, ranking or clustering problem by comparing the alternatives. It has to take into account the following, when it makes this:

– their fuzzy performances on all criteria;

– the weights attached to each criterion;

– the possible difficulties of comparing two alternatives, when one is significantly better than the other on a subset of criteria, but much worse on at least one criterion from the complementary subset.

The decision making problems under uncertainty may be classified into two groups:

i) The alternative estimations by criteria are crisp, but procedures for decision making imitate the human behavior, i.e. it uses the fuzzy sets theory;

ii) the criteria are fuzzy, i.e. the alternatives estimations are linguistically variables and the decision making may be realized applying traditional or fuzzy methods.

By no doubt, the problems from the second group may be reduced to the ones from the first group, if the linguistically estimations are transformed into quantitative ones. For example, the linguistically variables may be represented as fuzzy numbers. Then a function mapping of each fuzzy number on the real line may be determined.

2. Fuzzy models of multicriteria decision making by crisp criteria

The problem is defined in the following way: a finite set of alternatives is evaluated from several nonfuzzy criteria (utility functions, nonfuzzy orderings). The alternatives have to be compared in such a way that solutions of the problems for: choice of a subset from the "best", in some sense, alternatives; ordering over the whole set of alternatives; partition the set of alternatives of the subsets from the similar, close ones, i.e. partition from clusters, are obtained. The information about the alternatives can be supplied in different scales. In this case, it is required to make the information uniform. One basic approach to make this is to use a fuzzy relations over the set of alternatives as the main element of uniform representation. Therefore, it needs some transformation functions, which define the relations between the couple of alternatives by each criterion. These functions define relations with different properties, for example similar or preference relation. It is more realistic to use fuzzy relations because they appear as a more convenient and adequate form for representing the relationship between the alternatives then crisp relations. The fuzzy relations may model situations, whenever interactions between the alternatives are not exactly determined. Besides that, they reflect the interests of the experts or decision maker. The fuzzy relations and their properties are investigated by many authors [18, 32, 37, 53, 58, 59, 61]. Taking this into account, the defined problem is solved in the following three stages [11]:

A. Uniform stage. It derives an individual fuzzy relation for each criteria. Different transformation functions are used to do this.

B. Aggregation stage. A purposeful approach for uniting individual fuzzy relations is to use the aggregation procedures that realize the idea of compensation and compromise between conflicting criteria, when compensation is allowed. Using the concept of fuzzy majority represented by a linguistic quantifier and applying some aggregation procedure, an aggregated fuzzy relation is obtained from the individual fuzzy relations.

C. Exploitation stage. The problems of choice, ranking or clustering have to be solved in this stage on the base of aggregated fuzzy relation.

A. Information uniform stage

A fuzzy multicriteria decision making problem is investigated in [11], when the information about the alternatives can be represented by means of nonfuzzy preference ordering, utility functions and fuzzy preference relations. The purpose is to establish a general model for making the information uniform, which cover all possible representations. Firstly, the relationship between the utility values, given on the base of a positive ratio scale and fuzzy preference relations is study. Let a_{ik} and a_{jk} be the utility values of the alternatives a_i and a_j according to the criterion K_k and these values belong to the interval [0,1]. Then any possible transformation function h: $[0, 1]\times[0, 1] \to [0, 1]$, depending only on the values a_{ik} and a_{jk} presents a fuzzy preference relation, i.e. $p_{ij}^k = h(a_{ik}, a_{jk})$. This function h must by a non-decreasing one in the first argument and a non-increasing one in the second argument, i.e. $h(a_{ik}, a_{jk}) = l\left(\frac{a_{ik}}{a_{jk}}\right)$, where l is a nondecreasing function. The function h has to verify the following properties:

(1) $h(x, y) + h(y, x) = 1, \forall x, y\in[0, 1]$,

(2) $h(x, x) = 0.5, h(x, 0) = 1, \forall x\in[0, 1]$,

(3) $h(x, y) > 0.5$ if $x > y$, $\forall x, y\in[0, 1]$.

The type of transformation functions l are investigated and several examples of functions l are defined.

Several fuzzy preference relations derived from utility values of the alternatives are given in the sequel. Let [53] $a_{ik}, a_{jk}, i, j = 1,\dots,n, k = 1,\dots,m$, be positive integers. A fuzzy relation R on A for each criterion is defined as follows:

$$\mu_R(a_{ik}, a_{jk}) = \begin{cases} 0 & \text{if} \quad a_{ik} > a_{jk} \\ 1-[a_{ik} \mid a_{jk}]/a_{jk} & \text{if} \quad a_{ik} \le a_{jk} \end{cases},$$

where $\mu_R(a_{ik}, a_{jk})$ is the membership degree to the defined relation R and $[a_{ik} | a_{jk}]$ is the remainder, when a_{ik} is divided by a_{jk}. It is proved, that R is reflexive, perfect antisymmetrical and Δ-transitive, i.e.

(4) $\mu_R(x,x) = 1, \forall x \in [0,1]$,

(5) if $\mu_R(x,y) > 0$, then $\mu_R(y,x) = 0, \forall x, y \in [0,1]$,

(6) $\mu_R(x,z) \ge \max(0, \mu_R(x,y) + \mu_R(y,z) - 1), \forall x, y, z \in [0,1]$.

A pairwise preference approach which permits a homogeneous treatment of different kinds of evaluations is suggested in [49]. It supposes that greater $a_{ik}, i = 1,\dots,n$, corresponds to the better alternative. The degree to which the alternative a_i is not worse then a_j for the criterion k is defined with the help of two thresholds: an indifference threshold $\mathrm{IT}[a_{ik}]$ and a preference one $\mathrm{PT}[a_{jk}]$. A membership function related to a fuzzy interval (fuzzy number) $\tilde{a}_{ik}$ may be defined with the help of these thresholds (PT>IT). By comparing of two fuzzy numbers the degree of credibility for the preference of a_i over a_j for the criterion K_k is obtained. The related structure is called a fuzzy interval order.

Another transformation function is suggested in [41,43]. The degree of preference is defined as:

$$\mu_k(a_i, a_j) = \begin{cases} 1 & \text{if } i = j, \\ 0.5 + \dfrac{a_{ik} - a_{jk}}{2(\max\limits_i\{a_{ik}\} - \min\limits_i\{a_{ik}\})} & \text{if } i \neq j. \end{cases}$$

The preference relation given by this function is reflexive (4), reciprocal (1) and max-min transitive, i.e.

(7) $$\mu_R(x,z) \geq \min(\mu_R(x,y),\, \mu_R(y,z)),\ \forall x, y, z \in [0,1].$$

Therefore, this relation is a fuzzy total ordering according to the definition given in [58].

In the case, when the information about the alternatives by the criteria are nonfuzzy orderings [11] O^k, $k = 1,\dots,m$, it is supposed that the lower the position of an alternative in a preference ordering is, the "better" alternative and vice versa. It asserts the existence of a transformation function f that assigns a credibility value of preference of any alternative over any other one from any preference ordering, $p_{ij}^k = f(o^k(i), o^k(j))$. This transformation function must be a non-increasing one in the first argument and a non-decreasing in the second one. The function f has to satisfied the properties (1), (2) and (3). The examples of transformation functions f are given as well.

B. Aggregation stage procedures

The aggregation of the individual relations at the second stage of the fuzzy models, considered here, may be realized with the help of procedures, satisfying the requirements mentioned above. These procedures may be performed by using aggregation (fuzzy logic) operators (FLOs). The first ones introduced by L. Zadeh are for the logical operations AND, OR and NOT as extensions of their Boolean origins. These operators are Min, Max and $1-\mu$ (μ is the membership degree to a given fuzzy subset). Some research works reveal that the degree of compensation through which humans aggregate criteria is not expressed only by these operators. There exist some operators which represent human decision making more accurately. The Weighted Mean [3, 7, 11, 56, 57], Weighted Geometric [12], Weighted MaxMin and Weighted MinMax [21, 49] operators e.g., use the importance of the criteria, given as weights. In order to decide a variety of phenomena in decision situations, several operators with parameters are introduced. For example, such operators are MaxMin, MinAvg and Gamma [54, 62]. These parametrical operators give the possibility to the decision maker by means of the parameters values changes to take part in the process of decision making.

There are aggregation operators which are suitable for combining scores in multicriteria evaluation problems. These averaging operators compensate a bad score for one criterion by a good one for another criterion. All these operators represent particular cases of the Generalized Mean operator [17, 25, 51]. A very good overview of the aggregation operators, by presenting the characteristics, the advantages and disadvantages of each operator and the relations between them, is available in [17, 25]. There is a large range of operators, which can be advantageously used in the confluence of fuzzy criteria. The choice of an operator for specific application depends on various factors. In fact, some choices have to be made according to, e.g.:

– the mathematical model of the operators;

– the properties of the operators for deciding problems of ranking or choice, or clustering of the alternatives' set;

– the sensitivity of the operators for small variations of their arguments.

The dependence between the properties of the aggregated relation and the properties of the individual relations by each fuzzy criterion for the above operators are proved in [38, 39, 40, 42, 43, 44, 47]. In [47] these connections are summarized and presented in a table.

The sensitivity of the operators with respect to variations in their arguments is defined and computed in [46].

The list of aggregation operators includes the following ones besides:

– the ordered weighted average (OWA) operator as a generalization of the weighted mean and which has as particular case the operators Min and Max;

– the discrete fuzzy integrals – Choquet and Sugeno. The Choquet integral generalizes the OWA operator, while the Sugeno one generalizes the weighted maximum and the weighted minimum operators;

– the *t*-norms and the *t*-conorms, which compute the intersection and union (respectively) of fuzzy sets;

– the uninorms, which solve another problem connected with the lack of full (downwards and upwards) reinforcement.

The complete references for these operators is given in [17].

Continuity of an operator is a genuine property of practical application procedures in which a "small" difference in input values cannot cause a "big" difference in the output values. The associativity on the other hand, models the independence of the aggregation on the grouping of the input values. An investigation of continuous associative aggregation operators is made in [6].

One different approach of aggregation is investigated in [22]. Four classes of aggregation procedures of individual relations with weights are examined. These procedures are based on the use of a valued implication or coimplication. The existence of a hierarchy among them is showed. These results are applied for the definition of two classes of similarity measures between fuzzy sets, providing in general a pessimistic degree and an optimistic degree of similarity.

C. Exploitation stage

The aggregated degree to which "a is not worse then b" obtained at the end of the aggregation process does not always present any ordering properties (except reflexivity), having in mind mostly the max-min transitivity (7). Then the aggregated graph $G(A, R)$ (vertices correspond to the alternatives' set A and valued arcs support the aggregated relation R) cannot be interpreted in terms of ranking or choice [23]. But the relation R may be transformed in such a way to obtain a modified max-min transitive relation. For example, every α – cut of R is transitive in the crisp sense and corresponds to a quasiorder, which can be represented by Hasse diagram [18]. Other transitive relations close to a given preference relation are given in [49]. The general expression for a reflexive (4) and transitive relation (7) is defined in [52].

The main contribution of the notion of fuzzy preorder [18], i.e. the relation which posses the properties (4) and (6), consists in a membership degree proposal for a preference relation without violating the choice problem itself. Any antisymmetrized fuzzy preorder relation R', i.e. the relation with properties (4), (5), (6), is a fuzzy partial ordering. It is possible to represent R' as a triangular matrix. Due to perfect antisymmetry and transitivity, the graph corresponding to this matrix has no cycle and α – cut of R' is a nonfuzzy partial ordering.

The ways for solving the ranking and choice problems are introduced in [49]. Fuzzy strict order relations and the notion of their reduction are defined in [8]. A necessary and sufficient condition is obtained for the transitive closure of the reduction and some possible graph-theoretic significance's of the results are discussed. Orlovsky's concept of decision making on a finite set of alternatives with a fuzzy preference relation is analyzed in [20, 29]. The application of that concept for optimization of many decision problems is formulated and proved. Two quantifier guided choice degrees of alternatives are used in [11]: a dominance degree used to quantify the dominance that one alternative has over all the others, in fuzzy majority sense, and a nondominance degree, which generalizes Orlovsky's nondominated alternative concept. The application of the two choice degree can be carried out according to two different selection processes, a sequential selection process and a conjunction selection process. A systematic study of fuzzy ordered sets and an intrinsic fuzzy topology on them is given in [53].

If the aggregated relation is a similarity or likeness one, the problem of clustering may be solved. A similarity relation over a finite set of alternatives can be represented as a similarity tree of a dendogram type [60], where each tree level represents an α -cut of this relation. The set of elements on a specific α-level can be considered as similarity classes (fuzzy clusters) of α-level. A method for comparison of fuzzy clusters is given in [41]. The proposed algorithm is based on the assertion that the comparison between two fuzzy clusters can be made of comparing only fuzzy clusters' α-scores as it is proved in [48].

3. Fuzzy models of multicriteria decision making by fuzzy criteria

The problem under consideration is the following: a finite set of alternatives is evaluated by several fuzzy criteria, the estimations being fuzzy numbers or fuzzy relations. The alternatives have to be compared in a way to have the problems of ranking, choice or clustering solved. The case of fuzzy relations is already considered in section 2, using the stages B and C. That's why, only the case of fuzzy numbers will be presented in this section. In this case, the comparison between the alternatives consists in the comparison of fuzzy numbers or m-tuple of them.

3.1. Monocriterial comparison of fuzzy numbers

Different methods for comparing or ordering of fuzzy numbers exist. They can be classified according to two different approaches:

– using a crisp relation with the help of ranking function;

– using a fuzzy relation on the set of fuzzy number, computing a comparison index for each pair of them.

3.1.1. Methods with crisp relations. Let $\tilde{a}_i, i = 1,\dots,n$, be n normal convex fuzzy subsets (fuzzy numbers), i.e.

$$\tilde{a}_i = \{x, \mu_i(x)\}, \ x \in I_i \subset I, \ I \equiv [0,1],$$

where $\mu_i(x)$ is the membership function of the fuzzy number $\tilde{a}_i$. A simple method for ranking $\tilde{a}_i$ consists in the defining of a ranking function *F*, mapping each fuzzy number onto the real line, where a natural order exists. This function is such that if $F(\tilde{a}_i) <=> F(\tilde{a}_j)$, then $\tilde{a}_i <=> \tilde{a}_j$. This approach has been followed by several authors, e.g. in [1, 9, 10, 19, 20, 24, 28, 30, 41, 55]. It is a relatively simple and easy one for application, but it reduces the whole information about the fuzzy number into a real one. Several well known ranking functions are tested on selected examples of fuzzy numbers in [1, 28, 41].

3.1.2. Methods with fuzzy relations. These methods are based on the idea, that the property “ to be greater (less) than a fuzzy number” is a linguistic property and every decision maker handles such a property in a personal way, measuring it according to internal and external factors. Many methods for comparing fuzzy numbers with the help of fuzzy relations have been proposed in the literature [14, 15, 23, 29, 31, 32, 34, 58, and etc.]. Each method has its own advantages and disadvantages, hence it should be chosen for each particular problem. An attempt for evaluating these ranking methods is proposed in [58]. Four criteria for this evaluation are suggested: fuzzy preference representation, rationality of fuzzy ordering, distinguishability between fuzzy numbers and robustness by small changes in the membership function of the fuzzy number. Based on these criteria, two existing ranking methods (Baas and Kwakernaak’s, [28] and Nakamuras’ [32]) are evaluated.

3.2. Comparison of *m*-tuple of fuzzy numbers

Let a *m*-tuple of fuzzy numbers corresponds to given alternative from the set *A*, which is the evaluation of this alternative by the m fuzzy criteria. These m-tuple have to be compared to solve the problems of ranking, choice or clustering of the set of alternatives. The existing approaches to solving this problem may by classified into two groups:

– using distances between the m-tuples of fuzzy numbers;

– aggregating every m-tuple of fuzzy numbers by each alternative to a fuzzy number and then comparing the aggregated fuzzy numbers.

3.2.1. Comparing by distances. The decision maker often has suppositions about the best or the worst alternative, i.e. the upper or lower horizon. Hence, it seems to be quite natural to order alternatives according to the distance between each alternative and the fixed horizon. The family of distances on the space of all trapezoidal fuzzy numbers treated as elements of four-dimensional space are introduced and investigated in [27]. These distances, depending on a parameter, generate the class of linear orderings under fixed upper horizon. It is proved, that such an ordering does not depend on the chosen horizon.

A development and a generalization of the above method is presented in [26]. The two classes of metrics introduced provide a possibility to extend the application of this approach not only to trapezoidal fuzzy numbers, but to left-sided or right-sided fuzzy numbers.

3.2.2. Aggregating of m-tuple of fuzzy numbers. The idea for aggregating a m-tuple of fuzzy numbers corresponding to an alternative is applied in [5]. These m fuzzy numbers are aggregated using fuzzy arithmetic into fuzzy weights $\tilde{w}_i$, where $\tilde{w}_i$ is the fuzzy ranking assigned to alternative a_i. In [45] the aggregation of the sequences of fuzzy numbers, representing the alternatives, is done with the help of fuzzy logic (aggregation) operators in such a way that the aggregated evaluations be fuzzy numbers, as well. The methods for getting the aggregated fuzzy numbers are different depending on the selected operator and the type of fuzzy numbers. The arithmetic operations between fuzzy numbers and operations Max and Min are used for the computations of the aggregated fuzzy numbers. A new method implementing the operations Min and Max over sets of fuzzy number is proposed in [13].

The methods from section 3.1 may be used for comparison of the aggregated fuzzy numbers to solve the choice, ranking or clustering problems of the alternatives after that.

4. Concluding remarks

The purpose of this investigation is directed towards researching the models for decision making support in multicriteria problems under uncertainties from fuzzy type. The models have to simulate (approximate) human decision making by means of applying one of the basic elements of soft computing – fuzzy logic and more precisely – the fuzzy sets theory. Multicriteria fuzzy decision making problems are considered in cases of fuzziness present in initial information and at the stages of problem's solutions, as well. Our investigations and results solving these problems are presented, too. Some numerical examples deciding the suggested here problems and using the methods proposed from us, are given in [41, 43, 48, 47].

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Размити многокритериални задачи за вземане на решения

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Целта на изследването е свързана с моделите за подпомагане вземането на решения при многокритериални задачи в условия на неопределеност от размит вид. Тези модели симулират (апроксимират) вземането на решения от човека, като за целта се използва един от основните елементи на soft computing – размитата логика, а в по-широк аспект – теорията на размитите множества. Изследвани са многокритериални задачи за вземане на решения, при които размитостта е както в началната информация, така и при етапите от решаването на задачите. Моделите са класифицирани в зависимост от началната информация и от методите за тяхното решаване. Показан е и приносът на авторите при разработване на модели за размито многокритериално вземане на решения.