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An Optimizationally Motivated Interactive Method for Solving a Class of Discrete Multicriteria Choice Problems

Mariyana Vassileva

Institute of Information Technologies, 1113 Sofia

Abstract: *An optimizationaly motivated learning-oriented interactive method for solving a class of discrete multicriteria choice problems with a large number of alternatives and a small number of quantitative criteria is proposed in the paper. At each iteration the decision maker (DM) can set his local preferences in terms of desired improvements or relaxations of the criteria. On this basis a discrete optimization scalarizing problem is constructed. A small ranked set of relatively close alternatives is defined with the help of this scalarizing problem. The ranked set is presented to the DM for selection of the most preferred alternative or for entering his/her new local preferences.*

Keywords: *discrete multicriteria choice problems, quantitative criteria, scalarizing problem.*

1. Introduction

Multiple criteria decision-making problems can be divided [19] into two classes according to their formal statement: a finite number of explicitly stated constraints implicitly determine an infinite number of feasible alternatives in the first one whereas in the second a finite number of alternatives are stated explicitly. The first class of problems is called multiple objective mathematical programming (MOMP) problems or multicriteria choice problems with continuous alternatives. The multiple criteria decision analysis (MCDA) problems that are also called discrete multiple criteria problems or multi attribute analysis problems belong to the second category.

The so-called interactive methods have been most widely used in the solution of MOMP problems. In each of these methods the phases of decision and computation are iteratively executed. In the computation phases, when a certain type of a scalarizing problem is solved, one or several nondominated alternatives are generated, which satisfy to the greatest extent the local preferences of the decision maker (DM) in the decision phase. The DM realizes selection and choice of the best local alternative (the preferred alternative). In case this alternative satisfies his global preferences also, it becomes the best global alternative (the most preferred alternative). Otherwise the DM enters additional information, corresponding to his new local preferences, which is used in the next computation phases searching for new better alternatives. During the search of the best preferred alternative in these methods it is assumed that the DM optimizes an explicit value (utility) function or that by learning during the search process he/she tries to satisfy to the highest degree his/her aspiration concerning the values of the separate criteria (the aspiration level of the criteria). Convergence of the solution process is presumed in both types of methods. Mathematical convergence of the computing process is ensured in the first type of methods called "search - oriented methods". In the second type of methods, named "learning oriented methods" behavioral or intuitive convergence of the solution process is expected, which is ensured by the DM. In the two types of methods it is assumed that the DM can easily compare two alternatives and in this way can estimate whether to prefer one of them or whether the two are equivalent for him/her (indifference). Because of the fact when solving MOMP problems the purpose is the find the best preferred solution, MOMP problems can be called continuous multicriteria choice problems (CMCCP).

The problems of MCDA can be classified in three main groups. The best preferred nondominated alternative is searched for in the first group of problems (discrete multicriteria choice problem – DMCCP). In the second group of problems, ranking of the nondominated alternatives is established, starting from the best towards the worst one (ranking problem). In the third group of problems the set of alternatives is divided into separate groups (sorting problem). The MCDA problems usually contain a finite number of alternatives and criteria, but they can be of considerable number as well. When the alternative number is large, the DM can hardly perceive these alternatives as a whole, which makes these problems relatively close to MOMP problems. When the criteria number is large, the DM cannot perceive these criteria as a whole either, particularly when some of them are contradicting. That is why some procedures (techniques) are included in the main MCDA methods, which compare two nondominated alternatives based on preference information set by the DM or more exactly based on DM preference model. Two types of DM preference models are used in the main MCDA methods. The first type of a preference model enables the comparison of every pair of alternatives. This type of a preference model is the utility function. The main methods using this type of DM preference model to different extent are the multiattribute utility theory methods and the analytical hierarchy process methods (see [2, 3, 4, 6 and 14]). The second preference model allows the existence of incomparable alternatives, when the preference information obtained by the DM is insufficient to determine whether one of the alternatives is to be preferred or whether the two alternatives are equal for the DM. Such a preference model is the outranking relation. The methods using this type of a preference model are called outranking methods (see [1, 11, 12 and 13]).

In order to solve multicriteria choice problems with a large number of alternatives and a small number of quantative criteria, which can be regarded by the DM as problems close to MOMP problems, the so called "optimizationally motivated" interactive methods, inspired by MOMP methods, have been suggested. (see [5, 7, 8, 9, 10, 15 and 16]).Because the DM is engaged in the interactive solution process, this creates difficulties connected with:

• the design of user-friendly procedures. As the DM is responsible for the final solution, it is desirable that he/she is able to, to some extent, comprehend the problem and these procedures to have confidence in the quality of the selected solution.

• training the DM in solving the problem especially with a large number of alternatives and criteria. This refers to the relative importance of the criteria, the possibilities for compensation, scaling effects, amplitude of deviations among the criteria values and others

The paper offers learning oriented interactive method designed to solve multiciteria choice problems (MCCP) with a large number of alternatives and a small number of quantative criteria, which is similar to the above given optimizationally motivated methods and is inspired by MOMP method, published in [17, 18]. In this method the DM interference is considerably decreased. This is achieved introducing two main alterations. The first one consists in the fact, that to improve the alternative currently found, the DM is obliged to give desired values of some criteria, desired directions of other criteria alteration and preservation of the current value for the rest. The second change is that the DM is provided with a few (sometimes only two) close or comparable alternatives for comparison. In this way he can take into consideration more definitely and realistically the criteria importance, their correlation, the possibilities for compensation among them, as well as to estimate better the criteria values in the alternatives being compared. Besides this, taking into account the current preferred solutions at each iteration, the DM can compare the distributed alternatives also. In this way the interactive method suggested enables also the combination of efficient search and DM's learning in the set of the nondominated alternatives.

The paper is organized in the following way. Some notations and definitions are introduced in the next section. A general description of the method is given in Section 3 and its algorithm scheme is shown in Section 4. An illustrative example is represented in Section 5. The advantages of the method are summarized in the Conclusion.

2. Preliminary considerations

The discrete multiple criteria decision analysis problem is defined as follows: Given a set I of n (>1) deterministic alternatives and a set J of k ($\geq$2) quantitative criteria, we define an $n\times k$ decision matrix A. The element a_{ij} of the matrix A denotes the evaluation of the alternatives $i \in I$ with respect to the criterion $j \in J$. The vector (a_{i1}, a_{i2}, ..., a_{ik}) shows the evaluation of alternative $i \in I$ with respect to all the criteria in the set J. The column vector (a_{1j}, a_{2j} , ..., a_{ij}) gives the assessment of all the alternatives in set I for criterion $j \in J$. The objective is to search for a non-dominated alternative that satisfies the DM mostly with respect to all the criteria simultaneously.

The alternative $i \in I$ is called non-dominated if there is no other alternative $s \in I$ for which $a_{sj} \geq a_{ij}$ for all $j \in J$ and $a_{sj} > a_{ij}$ for at least one $j \in I$.

Because it is comparatively simple to identify dominated alternatives, in the rest of the paper, we shall assume that matrix A contains only nondominated alternatives.

A current preferred alternative is a non-dominated alternative chosen by the DM at the current iteration. The most preferred alternative is a preferred alternative that satisfies the DM to the greatest degree.

The reference neighborhood is defined by current preferred alternative, the desired changes in the criteria values of some of the criteria and the desired directions of change of a few of the remaining criteria as specified by the DM. The desired changes in the criteria values are the amounts by which the DM wishes to increase criteria compared to their values in the current preferred alternative. The desired directions of change of the criteria are the directions in which the DM wishes to improve or worsen the criteria with respect to their values at the current preferred alternative.

A current ranked sample of alternatives is a subset of the nondominated alternatives that includes the current preferred alternative and $(l-1)$ alternatives (l being set by the DM), that belong to the reference neighborhood and which are obtained after a discrete optimization scalarizing problem is solved.

3. Method description

The idea of the method presented is to generate at each iteration *iter* a ranked set $M=\{i_1, i_2, ..., i_l\}$ of alternatives, the first alternative being the current preferred alternative and l being the number of generated alternatives which the DM is willing or is able to evaluate at the this iteration. The DM has to estimate the relatively close alternatives and to choose one of them either as a current preferred or as the most preferred alternative. In the second case the discrete multicriteria choice problem is solved. In the first case the DM sets the desired changes of the criteria (desired values or desired directions for improving(relaxing)) in order to search for a new better alternative in the reference neighborhood of the current preferred alternative.

Let h denotes the index of the current preferred alternative. Let us introduce the following notations in relation to the current preferred alternative:

$K_h^{\geq} \cup K_h^{>}$ is the set of indices $j \in J$ of the criteria for which the DM wishes to increase their values compared to their values in the current preferred alternative, where:

$K_h^{\geq}$ – the set of indices of the criteria $j \in J$ that the DM wants to improve by desired (aspiration) values Δ_{hj};

$K_h^{>}$ – the set of indices of the criteria $j \in J$ that the DM wants to improve and for which he/she is able to provide only the desired changes in direction;

$K_h^{\leq} \cup K_h^{<}$ – the set of indices of the criteria for which the DM agrees to deteriorate their values compared to their values in the current preferred alternative, where:

$K_h^{\leq}$ is the set of indices of the criteria $j \in J$ that the DM agrees the values of the criteria to be deteriorated by no more than δ_{hj};

$K_h^{<}$ – the set of indices of the criteria $j \in J$ that the DM agrees to be worsened;

K_h^{0} – the set of indices of the criteria $j \in J$ in which the DM is not interested concerning alteration at the moment and these criteria can be freely altered;

$\bar{a}_{hj}$ – the desired (aspiration) value of the criterion with an index

$$\bar{a}_{hj} = a_{hj} + \Delta_{hj}\ ; \quad j \in K_h^{\geq}\ .$$

a_{hj} – the value of a criterion with an index $j \in K_h^{\geq}$ in the current preferred alternative;

Λ_j – the difference between the maximal and minimal value for the criterion with an index j;

$$\Lambda_j = \max_{i \in I} a_{ij} - \min_{i \in I} a_{ij}\ .$$

The set $M = (i_1, ..., i_l)$ is computed solving the following discrete scalarizing problem:

(S):

$$\min_{i \in I} S(i, h) = \min_{i \in I}\{\max[\max_{j \in K_h^{\geq}}(\bar{a}_{hj} - a_{hj})/\Lambda_j\ ,\ \max_{j \in K_h^{\geq} \cup K_h^{\leq}}((a_{hj} - a_{ij})/\Lambda_j\ ] + \max_{j \in K_h^{\geq}}(a_{hj} - a_{ij})/\Lambda_j\},$$

subject to

$$a_{ij} \geq a_{hj},\ j \in K_h^{>};$$
$$a_{ij} \geq a_{hj} - \delta_{hj},\ j \in K_h^{\leq}.$$

When solving a discrete optimization problem S the value of $S(i,h)$ is computed for all alternatives for which the conditions $a_{ij} \geq a_{hj}$, $j \in K_h^{>}$, and $a_{ij} \geq a_{hj} - \delta_{hj}$, $j \in K_h^{\leq}$, are satisfied. The function $S(i,h)$ denotes the distance between alternatives i and h with respect to the "modified" Chebychev norm.

4. The algorithm scheme

On the basis of the discrete optimization scalarizing problem S an algorithmic scheme can be designed for solving discrete multicriteria choice problem with a small number of quantitative criteria and a large number of alternatives. At each iteration the DM has the possibility to estimate a small (specified by the DM) ordered set of alternatives obtained by solving the scalarizing problem S. These alternatives belong to the reference neighborhood of the current preferred alternative defined by the local preferences of the DM. They are to some extend close to the current preferred alternative. Thus in the learning process and after that the DM can take into mind such factors that can be hardly formalized.

The main steps of the algorithm are:

Step 1. Reject all dominated alternatives and define the decision matrix A. Set $iter = 1$ and ask the DM to choose an initial preferred alternative, denoted by h.

Step 2. If the DM wants to store the current preferred alternative h, check if it has been saved before. If "not" – add h to LIST– a set of stored preferred alternatives.

Step 3. Ask the DM to define the desired change of the criteria values with respect to the current preferred alternative h. Ask the DM to specify a parameter l – the number of generated alternatives he/she would like to see in the next iteration.

Step 4. Define the parameters $K_h^{>}$, $K^{\geq}$, $K_h^{<}$, $K_h^{\leq}$, K_h^{0}, $\bar{a}_{hj}$, Λ_j and δ_{hj} of the discrete optimization problem S, solve it and determine the current ranked set of alternative M.

Step 5. Present the set M to the DM for evaluation. If the best compromise alternative has been found– Stop, otherwise – update $iter := iter + 1$.

Step 6. If the DM selects the new current preferred alternative– assign it to h and go to Step 2.

Step 7. If the DM wants to return to one of the stored alternatives – assign the selected stored alternative to h and go to Step 3.

Remark. The rejecting of dominated alternatives is done once they are known in the initial phase of the algorithm [15]. Their complexity is measured by $O(kn^2)$.

5. Illustrative example

In order to illustrate the method proposed we shall use an example with 5 alternatives and 3 quantative criteria, in which the dominated alternatives are apriori removed from consideration. The decision matrix A has the type as in Table 1.

Table 1

i \ j	1	2	3
1	2.0	15	6.5
2	2.5	20	4.5
3	1.8	27	5.5
4	2.2	18	6.0
5	2.3	21	5.0
Λ_j	0.7	12	2.0

We are looking for a maximal value of each of the criteria. We choose i=2 as an initial preferred alternative, for which the first criterion has maximal value. *iter* =1, h=2, l=3.

For each one of the criteria the DM sets his/her wishes concerning their values alteration with respect to these for alternative h=2. Deterioration by no more than 0.5 units is feasible for the first criterion $K_h^{\leq}$= {1}, δ_{21} = 0.5; for the second criterion no alteration is defined – K^0 = {2}, and he/she wishes to improve the third one – $K_h^{>}$ ={3]. In conformance with the parameters thus set, scalarizing problem S is stated and solved (Table 2).

Table 2

i	1	2	3	4	5
$S(i,h)$	–0.286	*	**	–0.32	0.036

* – current preferred alternative

** – the constraints of scalarizing problem S are not satisfied for this alternative.

M = {2, 4, 1}.

The DM chooses alternative i = 4 as a current preferred alternative from the ranked set M, h = 4 and l=3. At the next iteration *iter* = 2 he/she sets again his/her preferences for alteration of the criteria values compared to those of alternative h = 4. For the first criterion – feasible reducing $K^{<}$ = {1}, for the second criterion a desired value is a_{42} = 20, $K^{\geq}$ = {2}, and there are no requirements towards the value of the third criterion K^0 = {3}. Scalarizing problem S is again formed and solved.

Table 3

i	1	2	3	4	5
$S(i,h)$	0.416	0	0.571	*	–0.08

Here M = {4, 5, 2}.

The set M is represented to the DM for evaluation. He/she chooses i=5 as the best compromise alternative. This concludes the algorithm functioning.

6. Conclusion

An optimizationally-motivated learning-oriented interactive method for solving a class of discrete multicriteria choice problems with a large number of alternatives and a small number of quantative criteria is proposed. The method enables the DM to evaluate systematically and successively the set of the alternatives. The method proposed has several advantages:

– it is easy to understand and relatively simple to use;

– it is flexible as it allows the DM to provide the information with which he/she feels comfortable;

– it gives possibility to the DM at each iteration to compare quite close alternatives, which does not engage him/her too much, but enables the more realistic account of his/her preferences.

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Оптимизационно мотивиран интерактивен метод за решаване на клас дискретни задачи за многокритериален избор

Марияна Василева

Институт по информационни технологии, 1113 София

(Р е з ю м е)

В статията е предложен оптимизационно мотивиран ориентиран към обучение интерактивен метод за решаване на задачи за многокритериален избор при зададени голям брой алтернативи и малък брой количествени критерии. На всяка итерация лицето, вземащо решение (ЛВР), има възможност да задава своите локални предпочитания по отношение на желаните подобрения или възможните влошавания на критериите. На основата на тази локална информация, задавана от ЛВР, се конструира дискретна оптимизационна скаларизираща задача, с помощта на която се получава подредено подмножество от сравнително близки алтернативи. Това подмножество е представяно на ЛВР за избор на най-предпочитаната алтернатива или за задаване на неговите нови локални предпочитания.