BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **25**, No 2
Sofia • 2025 Print ISSN: 1311-9702; Online ISSN: 1314-4081
DOI: 10.2478/cait-2025-0013

Robust Performance Assessment of Control Systems with Root Contours Analysis

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Abstract: *Often, in a real practice environment, classical methods for tuning controllers do not provide the desired performance of the control strategy, as they do not account for changes in the parameters of the controlled process. Sometimes, there is no possibility for the rapid implementation of a new control algorithm, and the designer only has access to the current tuning of the introduced parameters. This paper presents an approach for assessing control performance in cases where the controller is designed for nominal operating conditions, aiming to reduce the control error caused by changes in the parameters of the controlled process. A controller with three tuning parameters is considered, with research conducted on the possible correction of the overall proportionality coefficient. The issue of insensitivity and the performance of the transient processes of the closed-loop system is also addressed.*

Keywords: *Control systems, Performance criteria, Robustness, Root contours.*

1. Introduction and research aim

The practical need for rapid decision-making in control strategy and the introduction of intelligent elements of industrial automation for their implementation have led to the diminishing significance of classical control theory. Today, efforts are focused on the relatively global integration of the entire control system. This has prompted the scientific community to seek transformations of classical control theory by adapting it to the new situation.

The advantage of transforming classical methods lies in the convenience of well-established scientific and applied knowledge and their known applications to different classes of controllable processes that fulfill complex performance criteria. This transformation enables a new interpretation of classical control theory through novel visualizations, new model representations, and the discovery of hidden beneficial relationships. These developments enhance engineering intuition, inspire the combination of various classical approaches, and ultimately lead to practically effective results [4, 12, 18].

This paper aims to propose a new approach for assessing control performance by adapting the capabilities of the complex plane to the study of system robustness.

This approach can be used for the rapid retuning of controller parameters within the system in situations where a parametric change in the controlled process has not been accounted for in the control strategy.

The primary task of this research is to mathematically track and graphically represent the relationship between the proportionality coefficient and the robustness properties in the complex plane, as well as the system's performance in the time domain.

1.1. Brief overview

For over a century, the PID controller has been the focus of theoretical research and practical implementation using different physical technical bases [7, 14]. Its application in automation systems has given rise to numerous methods for determining tuning parameters to satisfy complex performance criteria concerning the behavior of transient response processes [8, 9, 22]. It turns out that during the tuning procedures [8], which are carried out quickly by a human operator, the risk of accumulating inaccurate data should be minimized. Such data could be used by training algorithms for maintenance and decision-making in industrial production.

In practice, the theoretical analysis of automation processes controlled by PID regulators is rarely used. The difficulty arises from the fact that the mathematical model of the controlled process is not always accurate. Consequently, when a situation occurs, tuning is almost always performed experimentally and most often involves tuning the proportional gain. Typically, the mathematical models of controllers in modern SCADA systems implement form representation in which the proportional gain affects all control parameters [10].

There are various methods for analyzing and representing uncertainty in the parameters of the controlled process, as well as numerous approaches for tuning controllers for systems with uncertain parameters. Often, these methods rely on a priori known variations in the parameters of the controlled process, meaning the controller does not adapt to these changes but accounts for them in advance [1, 23]. Fundamentally, these methods are typically derived by transforming classical control approaches into modern control methods that consider changes in the controlled object [9, 16]. One such possibility is the development of methodologies that combine robust principles with the root locus method, where the beneficial effect is directly observable and gives meaningful application to the scientific-theoretical foundation in a practical environment [19, 22]. The root locus method is widely used as it enables the real design of automation systems without relying on experimental methods based on extrapolation practices, which would otherwise lead to time loss due to returning [2, 13, 24].

1.2. Brief comparison and explanation

The use of the complex plane for analyzing control systems with uncertain parameters requires the discovery of transformations of well-known methods, providing solutions that incorporate interval polynomials [6, 12].

In [15], an LMI technique for root positioning in second-order linear systems affected by uncertainty is described. In [20, 21], the problem of synthesizing stable characteristic polynomials that describe the dynamics of control systems under interval uncertainty is examined through the lens of the root locus. [11] presents an extended representation of the root locus method, incorporating polynomials with complex coefficients. In reference [4, 15], the concept of a robust root locus is introduced, providing a broader view of uncertainty in the control system, while in [9], root contours obtained from the components of the control algorithm are studied, but for systems with certain parameters. In [17], the authors derive another group of root contours, with the focus of the development being directed towards stability margins – gain and phase margin for control systems with certain coefficients.

These scientific studies provide the basis for further developing the concept of a robust root locus and the use of root contours in the present paper. Original research proposed in this paper is the consideration of changes in the parameters of the controlled process and the subsequent performance analysis of the system's robustness when the proportional gain coefficient in the control algorithm changes. The proposed research solution offers a canonical approach for the rapid retuning of the proportional gain, thus preventing the accumulation of unreliable data for training and processing in many industrial applications.

2. Root contours

2.1. Problem formulation

The engineering tool for analysis presented in this paper is conceptually related to the root locus method [4, 5, 12, 16]. This classical method is used for the analysis and synthesis of control systems, assuming a system with certain parameters. It is also indirectly used for evaluating key performance specifications of the transient responses by deriving geometric relationships in the complex plane based on the location of the dominant roots. When the parameters of the controlled process change, classical methods assume practice with a family of characteristics or a family of root loci.

In Fig. 1, the configuration of a closed-loop control system is presented, consisting of $C(s)$ – the transfer function of the controller; $G(s)$ – the transfer function of the controlled process; k_e is a free parameter that varies from 0 to infinity. The set of solutions to the characteristic equation (1), for $0 \leq \infty$ gives the values of the roots of the closed-loop system and represents the traditional root locus.

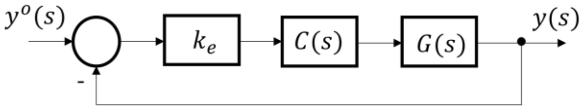

Fig. 1. Block-diagram of feedback control system

(1) $$H(s) = 1 + k_e C(s) G(s) = 0.$$

By limiting the coefficient k_e to a finite interval $k_e \in a$ segment of the root locus is obtained. It should also be noted that this formulation can be applied to any parameter of the control system, belonging to both the controller and the controlled process. The visualization of segments obtained through the solution of (1) for arbitrary k_e is known in references as root contours [12].

Let the transfer function of the controlled process $G(s)$ have the following form (2), and the controller $C(s)$ be represented by the following transfer function (3), Fig. 1,

(2) $$G(s) = \left((s+a)(s+b)\right)^{-1},$$

(3) $$C(s) = (k_e(s+a_i)(s+b_i))s^{-1}.$$

The tuning parameters for the controller $C(s)$ are a_i, $b_{i,}$ and k_e. They are determined based on the desired performance criteria of transient responses (PC) (4), and the nominal values of the parameters of the controlled process G (2), where $a = a^*$ and $b = b^*$, $a_i = a^*$ and $b_i = b^*$,

(4) $$\text{PC: } \left\{\sigma = \text{const},\ t_s^{2\%} = \text{const},\ [t],\ \varepsilon(\infty) = 0\right\}.$$

An interesting case is when the parameters of the controlled process $G(s)$, (2) change within an a priori known range,

(5) $$a \in [a^-,\ a^+] \text{ and } b \in [b^-,\ b^+].$$

In that case, the chosen tuning of the controller parameters will not be able to satisfy the specified performance criteria (4) for all possible combinations of changes in the controlled process parameters (5). The characteristic equation of the closed-loop system in the situation arising from (5), where $a_i \neq a^*$, and $b_i \neq b^*$, will have the form

(6) $$H(s) = s^3 + (a+b)s^2 + abs + s^2 + (a_i + b_i)s + a_i b_i = 0.$$

In the root locus plane, a set of root loci will be observed, providing solutions to (6), obtained as a result of the change in the controlled process parameters (5), while the controller parameters remain constant.

2.2. Generation of root contours

An original analysis of the situation that has arisen, which helps in taking further actions, is the visualization of segments of the root trajectory obtained based on the a priori information about the change in the parameters (5). For this purpose, the characteristic equation (6) is modified into two equations (7) and (8) concerning the variable parameters (5) of the controlled process (2),

(7) $$H(s) = 1 + a\frac{s^2+bs}{s^3+bs^2+s^2+(a_i+b_i)s+a_ib_i} = 0,$$

(8) $$H(s) = 1 + b\frac{s^2+as}{s^3+as^2+s^2+(a_i+b_i)s+a_ib_i} = 0.$$

The variants of the characteristic equations (7) and (8) are used to outline the root contours in the complex plane by specifying the ranges of variation of the variable parameters a and b (5). In this way, this change is modeled and graphically represented in the complex plane.

The root contours can conveniently be denoted as $C_s^{z,p}$, where z indicates that there is uncertainty, p is the parameter for which the contour is visualized, and s is the complex plane.

These contours can take a variety of shapes, sizes, and positions in the complex plane, depending on the specific model of control system and the uncertainty in the parameters of the controlled process.

3. Study of robustness

The positioning of the root contours, obtained by modeling the uncertainty in the control system due to changes in the controlled process parameters (2) and (5) within the complex plane, allows for a new evaluation of the system's robustness from the root locus perspective. This relationship enables a swift assessment of control transient performance response in the time domain.

The primary measure of the robust properties of systems is mathematically defined by the sensitivity function $S(s)$ and the complementary sensitivity function $T(s)$, as shown in Fig. 1:

(9) $$S(s) = (1 + k_e G(s) G(s))^{-1},$$

(10) $$T(s) = k_e G(s) G(s) (1 + k_e G(s) G(s))^{-1}.$$

Robustness, as a property of control systems, is synonymous with insensitivity or reduced sensitivity of the control system to parametric changes in the controlled process. The mathematical expressions that represent this property use the sensitivity function $S(s)$ and the complementary sensitivity function $T(s)$ (9), (10). These functions are defined in the complex domain and are directly represented in the frequency domain ($s = j\omega$), for a given frequency range dependent on the dynamics of the control system, where $\omega_l < \omega < \omega_h$ [8],

(11) $$\mathrm{NP} = \sup_{\omega} |y^0(j\omega) S(j\omega)|, \quad \mathrm{NP} \leq 1, \;\; y^0(j\omega) = (j\omega)^{-1},$$

(12) $$\mathrm{RS} = \sup_{\omega} |w_T(j\omega) T(j\omega)|, \;\; \mathrm{RS} \leq 1, \;\; w_T(j\omega) = G(j\omega) - G^*(j\omega),$$

(13) $$\mathrm{RP} = \sup_{\omega} (|y^0 S(j\omega)| + |w_T T(j\omega)|), \quad \mathrm{RP} \leq 1, \forall \omega \epsilon [\omega_l : \omega_h].$$

Condition (11) is known as the nominal performance condition, while conditions (12) and (13) are respectively known as the conditions for robust stability and robust performance. In expressions (9) and (10), the denominator coincides with the characteristic equation (1). This means that the sensitivity function $S(s)$ and the complementary sensitivity function $T(s)$ have a direct representation in the complex plane through the mobility of the roots obtained from the characteristic equation (1) and, accordingly, through the root contours $C_S^{z,p}$. From (9), it follows:

(14) $$1 + k_e G(s) G(s) = S^{-1}(s).$$

The equation shows that each position of the closed-loop system roots, determined by $k_{e,}$ is directly related to the inverse sensitivity $S^{-1}(s)$ of the control systems. From the robust stability condition (12), it can be written as

(15) $$w_T(s) = T^{-1}(s).$$

This equation shows that the difference in the control system parameters $w_T(s)$ s is directly related to the inverse function of the complementary sensitivity $T^{-1}(s)$. In turn, the complementary sensitivity function $T(s)$ from (10) can be expressed through the equation

(16) $$T(s) = S^{-1}(s) k_e G(s) G(s).$$

From which (15) takes the form of the next equation:

(17) $$w_T(s) = \left(S^{-1}(s)k_eG(s)G(s)\right)^{-1}.$$

The equation shows that the difference in the control system parameters $w_T(s)$ is directly related to the inverse sensitivity function $S^{-1}(s)$. Analyzing expressions (14)-(17) and their expected development in the complex plane leads to the following conclusions that must be considered when making decisions regarding changes in the parameters of controlled process $G(s)$, (5) and controller $C(s)$, tuned for the nominal operating mode of the control system.

- The sensitivity function$S(s)$ and the complementary sensitivity function $T(s)$ depend on the free parameter of the control system k_e.
- At small values of the coefficient k_e, the magnitude of the sensitivity function S(s) increases. This means that the control system is sensitive to changes in the parameters of the controlled process. On the other hand, at large values of the coefficient k_e, the magnitude of the sensitivity function $S(s)$ will decrease, which means that the system will become insensitive to changes in the parameters of the controlled process.
- Since $S(s) + T(s) = 1$, the magnitude of the sensitivity function $S(s)$ and the magnitude of the complementary sensitivity function $T(s)$ can range from 0 to 1. The goal of any control system is to achieve $S(s) \approx 0$ and $T(s) \approx 1$.
- Regarding the complementary sensitivity function $T(s)$, for large values of k_e, its magnitude approaches 1, whereas for small values, it tends to 0.

These conclusions are fully confirmed when verifying the strictness of the conditions in the frequency domain, given by (11)-(13). In the time domain, increasing the coefficient k_e can lead to transient responses with significantly higher time response speed, oscillations, and even loss of stability, especially when it is associated with the proportional gain of the controller. This is independent of the fact that the control system is characterized by low sensitivity.

4. Numerical example

The performance criterion (4) of the system studied from Fig. 1 has a specific form in the presented numerical example:

(18) $$\text{PC: } \left\{\sigma = 0, [\%],\ t_s^{2\%} = 4, [s],\ \varepsilon(\infty) = 0\right\}.$$

The parameters of the controlled process (2) change within the range,

(19) $$a \in [1.5\colon 2.5] \text{ и } b \in [0.5\colon 1.5].$$

In the nominal mode, the values are $a^* = 2,\ b^* = 1$. The controller $C(s)$ is tuned for the nominal mode as follows, by matching the poles of the transfer function of the controlled process $G(s)$ with the zeros of the transfer function of the controller $C(s)$,

(20) $$a_i = a^* = 2,\ b_i = b^* = 1,\ k_e = 1.$$

The characteristic equation, which corresponds to the nominal operating mode, has the form

(21) $$H(s) = 1 + s^{-1} = 0.$$

Fig. 2 shows a set of root locus plots with changes in the parameters of the controlled process (19), controller parameters are $a_i = 2,\ b_i = 1$, and constructed for $0 \leq k_e \leq \infty$. The points mark the values for the nominal tuning of the controller in the control system, with a proportional gain coefficient $k_e = 1$.

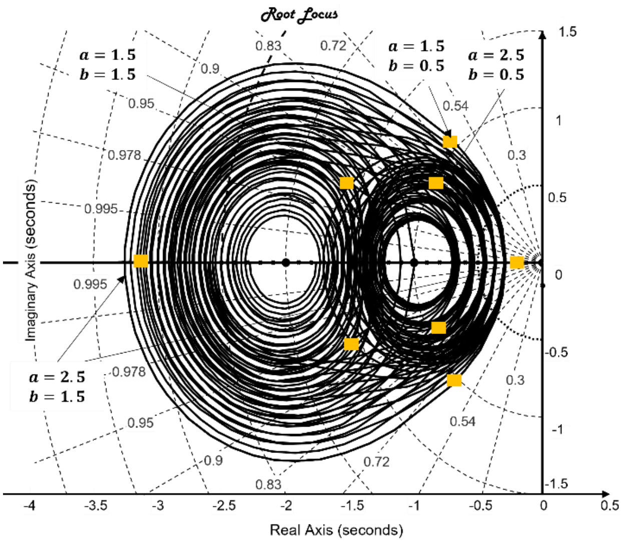

Fig. 2. Root Locus of control systems (uncertainty parameters)

The visualization in Fig. 2 shows that, with changes in the parameters of the controlled process (19), the system with the settings (20) will maintain stability, but the performance in the time domain will not meet the performance criterion (18).

The indirect performance specifications of the transient responses, which are considered in the complex plane, lie within the boundaries $\xi \in [0.72: 1]$ and $\omega_n \in [1: 3.2]$, as shown in Fig. 2, and explain the variations in the transient responses of the closed-loop control system.

Fig. 3 shows the transient responses with changes in the parameters of the controlled process (19), where the nominal operating mode (20) is marked with a bold line, corresponding to the points and roots of the closed-loop system from Fig. 2.

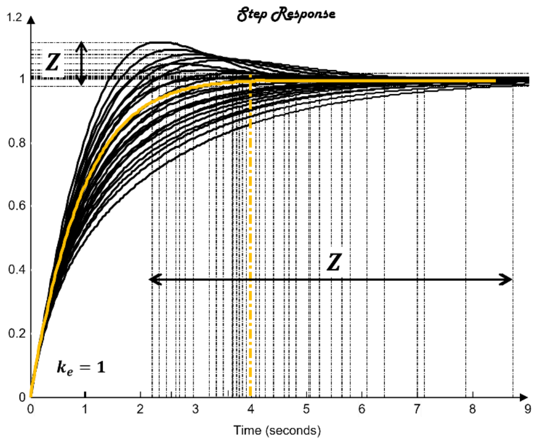

Fig. 3. Transient performance of control systems (uncertainty parameters, $k_e = 1$

The visualization in Fig. 3 demonstrates a significant change in the control performance in the time domain with the variation of the parameters of the controlled process $G(s)$.

The zone of variation in the direct performance specifications, overshoot $\sigma(\%)$, and settling time $t_s^{2\,\%}(\mathrm{s})$, is conditionally marked as Z in Fig. 3.

It is formed by the change in the parameters of the controlled process $G(s)$, (19) and will further be used for a visual quantitative-qualitative assessment of the magnitude of the uncertainty in the parameters of the controlled process in the time domain.

Along the segment of Z there will be a value determined by the change in the settling time $t_s^{2\%}$, while in height, it will have a value determined by the length of the segment defined by the change in the maximum value of the output $y_{\max}$, where:

$$Z:\{|t_s^{2\%}|, |y_{\max}|\},\ |t_s^{2\%}| = \overrightarrow{t^{2\%}_{s_{\max}}} \cap \overrightarrow{t^{2\%}_{s_{\min}}},\ |y_{\max}| = \overrightarrow{y^{\max}_{\max}} \cap \overrightarrow{y^{\min}_{\max}}.$$

For a proportional gain coefficient $k_e = 1$, the zone will have a value of $Z:\{6.35, 0.12\}$, as shown in Fig. 3.

In the complex plane, the zone Z can also be interpreted and manipulated through the root contours (C_s^{z,k_e}), obtained by considering the uncertainty in the parameters of the controlled process (19) and concerning the proportional gain coefficient k_e.

Fig. 4 shows the root locus for the studied system $C_s^{z,1}$, described in Fig. 1 and by (2), (3), and (19), with, $k_e = 1$.

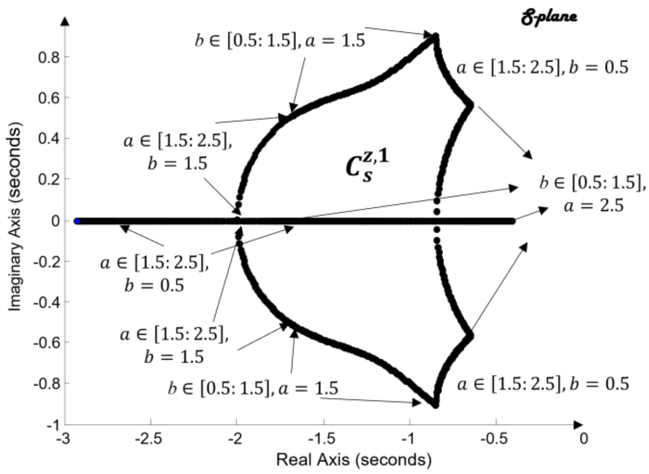

Fig. 4. Root contour of closed loop system, $k_e = 1$

To plot the root contour $C_s^{z,1}$ in the complex plane in Fig. 4, the following equations are required, which represent modifications of the characteristic equation of the closed-loop control system (6):

(22) $$H_{a_{b^-}}(s) = 1 + a\frac{s^2+0.5s}{s^3+1.5s^2+3s+2} = 0,\ a \in [1.5{:}2.5]\ ,\ b = 0.5,$$

(23) $$H_{a_{b^+}}(s) = 1 + b\frac{s^2+1.5s}{s^3+3.5s^2+3s+2} = 0,\ a \in [1.5{:}2.5],\ b = 1.5,$$

(24) $$H_{b_{a^-}}(s) = 1 + a\frac{s^2+1.5s}{s^3+2.5s^2+3s+2} = 0\ ,\ b \in [0.5\colon 1.5],\ a = 1.5,$$

(25) $$H_{b_{a^+}}(s) = 1 + b\frac{s^2+2.5s}{s^3+3.5s^2+3s+2} = 0\ ,\ b \in [0.5\colon 1.5],\ b = 2.5.$$

The zone, defined by the root contour in Fig. 4, represents segments of the root loci obtained from the solutions of equations (22)-(25). It provides information on whether, with the change in (19), the system will remain within the stability zone in the complex plane.

In the example considered, the system retains stability when the parameters of the controlled process G are changed, as described by (19), which indicates that the condition for robust stability (12) will be satisfied in the frequency domain.

The size of this zone, defined by the root contour $C_S^{z,1}$ is an indication of the varying performance of the transient responses, as shown in Fig. 3. The opposite statement is also valid.

The graphical representation of the mathematical conditions for robust stability and robust performance (11)-(13), with a proportional gain coefficient $k_e = 1$ is shown in Fig. 5.

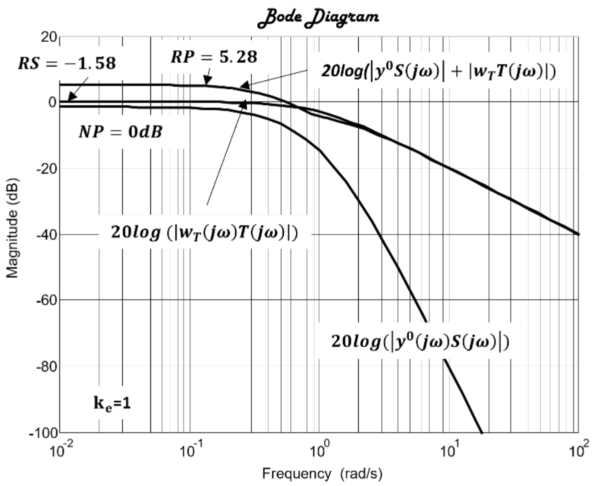

Fig. 5. Bode diagram of nominal performance, robust stability, robust performance, $k_e = 1$

From the graph shown in Fig. 5, it can be seen that the considered control system with the tuned parameters (20) has nominal performance, NP = 0 dB and robust stability, RS = −1.58 dB, for frequency values within the entire range of the study $\omega_l < \omega < \omega_h$. However, it does not have robust performance, RP = 5.28 dB. This fact is indirectly reflected in Fig. 3, with the zone $Z\colon\{6.35, 0.12\}$, and in Fig. 4, with the root contour $C_S^{z,1}$. Equations (14)-(17) are generally valid for all control systems and show that, to reduce the uncertainty (19) and consequently the difference in performance between the time and frequency domains, this is achieved by increasing the value of the proportional gain coefficient k_e.

Furthermore, as the value of k_e increases, a qualitative study of the control system's properties in the complex plane is conducted through the root contours C_S^{z,k_e}

(22)-(25). In the frequency domain, this is done by checking the conditions for robust stability and performance (11)-(13). A comparison of the performance in the time domain is also made, using the information provided by the zone $Z: \{|t_s^{2\%}|, |y_{\max}|\}$.

- Case $k_e = 2$.

The root contour $C_s^{z,2}$, compared with the root contour $C_s^{z,1}$ from Fig. 4, shows a change in shape. The continuation of the contour to the left of the imaginary axis indicates that, in the time domain, the system will exhibit greater time response speed, as shown in Fig. 6.

The check of the robust conditions (11)-(13) and their comparison with those from Fig. 5 shows that robust performance $\mathrm{RP} = 2.5$ dB has not been achieved for $\omega_l < \omega < \omega_h$, but the maximum value of condition (13) decreases. Robust stability, $\mathrm{RS} = -1.58$ dB and nominal performance, $\mathrm{NP} = -6.02$ dB are present, with a noticeable decrease in the maximum value of condition (11), as shown in Fig. 7.

Zone Z has values $Z: \{4.05, 0.11\}$. Compared to Fig. 3, with a higher proportional gain coefficient, the uncertainty zone decreases, as shown in Fig. 8.

- Case $k_e = 3$.

The root contour $C_s^{z,3}$ changes its shape with an increase in k_e. Overall, its area visually decreases, while its extension to the left of the imaginary axis increases. In the time domain, this is expected to result in a faster time response speed and closer performance specifications for the transient responses of the closed-loop control system, as shown in Fig. 9.

In the frequency domain, the following maximum values of conditions (11)-(13) are recorded: $\mathrm{NP} = -9.54$ dB, $\mathrm{RS} = -1.58$ dB, $\mathrm{RP} = 1.34$ dB. The condition for robust performance is violated, and the maximum value of nominal performance continues to decrease, as shown in Fig. 10.

As the coefficient increases, zone Z has values $Z: \{2.51, 0.1\}$. It decreases, corresponding to the smaller values of NP and RP and the reduced area of $C_s^{z,3}$. The transient responses are characterized by a faster time response speed, as shown in Fig. 11.

- Case $k_e = 5$.

The defined root contour $C_s^{z,5}$ is modified as lines, decreasing in size and extending along the real axis in a negative direction. In the time domain, a smaller uncertainty and greater time response speed are expected, while in the frequency domain, it approaches the fulfillment of condition (13), as shown in Fig. 12.

The maximum values of the conditions (11)-(13) recorded in Fig. 13 are: $\mathrm{NP} = -14$ dB, $\mathrm{RS} = -1.58$ dB, $\mathrm{RP} = 0.285$ dB. The condition for robust performance is still not met, but it shows that with an increase in the coefficient k_e, it will be satisfied, while the maximum value of nominal performance continues to decrease, as shown in Fig. 13.

The zone has values $Z: \{1.91, 0.08\}$, which indicates that the uncertainty has decreased, and the time response speed has increased, as shown in Fig. 14.

- Case $k_e = 10$

The significant increase in the proportional gain coefficient, $k_e = 10$ defines the contour $C_s^{z,10}$, in which uncertainty is virtually absent. This is an indication of insensitivity, both in the time and frequency domains, as shown in Fig. 15.

The graphical visualization of the conditions (11)-(13) in Fig. 16 shows that they are satisfied. Increasing the proportional gain coefficient k_e in the control system leads to insensitivity of the output signal to changes in the parameters of the controlled process. Accordingly, the robustness indicators have the following values: NP $= -20$ dB, RS $= -1.58$ dB, RP $= -0.599$ dB, as shown in Fig. 16.

The fulfillment of the robust conditions (11)-(13) leads to an even further reduction of the zone $Z\colon\{1.29, 0.05\}$. The direct performance specifications are very close to the transient responses family shown in Fig. 17, and it can be conditionally concluded that there is insensitivity of the transient responses of the closed-loop control system to changes in the parameters of the controlled process, as shown in Fig. 17.

The following general conclusions can be drawn:

- Figs 6, 9, 12, and 15 show that as the value of the proportional gain coefficient k_e increases, the zone defined by the root contours C_s^{z,k_e} changes shape. Graphically, it becomes smaller, and for $k_e = 10$ it virtually disappears in the complex plane (Fig. 15). This indicates the achievement of insensitivity in the control system to parametric uncertainty in the controlled process.
- Figs 7, 10, 13, and 16 show that the high values of the proportional gain coefficient k_e reduce the maximum value of the modulus RP (13), with the condition for robust performance (13) being fully satisfied at $k_e = 10$, as shown in Fig. 16.

These conclusions indicate that, in the time domain, the control system's performance will have similar direct performance specifications.

- Figs 8, 11, 14, and 17 show that the zone $Z\colon\left\{\left|t_s^{2\%}\right|, |y_{\max}|\right\}$, decreases, which means that increasing the proportional gain coefficient k_e virtually reduces the uncertainty in the control system caused by changes in the parameters of the controlled process. The direct performance specifications have very similar values, which is an indication of control system performance under uncertainty.

Table 1 summarizes all the indicators for clarity in the presented study and the conclusions drawn.

Table 1

k_e	NP (dB)	RS (dB)	RP (dB)	$Z\colon\{\lvert t_s^{2\%}\rvert, \lvert y_{\max}\rvert\}$
1	0	–1.58	5.28	$\{6.35, 0.12\}$
2	–6.02	–1.58	2.5	$\{4.05, 0.11\}$
3	–9.54	–1.58	1.34	$\{2.51, 0.1\}$
5	–14	–1.58	0.285	$\{1.91, 0.08\}$
10	–20	–1.58	–0.599	$\{1.29, 0.05\}$

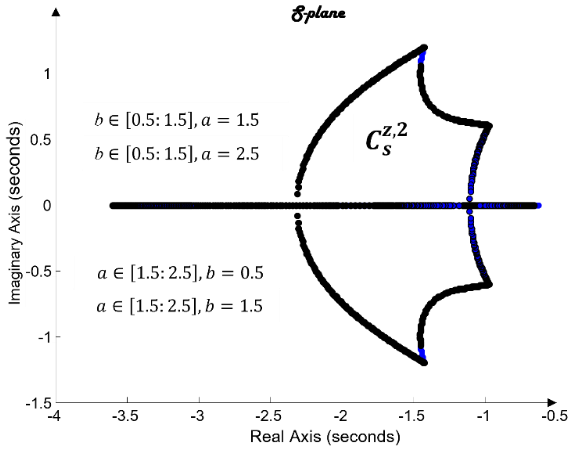

Fig. 6. Root contour for $k_e = 2$

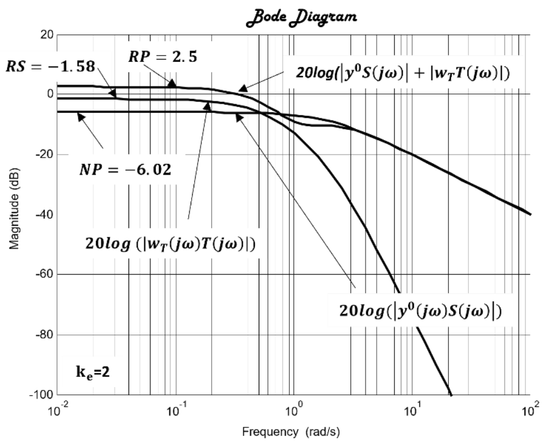

Fig. 7. Bode diagram of nominal performance, robust stability, robust performance, $k_e = 2$

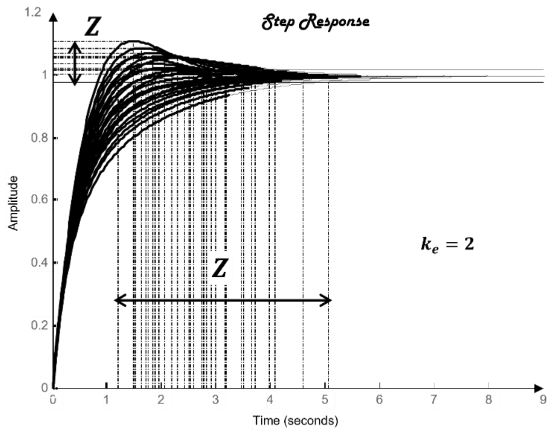

Fig. 8. Transient performance of control systems (uncertainty parameters), $k_e = 2$

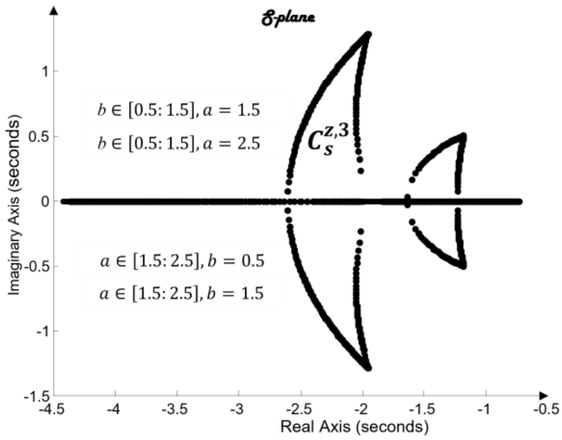

Fig. 9. Root contour for $k_e = 3$

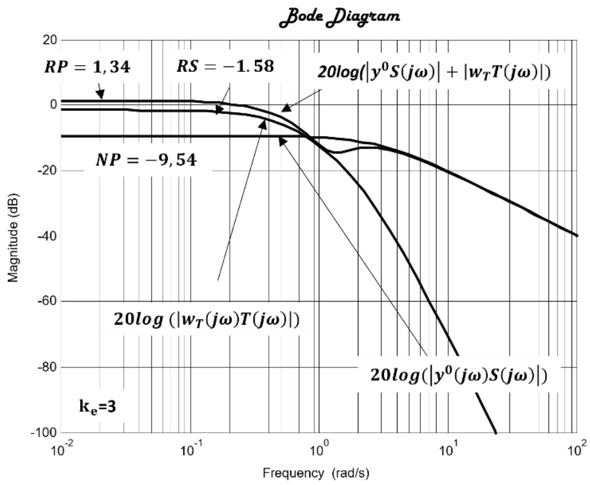

Fig. 10. Bode diagram of nominal performance, robust stability, robust performance, $k_e = 3$

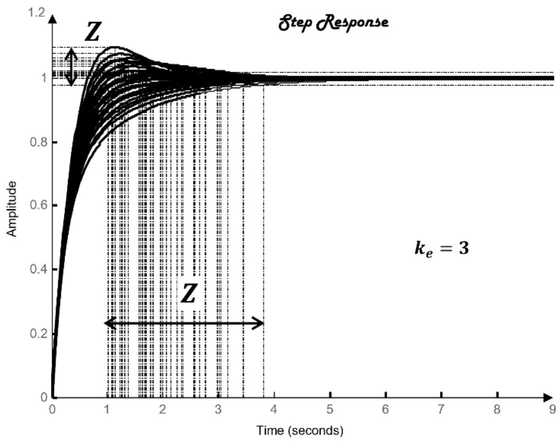

Fig. 11. Transient performance of control systems (uncertainty parameters), $k_e = 3$

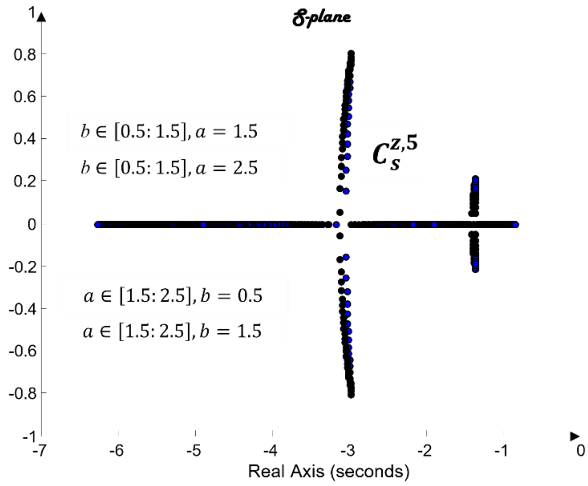

Fig. 12. Root contour $k_e = 5$

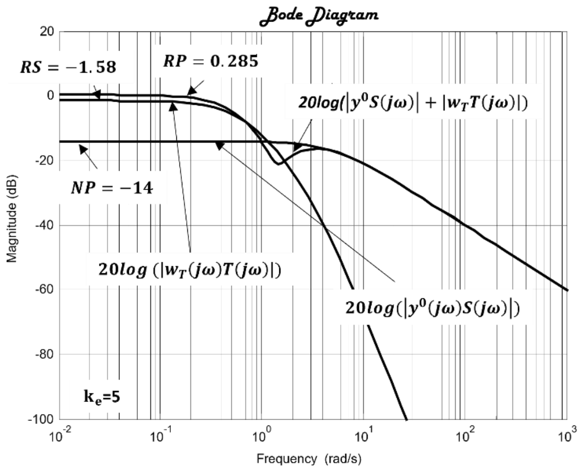

Fig. 13. Bode diagram of nominal performance, robust stability, robust performance, $k_e = 5$

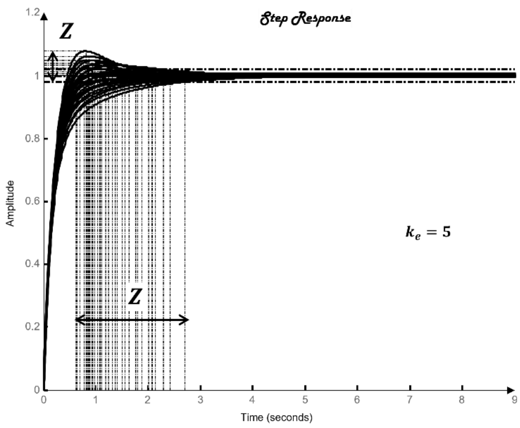

Fig. 14. Transient performance of control systems (uncertainty parameters), $k_e = 5$

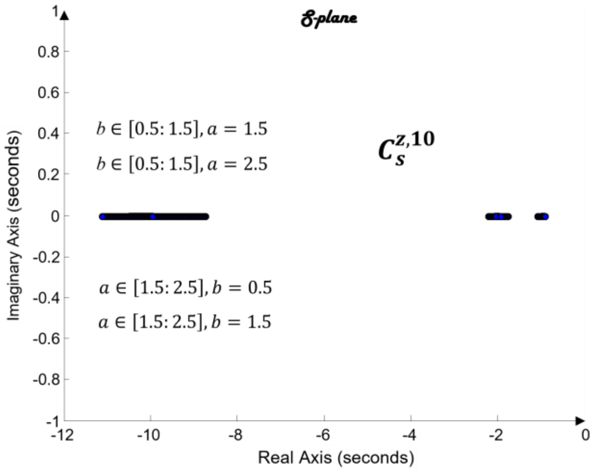

Fig. 15. Root contour, $k_e = 10$

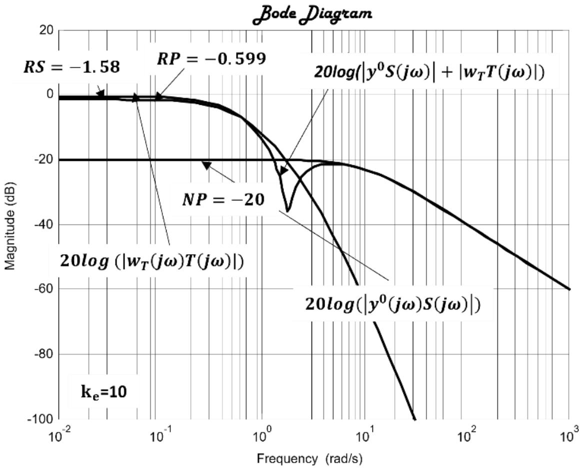

Fig. 16. Bode diagram of nominal performance, robust stability, robust performance, $k_e = 10$

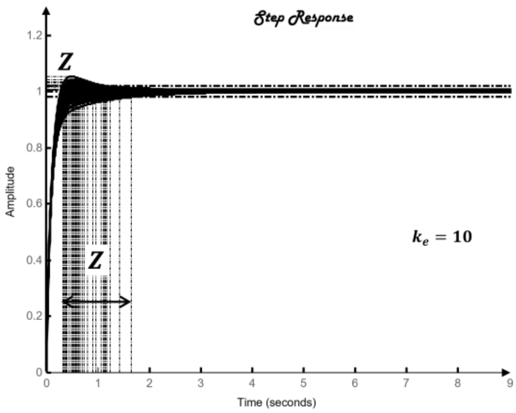

Fig. 17. Transient performance of control systems (uncertainty parameters), $k_e = 10$

5. Conclusion

The research showed that rapid current correction aimed at reducing uncertainty is achieved by increasing the proportional gain. This results in close transient response specifications and robust properties, but it leads to significantly faster dynamics, which is undesirable. Maintaining low sensitivity of the control system and acceptable performance of the transient responses requires a compromise or the introduction of algorithms that use robust, adaptive, or other mechanisms accounting for parametric uncertainty in the controlled process.

This paper provides a new perspective on the relationship between robustness representations in the frequency domain and the subsequent behavior of the system in the time domain, in terms of the overall proportional gain. The generalized conclusions are generally valid for control systems, regardless of their structure, control algorithm, and uncertainty caused by disturbances in the controlled process. A limitation may arise from the order of the modeling transfer function of the closed-loop control system in combination with the size of the uncertainty in the controlled process.

The conducted research provides a starting point for exploring innovative methodological and new combinations of methods that simultaneously reduce the region defined by the root contours while maintaining the desired performance criteria of transient responses, confirmed by the robustness requisites.

***Acknowledgements*:** The author/s would like to thank the Research and Development Sector at the Technical University of Sofia for the financial support.

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Received: 28.03.2025, Accepted: 11.04.2025