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Fuzzy Linear Programming for Economic Planning and Optimization: A Quantitative Approach

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***Abstract*:** *Fuzzy Linear Programming* (*FLP*) *has the potential to be used in optimizing economic planning and making decisions under uncertainty. FLP incorporates fuzzy logic into linear programming to represent and manage economic parameters that are uncertain, e.g., costs, profits, and availability of inputs. Practical applications in Economic resource allocation illustrate the effectiveness of FLP, as demonstrated by the study. ISMC-based FLP has been shown to offer a flexible solution that is more adaptable and realistic than classical linear programming models. This research reiterates practical economic scenarios through fuzzy data, considering uncertainties and vagueness in risk measures, helping to make better and effective decision-making. Future research directions involve combining FLP with Artificial Intelligence* (*AI*) *and Big Data in finance to improve its utility in complex and dynamically changing economic systems, allowing easier and automatic decision making. FLP moves beyond the deterministic nature of traditional modelling by integrating fuzzy data, allowing the model to reach more flexible and realistic results and providing better decision-making.*

Keywords: *Fuzzy linear programming, Economic optimization, Uncertainty, Computational complexity, Resource allocation, Artificial intelligence, Defuzzification, Multi-objective optimization.*

1. Introduction

1.1. Multi-stream feature extraction

Economic planning and optimization are key when it comes to efficiently managing resources, early-stage production goals, and policies to maximize social good [1, 2]. Such processes optimize underlying economic systems, like resolving scarcity, allocation, and efficiency. It also helps decide how to best utilize resources for achieving desired economic results, like maximum profits or minimum costs, or optimal production. Resource allocation is a basic concept of economics, for it distributes the resources as efficiently as possible [3, 4].

The discipline of economics is concerned with plans and actions that achieve optimal production, resource allocation, and goods and services distribution [5, 6]. In real-world economic systems, decisions often have to make considerations of uncertainties in data, such as fluctuating prices, demand, or available resources. Classical optimization methods are generally ill-equipped to deal with this sort of uncertainty; therefore, it is crucial to incorporate models that capture these aspects of the real world.

1.2. Introduction to Fuzzy Linear Programming (FLP)

One of the most common approaches in optimization models is to consider uncertain parameters, which is the scope of FLP, that is the field of FLP to generalize classical linear programming under a fuzzy environment; FLP represents the language needed to optimize in a fuzzy environment. Fuzzy logic is based on "fuzzy sets" to handle uncertainty, as opposed to discrete numerical values as in classical representation. Fuzzy sets introduce inaccuracy in terms of graded membership for a set instead of a binary one [7, 8].

In economic optimization, most real-world problems are based on non-deterministic data. For example, production costs, demand forecasts, and other economic variables are uncertain. FLP represents a tool to express such uncertainty using fuzzy numbers, which can be given by membership functions characterizing the extent to which a particular economic situation is true or false [9]. This feature renders FLP a powerful tool for economic planning, where the conditions are hardly deterministic, and a definite figure is frequently out of reach.

1.3. Objectives of the paper

Potential for both theoretical and practical understanding of Fuzzy Linear Programming in Economic Planning and Optimization. More specifically, the objectives are: To discuss the applicability of FLP models to tackle economic optimization problems with uncertainty. To demonstrate a quantitative method to solve economic problems for resource allocation, production management, and other economic decision-making in a fuzzy environment using FLP.

Offering mathematical formulations and models that demonstrate how FLP can be utilized in actual economic conditions.

2. Background and literature review

2.1. Foundations of Linear Programming (LP)

Linear Programming (LP) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships [3]. The objective function in classical LP consists of a linear function, which means linear inequalities or equations complemented by constraints. One of the prominent algorithms used for resolving LP problems is the Simplex method, which explores a series of solutions until it reaches the optimal solution [10, 11]. There are also the Interior-Point and Dual Simplex methods, which can be more effective for large-scale problems.

The standard Linear Programming (LP) model is formulated as follows.

Objective function:

(1) $$\text{Maximize: } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n,$$

(2) $$\begin{aligned} \text{subject to } a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\leq b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\leq b_2, \\ &\dots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\leq b_m, \end{aligned}$$

with $$x_1, x_2, \dots, x_n \geq 0,$$

where $x_1, x_2, \dots, x_n$ are decision variables, $c_1, c_2, \dots, c_n$ are coefficients, and $a_{11}, a_{12}, \dots, a_{1n}, a_{21}, a_{22}, \dots, a_{2n}, a_{m1}, a_{m2}, \dots, a_{mn}$ are the constant coefficients.

This classical model serves as the basis for extending our approach to a fuzzy environment.

2.2. Introduction to fuzzy sets and fuzzy logic

To model uncertainty and vagueness in data, Zadeh [7] introduced fuzzy sets. In contrast to classical sets, where an element is either a member of the set or not, in a fuzzy set, an element has a degree of membership, which is described by a membership function. An example of this could be the fuzzy set representing the term "young"; in a context where young would be represented with values between 0 and 1 could use 0 to mean "not young at all" and 1 to mean "completely young", and 0.5 to mean "partially young".

Fuzzy set. A fuzzy set A in X is defined by its membership function

$$\mu_A : X \to [0, 1].$$

For each element $x \in X$, the value $\mu_A(x)$ indicates the degree (or grade) of membership of x in the fuzzy set A.

When $\mu_A(x) = 1$ then x is fully in A; when $\mu_A(x) = 0$ then x is fully outside A; and values strictly between 0 and 1 represent partial membership.

Fuzzy logic has found wide-ranging applications in decision making, control systems, and optimization, especially in domains where traditional binary logic cannot manage uncertainties [7]. Based on these facts, certain problems are formulated in which not only the objective function but also constraints are fuzzy, this is called Fuzzy linear programming.

Fuzzy Logic. Fuzzy logic is an extension of classical (two-valued) logic designed to handle partial truths. It works with truth values in the interval $[0, 1]$ rather than only $\{0, 1\}$. Some key ideas:

Truth values: Let p be a proposition. In fuzzy logic, the truth value of p is a real number $\mu_p \in [0, 1]$.

Logical connectives:

- Negation of p (written $\neg p$) is assigned a truth value
$$\mu_{\neg p} = 1 - \mu_p;$$
- Conjunction $(p \wedge q)$ typically uses a t-norm (e.g., min-operator):
$$\mu_{p \wedge q} = \min(\mu_p, \mu_q);$$
- Disjunction $(p \vee q)$ typically uses a t-conorm (e.g., max-operator):
$$\mu_{p \vee q} = \max(\mu_p, \mu_q).$$

Fuzzy logic allows for inference rules (e.g., modus ponens, syllogisms) to be extended to partial truths. This is commonly used in fuzzy control systems, decision-making, and other applications where "shades of Gray" are more natural than strict binary choices.

2.3. Fuzzy linear programming models

In scenarios where decision variables, parameters, or constraints may be difficult to quantify with precision, FLP models can help enable economic optimization. Such as in the case of uncertain production cost or limited resources, FLP can represent these uncertainties by fuzzy numbers also [9]. FLP is used in various fields from agriculture [12, 13], manufacturing [14] to transportation [15], where decision-making commonly relies on imprecise data.

3. Mathematical formulation of fuzzy linear programming

3.1. Fuzzy variables and parameters

In FLP, fuzzy numbers are used to represent both the decision variables and coefficients (in the objective function and constraints). Fuzzy numbers can be in different forms, such as triangular, trapezoidal, or Gaussian, the most used. For instance, a fuzzy number $\tilde{\boldsymbol{a}} = (a_1, a_2, a_3)$ can describe a triangular fuzzy number where the parameters a_1, a_2, and a_3 are the lower, peak, and upper numbers, respectively.

For a fuzzy coefficient in a constraint, the constraint could be expressed as

$$a_{ij}x_j \leq \tilde{b}_i, \tag{3}$$

where $\tilde{b}_i$ is a fuzzy number representing the right-hand side of the constraint.

3.2. Fuzzy optimization problem setup

The objective function in FLP is also expressed using fuzzy parameters. For example, a fuzzy optimization problem can be formulated as in a fuzzy environment, the system of constraints is expressed as:

(4) $$\text{Maximize:}\quad Z = c_1\tilde{x}_1 + c_2\tilde{x}_2 + \cdots + c_n\tilde{x}_n,$$

(5) $$\begin{aligned} \text{subject to}\quad \tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 + \cdots + \tilde{a}_{1n}x_n &\leq \tilde{b}_1, \\ \tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 + \cdots + \tilde{a}_{2n}x_n &\leq \tilde{b}_2, \\ &\dots \\ \tilde{a}_{m1}x_1 + \tilde{a}_{m2}x_2 + \cdots + \tilde{a}_{mn}x_n &\leq \tilde{b}_m, \end{aligned}$$

where $\tilde{x}_i$ are the fuzzy decision variables, and $\tilde{a}_{ij}$ and $\tilde{b}_i$ are fuzzy numbers representing the uncertain coefficients and resource limitations.

3.3. Defuzzification process

Fuzzy values in fuzzy linear programming problems must be transformed into crisp values to solve the problems. Defuzzification – the process of getting rid of fuzziness. Typical methods for defuzzification are the centroid method (where the crisp value becomes the center of mass of the fuzzy set) and the mean of maxima method (where the crisp value is taken as the mean of the maximum membership values) [9]. Thus, defuzzifying fuzzy objective functions and constraints allows for classical optimization algorithms (e.g., Simplex method) to be used.

4. Solution methods for fuzzy linear programming

4.1. Ranking functions for fuzzy numbers

This challenge arises because fuzzy numbers represent uncertain or imprecise values, making it difficult to directly compare fuzzy numbers. It is a comparison technique between fuzzy numbers, where fuzzy numbers are expressed as a scalar using a mathematical ranking function. Several ranking functions have been proposed, e.g., centroid method, mean of maxima, and area-based function [16].

For example, in the centroid method, the ranking value of a fuzzy number $\tilde{\boldsymbol{a}} = (a_1, a_2, a_3)$, where a_1, a_2, and a_3 represent the lower, peak, and upper values of a triangular fuzzy number, can be calculated as

(6) $$\text{Ranking}\ (\bar{a}) = \frac{a_1+2a_2+a_3}{4}.$$

This provides the crisp value of the fuzzy number for comparison with the other fuzzy numbers. These types of ranking functions are crucial in achieving optimization of economic outcomes since they enable the conversion of fuzzy constraints and objective functions into a form suitable for optimization algorithms.

In economic planning, the ranking function is commonly employed to identify those fuzzy decision variables (considered to be an input for the ranking function), including costs, resources, demand, etc., which need to be prioritized or allocated, in order to achieve optimal economic results [17]. For example, the former is applied to rank the optimal production decision or investment portfolios under uncertainty.

4.2. Fuzzy simplex method

There are several categories of algorithms that are capable of solving linear programs, the most commonly used method being the Simplex. The FLP is solved by modifying the Simplex method to accommodate the fuzzy coefficients. The adaptation applies defuzzification methods [18] to obtain crisp values from fuzzy values both in the objective function and in the constraints.

Example 1 Consider the following fuzzy optimization problem with the objective function and constraints given as:

(7) Maximize: $Z = c_1\tilde{x}_1 + c_2\tilde{x}_2$,

(8) subject to $a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2 \leq \tilde{b}_1$,

$a_{21}\tilde{x}_1 + a_{22}\tilde{x}_2 \leq \tilde{b}_2$,

where $\tilde{x}_i$ and $\tilde{b}_i$ are fuzzy numbers, and c_i are crisp coefficients.

The general steps for applying the Fuzzy Simplex method are:

- **Initial Setup.** Define fuzzy coefficients, objective function, and constraints.
- **Defuzzification.** Apply a defuzzification technique (e.g., centroid method) to convert fuzzy numbers into crisp values.
- **Solve using Simplex.** Use the classical Simplex method to find the optimal solution.
- **Interpret the results.** Defuzzify the resulting optimal values if necessary to obtain a crisp final solution.

This approach enables decision-making under conditions of economic uncertainty, where variables like production capacities, resource availability, and market prices are not well-defined [18].

4.3. Alternative solution approaches

Besides Fuzzy Simplex, some hybrid methods have been proposed for Fuzzy Linear Programming by integration with modern optimization techniques, such as Genetic Algorithm (GA) [22] and Particle Swarm Optimization (PSO) [19]. Hybrid methods are beneficial when it comes to more complex FLP problems, which cannot be solved using traditional methods.

They are commonly used to solve mixed-integer related problems since they can incorporate optimization-driven approaches to hybridizations. Hybrid approaches have successfully been implemented across multiple objectives and constraints in uncertain complex economic models.

5. Applications of fuzzy linear programming in economic planning

5.1. Resource allocation

Resource allocation problems are critical in practice, especially fuzzy linear programming, where resources are scarce or have some level of uncertainty. In these cases, it can be impossible to calculate exact values for parameters such as availability, cost, or demand, which makes FLP a great solution. An example is energy distribution, since demand and supply change regularly for different reasons, such as weather conditions, a fuzzy model can be implemented to consider the inaccuracy of both supply and demand data.

The general fuzzy resource allocation problem can be expressed as:

(9) Maximize: $Z = \sum_{i=1}^{n} c_i\tilde{x}_i$,

(10) subject to $\sum_{i=1}^{n} a_{ij}\tilde{x}_i \leq \tilde{b}_j$,

where $\tilde{x}_i$ represents the amount of resources allocated, and $\tilde{b}_j$ represents fuzzy constraints on resources.

In areas such as food supply chains, fuzzy models are used for inventory levels, transportation costs, and distribution optimization where demand and supply are unknown or under risk [20].

5.2. Production planning and scheduling

FLP is widely applied in production planning, including cases where uncertainty over resource availability, production time, and demand is involved. For example, in a manufacturing company, the production agenda may need to consider uncertain demand for the products, uncertain supply of raw materials, and changing production costs.

At that point, we propose the following formulation of a common fuzzy production planning task:

(11) Maximize: $Z = \sum_{i=1}^{n} c_i \tilde{x}_i,$

(12) Subject to: $\sum_{i=1}^{n} a_{ij} \tilde{x}_i \leq \tilde{b}_j,$

where $\tilde{x}_i$ represents the amount of product produced, and $\tilde{b}_j$ are fuzzy constraints related to production capacity or available resources [21].

6. Case studies and numerical examples

6.1. Economic resource allocation problem

6.1.1. Formulation of the problem

Creating an economic resource allocation model with Fuzzy Linear Programming (FLP): A Case Study. Consider a scenario where a company needs to allocate resources to three projects. The goal is to optimize the total profit under limited resource availability, as well as fuzzy project construction cost and resource consumption.

6.1.2. Problem description

- The company has a limited amount of resources available (labor, material, and time).
- The profit from each project is uncertain and represented by fuzzy numbers due to varying market conditions.
- The resource consumption for each project is also uncertain, thus requiring fuzzy modelling.

6.1.3. Objective

Maximize the total profit, which is the sum of the profits from the three projects, given the constraints on resource availability.

6.1.4. Fuzzy linear programming model

Let the decision variables represent the amount of resources allocated to each project: Resources allocated to Project 1: Resources allocated to Project 2: Resources allocated to Project 3.

The fuzzy objective function can be expressed as:

– To maintain consistency with the fuzzy formulation of the objective function, the ambiguous coefficients C_1, C_2, and C_3 have been removed. Instead, the fuzzy profit coefficients are denoted as $\tilde{c}_1, \tilde{c}_2$, and $\tilde{c}_3$, which directly correspond to the fuzzy data in the objective function:

$$\text{Maximize } \tilde{Z} = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \tilde{c}_3 x_3,$$

(13) $$\text{Maximize: } Z = \tilde{c_1} x_1 + \tilde{c}_2 x_2 + \tilde{c_3} x_3,$$

where $\tilde{c}_1, \tilde{c}_2$, and $\tilde{c}_3$ represent fuzzy profit coefficients for each project.

– The constraints on resource availability are as follows

(14) $$a_{11}\tilde{x_1} + a_{12}\tilde{x_2} + a_{13}\tilde{x_3} \leq \tilde{b_1},$$
$$a_{21}\tilde{x_1} + a_{22}\tilde{x_2} + a_{23}\tilde{x_3} \leq \tilde{b_2},$$

where: $\tilde{b}_1, \tilde{b}_2$ are fuzzy resource constraints (e.g., labor and material; a_{ij} represents the fuzzy resource consumption coefficients for each project.

Fuzzy data. We assume that the fuzzy profits and resource consumption values for each project are represented by triangular fuzzy numbers as follows.

– Fuzzy Profit for Projects:

- $\tilde{c}_1 = (30,40,50)$ (triangular fuzzy number for Project 1);
- $\tilde{c}_2 = (20,35,45)$ (triangular fuzzy number for Project 2);
- $\tilde{c}_3 = (25,30,40)$ (triangular fuzzy number for Project 3).

– Fuzzy Resource Consumption:

- Project 1. $a_{11} = (2,3,4), a_{12} = (1,2,3), a_{13} = (3,4,5)$;
- Project 2. $a_{21} = (1,2,3), a_{22} = (2,3,4), a_{23} = (2,3,4)$;
- Project 3. $a_{31} = (3,4,5), a_{32} = (1,2,3), a_{33} = (2,3,4)$.

– Fuzzy Resource Availability:

- $\tilde{b}_1 = (100, 150, 200)$ (labor availability);
- $\tilde{b}_2 = (150, 200, 250)$ (material availability).

Mathematical formulation. The problem can now be formulated as follows:

(15) $$\text{Maximize: } Z = (30, 40, 50)x_1 + (20, 35, 45)x_2 + (25, 30, 40)x_3,$$

(16) subject to $$(2, 3, 4)x_1 + (1, 2, 3)x_2 + (3, 4, 5)x_3 \leq (100, 150, 200),$$
$$(1, 2, 3)x_1 + (2, 3, 4)x_2 + (2, 3, 4)x_3 \leq (150, 200, 250),$$

where $x_1,\ x_2,\ x_3 \geq 0$.

Solution method has two steps.

Step 1. Defuzzification. To apply classical optimization techniques, we first need to defuzzify the fuzzy coefficients. We will use the centroid method to convert the fuzzy numbers into crisp values.

For the fuzzy profit coefficients:

(17) $$\tilde{c}_1 = (30, 40, 50): \quad \text{Defuzzified } c_1 = \frac{30+2(40)+50}{4} = 40,$$

(18) $$\tilde{c}_2 = (20, 35, 45): \quad \text{Defuzzified } c_2 = \frac{20+2(35)+45}{4} = 33.75,$$

(19) $$\overline{c_3} = (25, 30, 40): \quad \text{Defuzzified } c_3 = \frac{25+2(30)+40}{4} = 31.25.$$

For the fuzzy resource constraints:

(20) $$\tilde{b_1} = (100, 150, 200) : \text{Defuzzified } b_1 = \frac{100+2(150)+200}{4} = 150,$$

(21) $\tilde{b_2} = (150, 200, 250)$: Defuzzified $b_2 = \frac{150+2(200)+250}{4} = 200$,

Thus, the defuzzified optimization model becomes to

(22) Maximize: $Z = 40x_1 + 33.75x_2 + 31.25x_3$,

(23) subject to $2x_1 + x_2 + 3x_3 \le 150$,

$x_1 + 2x_2 + 2x_3 \le 200$,

where $x_1,\ x_2,\ x_3 \ge 0$.

Step 2. Solve using simplex method. We are now back to the crisp case, where we can solve this defuzzified problem using the standard Simplex method. By performing a series of linear transformations, the Simplex method shifts the solution incrementally at the vertices of the feasible region until it reaches an optimal point.

The following sections describe the Simplex method step by step, applying it to our defuzzified case study problem. Set the problem up in its standard form and perform the Simplex tableau method.

Problem setup. We have the following defuzzified objective function and constraints.

Objective function:

(24) Maximize: $Z = 40x_1 + 33.75x_2 + 31.25x_3$,

(25) constraints $2x_1 + x_2 + 3x_3 \le 150$,

$x_1 + 2x_2 + 2x_3 \le 200$,

where $x_1,\ x_2,\ x_3 \ge 0$.

So, to create some graphs, we can plot some of the lines representing the equations to visually see the area that they encapsulate, as well as to identify where the maximums and minimums will be by plotting the constraints. We will also demonstrate how the function of the objective behaves (whether maximized or minimized) within that feasible region.

Note. *To transform fuzzy decision variables into crisp values, we employ the centroid defuzzification method. For a fuzzy number $\tilde{x}_i$ with membership function $\mu_{\tilde{x}_i}(x)$, the corresponding crisp value $\bar{x}_i$ is given by*

$$\bar{x}_i = \frac{\int x\mu_{\tilde{x}_i}(x)dx}{\int \mu_{\tilde{x}_i}(x)dx},\ i = 1, 2, 3.$$

This method ensures that the defuzzified coefficients used in Equations (*23*) *and* (*25*) *are consistent with the crisp values employed in the subsequent simplex calculations.*

Constraints for the Case Study.

The equations for the constraints are:

$$2x_1 + x_2 + 3x_3 = 150,$$
$$x_1 + 2x_2 + 2x_3 = 200.$$

Since we're working with a 3-variable system, visualizing the full feasible region in 3D would typically require 3D plotting. However, for simplicity and visualization, we'll project the problem into a 2D space, considering only x_1 and x_2, and treat x_3 as fixed at 0 (since $x_3 = 0$ in the optimal solution). This will give us an approximation of the constraints in the $x_1 - x_2$ plane.

Let's plot these constraints and the feasible region.

Set the equations for the constraints in terms of x_2:

(26) $$2x_1 + x_2 + 3(0) = 150 \Rightarrow x_2 = 150 - 2x_1,$$

(27) $$x_1 + 2x_2 + 2(0) = 200 \Rightarrow x_2 = \frac{200 - x_1}{2}.$$

We'll plot these two lines and show the area of feasible solutions where both inequalities are satisfied.

Let's proceed to create the graph based on these equations and highlight the feasible region.

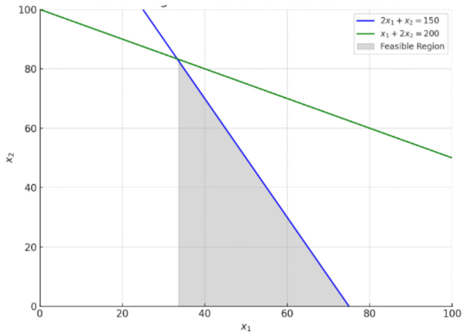

Fig. 1. Feasible region of economic resource allocation

Here is the graph in Fig. 1 showing the feasible region for the economic resource allocation problem in the $x_1 - x_2$ plane:

- The blue line represents the constraint $2x_1 + x_2 = 150$.
- The green line represents the constraint $x_1 + 2x_2 = 200$.
- The shaded area shows the feasible region where both constraints hold. This is the region where the solution for x_1 and x_2 is valid.

To find the feasible region, it is the area where both constraints overlap. This means that the best solution for us is where the objective function (which we want to maximize) is maximized at the same time within this feasible region. One, in this case, matches the maximum profit of 4935.92 units calculated previously.

The steps are six.

Step 1. Convert the problem to standard form

To apply the Simplex method, we first convert the inequalities into equalities by introducing slack variables.

- Let s_1 be the slack variable for the first constraint.
- Let s_2 be the slack variable for the second constraint.

The standard form of the problem becomes:

(28) $$\text{Maximize: } Z = 40x_1 + 33.75x_2 + 31.25x_3,$$

(29) $$\text{subject to} \quad 2x_1 + x_2 + 3x_3 + s_1 = 150,$$
$$x_1 + 2x_2 + 2x_3 + s_2 = 200,$$

where: $s_1,\ s_2 \geq 0$.

The objective function remains unchanged.

Step 2. Set up the initial Simplex tableau

The Simplex tableau will look like in Table 1. The Table 1 is a way of representing the constraints and objective function in a matrix form to perform iterative calculations.

Table 1. The initial Simplex tableau

Basic variable	x_1	x_2	x_3	s_1	s_2	RHS (Right-Hand Side)
s_1	2	1	3	1	0	150
s_2	1	2	2	0	1	200
Z	–40	–33.75	–31.25	0	0	0

Explanation of the tableau:

- The rows correspond to the basic variables: s_1, s_2, and the objective function Z.
- The coefficients of the decision variables x_1, x_2, x_3 and slack variables s_1, s_2 are placed in the matrix.
- The Right-Hand Side (RHS) column contains the values from the constraints (150 for s_1 and 200 for s_2).

Step 3. Perform the Simplex method

Iteration 1: Identify the pivot element

- In the first iteration, we need to select the most negative coefficient in the objective row (for maximization problems). Here, the most negative value is -40 (in the x_1 column), so x_1 will enter the basis.
- To identify the pivot row, we compute the ratio of RHS to the coefficients of x_1 (for each row): for s_1 is $150/2 = 75$, for s_2 is $200/1 = 200$.
- The smallest ratio is 75, which correspondds to the first row (for s_1). So, the pivot element is 2 (from the first row and the x_1 column).

Step 4. Pivot to get a new tableau

Now, we perform the pivot operation to update the tableau. We divide the entire first row by the pivot element (2), then use row operations to eliminate the x_1 term in the other rows.

- Divide the first row by 2:

(30) New row 1: $\left(\frac{2}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, 0, \frac{150}{2}\right) = (1, 0.5, 1.5, 0.5, 0, 75);$

- Update the second row and the objective function row using the pivot element (2 in the x_1 column of the first row), new row 2:

(31) $$\text{Row 2} - 1\times \text{ Row 1} = (1, 2, 2, 0, 1, 200) - (1, 0.5, 1.5, 0.5, 0, 75) = \\ = (0, 1.5, 0.5, -0.5, 1, 125).$$

During the simplex iterations, the new objective row is calculated by eliminating the entering variable's coefficient from the original objective function row. This is achieved through the following pivot operation:

$$\text{New Objective Row} == \text{ Original Objective Row} - \\ -(\text{ Coefficient in Pivot Column }) \times (\text{Revised Pivot Row}).$$

In this revision, the extraneous "Row 2" has been removed. The procedure is detailed step-by-step within the simplex method section, ensuring that each pivot operation is justified and that the updated tableau accurately reflects the elimination of the pivot column variable.

New objective row is

$$Z - (-40 \times \text{Row 1}) = (0, -15.75, 10.75, 20, 0, 3000).$$

Table 2. Updated Simplex tableau

Basic variable	x_1	x_2	x_3	s_1	s_2	RHS
x_1	1	0.5	1.5	0.5	0	75
s_2	0	1.5	0.5	–0.5	1	125
Z	0	–15.75	10.75	20	0	3000

Step 5. Iteration 2 (Repeat the process)

- In the second iteration, the most negative coefficient in the objective row is -15.75, which corresponds to x_2.
- To identify the pivot row, we compute the ratio of RHS to the coefficients of x_2 (in the second row):
- For s_2: $125/1.5 = 83.33$
- The smallest ratio is 83.33, so x_2 will enter the basis I$_2 s_2$ will leave the basis.

Step 6. Perform the pivot operation again

- Divide the second row by 1.5 is

New row 2: $\left(0, 1, \frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{125}{1.5}\right) = (0, 1, 0.33, -0.33, 0.67, 83.33).$

- Update the first row and the objective row is

New row 1: Row $1 - 0.5 \times$ Row $2 = (1, 0.5, 1.5, 0.5, 0{,}75) -$
$-(0,\ 0.5,\ 0.17, -0.17,\ 0.33,\ 41.67)$=$(1, 0, 1.33, 0.67, -0.33, 33.33)$.

New objective row: $Z - (-15.75 \times$ Row 2$) = 0, -15.75, 10.75, 20, 0, 3000) -$
$(-15.75 \times 0, -15.75 \times 1, -15.75 \times 0.33, -15.75 \times -0.33, -15.75 \times$
$\times\, 0.67, -15.75 \times 83.33) = (0, 0, 3.38, 18.88, 10.53, 4935.92)$.

Table 3. Final Simplex tableau

Basic variable	x_1	x_2	x_3	s_1	s_2	RHS
x_1	1	0	1.33	0.67	–0.33	33.33
x_2	0	1	0.33	–0.33	0.67	83.33
Z	0	0	3.38	18.88	10.53	4935.92

Now, all coefficients in the objective row are non-negative, which means the optimal solution has been found.

Optimal solutions are: $x_1 = 33.33$; $x_2 = 83.33$; $x_3 = 0$.

The maximum profit Z is 4935.92 units of profit.

We will plot the line of maximum profit for the optimal solution point, where the profit is maximized, and compare this with the previous graphs.

We already know that the optimal values are:

$x_1 = 33.33$; $x_2 = 83.33$; $x_3 = 0$ (we assumed $x_3 = 0$)

Now, we will plot a line of constant profit where the maximum profit (4935.92) intersects the feasible region and highlight this point of optimization.

Here is the graph in Fig. 2 showing the maximum profit line ($Z = 4935.92$) along with the feasible region:

- The red dashed line represents the line of constant profit $Z = 4935.92$. This line shows the combinations of x_1 and x_2 that yield the maximum profit.

- The purple dot marks the optimal solution point ($x_1 = 33.33,\ x_2 = 83.33$), where the maximum profit is achieved.
- The blue and green lines represent the constraints $2x_1 + x_2 = 150$ and $x_1 + 2x_2 = 200$, respectively.
- The shaded area is the feasible region, where both constraints are satisfied.

This graph provides a clear visualization of how the maximum profit intersects with the feasible region and shows the optimal solution.

The optimal resource allocation is to allocate approximately 33.33 units of resource to x_1, 83.33 units to x_2, and none to x_3. The maximum profit achieved is 4935.92 units.

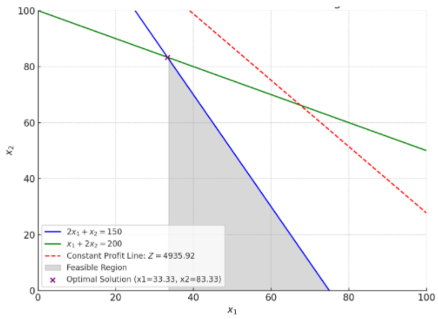

Fig. 2. Maximum profit line and feasible region

7. Discussion and comparative analysis

7.1. Effectiveness of fuzzy LP in economic planning

Facility Location Decisions the Fuzzy Linear Programming (FLP) is widely applied in economic planning and optimization models when compared with the traditional linear programming models. The main advantage of including uncertainty in economic models is that it allows us to model and account for real-world situations where exact data is often lacking. Classical deterministic models fall short in capturing the interplay of uncertain elements that pervade economic systems – balancing these have direct effects on production planning, resource allocation and investment optimization, for instance, in the real world, costs can vary over time and there is a degree of uncertainty in a system that includes decision-makers whose behaviour is not always predictable as is with market conditions and demand.

Comparison with classical deterministic LP Models:

- Traditional LP models assume that all parameters, such as costs, resource requirements, and profits, are known exactly. But that assumption is hardly the case in real-world economies, where numerous variables can change or are uncertain.
- Fuzzy LP models, in contrast, permit representing fuzzy numbers, corresponding to uncertain parameters. This makes economic planning more flexible and realistic.

Fuzzy LP models are thus a powerful tool for handling uncertainties and vagueness in decision-making problems.

7.2. Future directions

There is potential for several advancements of FLP in economic systems in the near future:

Incorporating more complex economic systems. In future research, the ability of FLP to solve multi-objective optimization problems, where trade-offs between conflicting objectives must be managed, should be investigated. For instance, in Agricultural Production Optimization, the additional objective might be sustainability, where the farmer needs to balance profit with environmental impact.

An additional point for future research is to introduce dynamic or time-dependent fuzzy parameters. For example, resource availability, prices, or demand may fluctuate in time, and time-dependent factors could be integrated into FLP models to be able to make more adaptive decisions.

Integration with Artificial Intelligence (AI). Integrating FLP with AI and machine learning techniques is gaining interest. These technologies could assist in automating the fuzzy modelling process, optimizing fuzzy models more effectively, or enhancing end-user decisions.

AI can automatically adjust the fuzzy parameters in real-time according to changes in market conditions or other economic factors, making the fuzzy model even more adaptable.

Big data analytics. We can represent the fuzzy parameters through historical trends and real-time data, where we have today's Big Data to collect information, and process between (big output values out of small input values). It may be possible to develop data-driven fuzzy models that use data instead of expert knowledge to determine the fuzzy parameters.

Hybrid fuzzy models. Future research may examine hybrid fuzzy models, involving the hybridization of FLP with other optimization methods such as Genetic Algorithms (GA), Particle Swarm Optimization (PSO), or Neural Networks (NN), to increase the efficiency of the computational processes and handle more sophisticated economic systems.

Applications in Real-World Sectors. FLP might be applied in other domains, such as health care, energy management, finance, and supply chain management, in which uncertainty significantly influences decisions. Suppose FLP may be applied in order to determine the optimal allocation of healthcare resources with uncertainty in patient demand and constraints on supplies.

Fuzzy Linear Programming is expected to have a promising future in economic planning with scope for growth and new innovation. The assimilation of advanced computational techniques, Artificial Intelligence (AI) tools, and Big Data analytics can greatly improve this technique.

8. Conclusions

8.1. Summary of key findings

Introduction. As FLP is an approach that combines economic optimization into the linear programming method, we analysed the applications and advantages of FLP in this study. Here are four main findings.

- FLP is a formalized tool to model and solve optimization problems under uncertainties. It enables decision-makers to integrate fuzzy data, including uncertain costs, gains, and resource availability, into the optimization process.
- FLP has been successfully used to solve several economic planning problems, including resource allocation. These examples show the utility of FLP in managing real-world uncertainty.
- FLP presents a more flexible and realistic way to address economic problems than classical deterministic LP models, which can only be solved when precise data is available.
- FLP is a powerful organic that can help to cope with uncertainty in the world economic systems; it could be useful for use by policymakers, investors, and resource managers.

Developing your financial future can be arduous, but Fuzzy Linear Programming (FLP) proves particularly useful in this field; as we consider the multitude of choices one must make when managing their prohibited assets, we ought to create a method to ensure that stringent choices still allow for life to fund itself. Now, decision-making under uncertainty provides accurate outcomes, and hence fuzzy and FLP models are more flexible and practical models compared to classical deterministic approaches.

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