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Maximal Generalized Network Flow Accounting for Motivation

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Abstract**:** *In the present work, the maximal generalized network flow, often referred to as the profit-loss flow, is examined, considering the motivation in the decision-making systems built on this flow. The general description of the features of motivation as a psychological process actively involved in decision-making systems through the generalized network flow is given. A method is proposed to embed motivation in a generalized network flow through the motivation coefficients on different network sections* (*arcs*)*. It is shown that the proposed method offers more possibilities than the classical network flow. It is proven that the initial resource in the source does not match the final resource in the consumer. This has been suggested to be due to the influence of motivation. The theoretical and experimental results convincingly prove the possibilities of considering motivation in decision-making systems when managing the transportation of resources in an arbitrary transport network.*

Keywords**:** *Intelligent systems, Generalized network flow, Decision-making systems, Motivation.*

1. Introduction

The topic of motivation is taking a more and more important place in the everchanging modern economic world. There is a search for and application of methods, techniques, and means for securing qualified, highly productive, and loyal personnel, capable of achieving the organization's goals in medium-term and long-term perspective. The accent is placed upon the need for people to be stimulated or motivated, which in turn determines specific directions in human behavior.

Psychology is one of the fastest-growing research fields of human behavior. In this behavior, an essential role is played by motivation, which is an important factor for human survival and progress. The rapid development of various fields of science and technology leads to the integration of one scientific field into another and to their mutual enrichment with modern research methods and means [9, 10].

In recent years, there has been a trend towards intensive use of mathematical methods, models, and software tools to describe various processes in psychology, including motivation. Decision-making systems play a very important role in this activity. In paper [13] analyzes the different definitions of a negator of a probability mass function and a basic belief assignment.

Based on the analysis, it is constated that discrete systems for decision-making based on network flows allow a precise and adequate accounting of the behavior of motivation in these systems. The object of this research is the influence of the subjective factor in decision-making systems and in particular that of motivation in human-machine decision-making systems.

The present work examines the problems of describing motivation from the point of view of decision-making capabilities. To solve this problem, a discrete mathematical model based on a certain class of network flows is proposed. It is shown that the min-cut-maxflow theorem holds in the maximal generalized network flow. It is proven that the number of minimal cuts in the generalized network flow is equal to or greater than in the analogous classical network flow. It is shown that the values of the different minimal cuts of the generalized maximal network flow can be different. The obtained theoretical results are confirmed by a numerical example for the determination of the maximal generalized network flow with motivation.

2. Motivation in decision-making systems

Motivation is an extremely complex psychological process that is involved not only in making operational and strategic decisions but also in a person's daily life [11, 12].

There are many different types of motivation. The most general is the classification of intrinsic and extrinsic motivation.

As a result of the research activity, various theories have been developed and applied, each of which seeks to explain this complex psychological process, which in certain situations can have a decisive influence on making a variety of decisions.

The advent of computer technology has led to the creation of information processing systems through which effective decisions can be made or to support the making of such decisions.

In this class, decision-making systems require careful selection of appropriate mathematical structures that best suit their operation.

In production, military, and transportation systems, the main features of these systems are embedded, as well as the methods to calculate the optimal or pseudo-optimal solutions. Motivation or other psychological processes should also be appropriately incorporated into these frameworks.

In most complex systems like these to manage large production complexes and traffic on complex transport networks, the man – dispatcher or manager – has a decisive influence. When he is well motivated, his human-machine decisions are better reasoned and effective. Insufficiently well-reasoned decisions lead to significant losses, especially in large controlled systems. All this leads to the need for preliminary formalization of motivation and to the possibilities of its being built into the basic model of the management system. It is necessary to transfer bridges between psychological processes – such as motivation and the deterministic models of control that are currently widespread.

Relatively good opportunities to embed motivation in mathematical management models offer the processes for managing the transportation of resources on production transport networks [1, 2].

Using network flows to build the control system proves to be an effective tool in many cases. It is relatively easy to incorporate the motivation processes, which can have different values for different sections of the networks. Generalized network flows, often called gain and loss flows, provide the best opportunities for this. There, the influence of motivation on decisions can be assessed both quantitatively and qualitatively. Decisions are made and implemented upon reaching the end of the section along which the resource is being transported and determining how much of this resource and which of the following sections should be taken [3].

Such consistent decisions on the transportation of the resource in the network are necessary, which allow reaching the maximal possible transportation of the product, taking into account the motivation and the costs of the respective sections. In the language of network flows, this means to determine the maximal generalized network flow with consideration of motivation.

3. Features of the maximal generalized network flow with consideration of motivation

For the formal description of the maximal generalized flow with consideration of the motivation, the following notations are needed [1-3]:

$\{ X, U, I, J \}$ – symbols of sets;

$\{ x_i, x_{ij}, (i, j), c_{ij}, a_{ij}, ... \}$ – symbols of set elements;

$| X |$ – power of the set *X*;

$\varnothing$ – empty set symbol;

I – set of the indices of the elements x_i of *X*;

$$I = \{ i_1, i_2, i_3, ..., i_k \}; i_j \in I;$$

X – set of all vertices of the graph $G(X, U)$:

$$X = \{ x_1, x_2, x_3, ..., x_n \}; x_i \in X; i \in I;$$

where x_i is a vertex of the graph;

U – set of all arcs of the graph $G(X, U)$ for which $x_{ij} = (x_i, x_j) \in U$;

J – set of all pairs of indices corresponding to the arcs.

(1) $$J = \{ (i, j)/i \in I; j \in I; x_{ij} \in U \},$$

where:

(2) $$X = \{ x_i / i \in I \}; \; U = \{ x_{ij} / (i, j) \in J \};$$

Γ_i^1 – direct image equal to the set of the indices of all vertices $\{ x_j \}$, that are terminal vertices of the arc $x_{ij} = (x_i, x_j)$, i.e.,:

(3) $$\Gamma_i^1 = \{ j/(i, j) \in J; j \in I \};$$

Γ_i^{-1} – inverse image showing the set of indices of all vertices that are terminal vertices of the arc $x_{ji} = (x_j, x_i)$:

(4) $$\Gamma_i^{-1} = \{ j/(j, i) \in J;\ j \in I \};$$

the graph $G(X, U)$ can be defined not only by the sets X and U but also by the set X and the images Γ^1 and Γ^{-1}, then written $G(X, \Gamma)$.

The following functions need to be defined on the graph structure:

f_{ij} – arc flow function on the arc $x_{ij} \in U$;

V_0 – initial flow in the source s:

$$s = x_0;\ x_0 \in X;$$

V – final flow in the consumer t:

$$t = x_0;\ x \in X;$$

$(V_0)_{\max}$ – maximal initial flow in source s;

$V_{\max}$ – maximal final flow in the consumer t;

C_{ij} – a function of throughput over the arc x_{ij};

a_{ij} – transport value per unit flow on the arc x_{ij}.

The following restrictions are observed when defining network flows:

(5) $$0 \le f_{ij} \text{ for each } (i, j) \in J;$$

(6) $$V_0 \ge 0;\ V \ge 0;$$

(7) $$(V_0)_{\max} \ge 0;\ V_{\max} \ge 0;$$

(8) $$0 \le f_{ij} \le C_{ij} \text{ for each } (i, j) \in J;$$

g_{ij} – profit and loss ratio or generalized ratio.

In the present work, g_{ij} will also be called the motivation coefficient:

(9) $$0 \le g_{ij};\ (i, j) \in J.$$

There are three possibilities:

(10) a) $$g_{ij} > 1;\ (i, j) \in J;$$

where the arc flow function increases with $g_{ij} f_{ij} - f_{ij} > 0$;

(11) b) $$0 < g_{ij} < 1;\ (i, j) \in J;$$

then the arc flow function is reduced by $f_{ij} - g_{ij} f_{ij} > 0;\ (i, j) \in J$;

(12) c) $$g_{ij} = 1;\ (i, j) \in J;$$

In this case, the arc flow function always retains its value.

The concept of cut plays an essential role in defining the maximal network flow. Usually, the cut is denoted by the symbol $(X_0, \overline{X}_0)$, where X_0 and $\overline{X}_0$ are subsets of X for which $X_0 \subset X$ and $\overline{X}_0 \subset X$. They obey the rules for breaking up the set X according to:

(13) $$X_0 \cap \overline{X}_0 = \varnothing \text{ and } X_0 \cup \overline{X}_0 = X.$$

Each arc of the cut $\left(X_0, \overline{X}_0\right)$ has a starting vertex x_i from X_0 and an ending vertex from $\overline{X}_0$, i.e., for each cut $\left(X_0, \overline{X}_0\right)$ and $\left(\overline{X}_0, X_0\right)$ can be written:

(14) $$\left(X_0, \overline{X}_0\right) = \left\{ x_{ij} / x_i \in X_0;\, x_j \in \overline{X}_0;\, (i,\, j) \in J \right\};$$

(15) $$\left(\overline{X}_0, X_0\right) = \left\{ x_{ji} / x_j \in \overline{X}_0;\, x_i \in X_0;\, (j,\, i) \in J \right\}.$$

On the sections, the total values of the flow function, the throughput function, and the price per unit of transported resource can be defined:

(16) $$\sum_{x_{ij} \in (X_0,\, \overline{X}_0)} f_{ij} = f\left(X_0, \overline{X}_0\right);$$

(17) $$\sum_{x_{ij} \in (X_0,\, \overline{X}_0)} C_{ij} = C\left(X_0, \overline{X}_0\right);$$

(18) $$\sum_{x_{ij} \in (X_0,\, \overline{X}_0)} a_{ij} = a\left(X_0, \overline{X}_0\right).$$

In a similar way, the dimensions for the cuts $f\left(\overline{X}_0, X_0\right); C\left(\overline{X}_0, X_0\right)$ and $a\left(\overline{X}_0, X_0\right)$ are determined.

The classic network flow between source *s* and consumer *t* can be defined as follows [4, 5, 6]:

(19) $$\sum_{j \in \Gamma_i^1} f_{ij} - \sum_{j \in \Gamma_i^{-1}} f_{ji} = \begin{cases} V_0, \text{ iff } x_i = s; \\ 0, \text{ iff } x_i \neq s, t; \\ -V_0, \text{ iff } x_i = t; \end{cases}$$

(20) $$f_{ij} = C_{ij} \text{ for each } (i,\, j) \in J;$$

(21) $$0 \leq f_{ij} \text{ for each } (i,\, j) \in J.$$

In this flow, the arc stream functions are not modified by the coefficients $\left\{ g_{ij} / (i,\, j) \in J \right\}$ and the right-hand side of the conservation equation (19) is always equal to zero and $V_0 = V$.

Greater modeling and management capabilities can be obtained using the so-called generalized network flow (or flow with gains and losses). It is defined as follows:

for each $i \in I$ and $(i,\, j) \in J$

(22) $$\sum_{j \in \Gamma_i^1} f_{ij} - \sum_{j \in \Gamma_i^{-1}} g_{ji} f_{ji} = \begin{cases} V_0, \text{ iff } x_i = s; \\ 0, \text{ iff } x_i \neq s, t; \\ -V_0, \text{ iff } x_i = t; \end{cases}$$

(23) $$f_{ij} \leq C_{ij} \text{ for each } (i,\, j) \in J;$$

(24) $$0 \leq f_{ij} \text{ for each } (i,\, j) \in J.$$

Moreover, in the most general case, for the generalized flow:

(25) $$V_0 \neq V.$$

The maximal possible generalized flow (resource) that can be delivered to the consumer *V*, will be determined by the following goal function:

(26) $$L = V \to \max.$$

If maximization of the resource from the source V_0 is required, then the goal function has the form [7, 8]:

(27) $$L' = V_0 \to \max.$$

A goal function in which maximization of the sum of the two parameters V_0 and V is sought is also possible, then:

(28) $$L'' = V_0 + V \to \max.$$

Determining the maximal generalized network flow is reduced to solving the linear programming problem (22) to (24) for one of the three-goal functions L, L' or L''. This is done using a standard linear optimization package.

When the cuts are obtained such that there is equality between the maximal flow over those cuts (the minimal cut), then for each such cut the well-known mincut-maxflow theorem of Ford and Fulkerson holds. According to this theorem, if it is such a cut, then the dependencies are observed for it:

(29) $$V_{\max} = f(X_0, \overline{X}_0) = C(X_0, \overline{X}_0)$$

where:

(30) $$f(X_0, \overline{X}_0) = 0.$$

A cut that satisfies the above two dependencies is called a minimal cut, and the corresponding flow on the same cut is called a maximal flow.

In the classical network flow from (19) to (21), if there are several minimal cuts, then they always have the same value among themselves. The same applies to the maximal flows in these same sections.

This is not the case with the generalized network flow. It is a result of the following.

***Statement*:** If there are several sections with saturated arcs in the generalized network flow, the throughput values on the different sections may not match each other. The same applies to the flows in these sections.

To prove this statement, it is sufficient to indicate at least two cuts in some generalized network flow for which the values of the corresponding maximal flows and minimal cuts do not coincide with each other.

Fig. 1 shows a generalized network flow with quadriminimal cuts that satisfy the above statement.

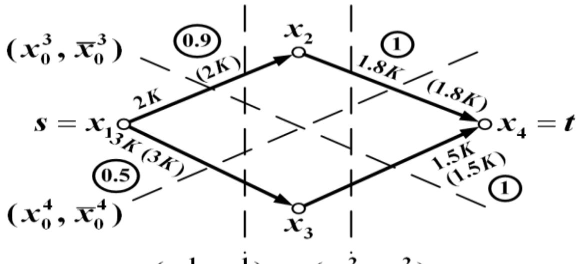

Fig. 1. Generalized network flow

These cuts will be denoted generally by $(X_0^r,\ \overline{X}_0^r),$ where r is the ordinal index of each of the cuts $r \in \{1, 2, 3, 4\}$.

The functions $\{f_{ij}\}, \{C_{ij}\}\ \{g_{ij}\}$ are respectively equal to:

$$f_{1,2} = 2K;\ f_{1,3} = 3K;\ f_{2,4} = 1{,}8K;\ f_{3,4} = 1{,}5K;$$
$$C_{1,2} = 2K; C_{1,3} = 3K; C_{2,4} = 1{,}8K; C_{3,4} = 1{,}5K;$$
$$g_{1,2} = 0{,}9;\ g_{1,3} = 0{,}5;\ g_{2,4} = 1;\ g_{3,4} = 1.$$

The quantity V_0 has an initial value of $5K$, where K is a positive finite number.

After solving the optimization problem from (22) to (24) under the goal function (27), a maximal generalized network flow with value $V = 3{,}3\,\mathrm{K}$ and with arc flow functions with the above values is obtained.

All the arcs of the graph of Fig. 1 have equality between the respective arc flow functions and throughputs, i.e. they are all saturated arcs. There are four cuts – $(X_0^1,\ \overline{X}_0^1), (X_0^2, \overline{X}_0^2), (X_0^3, \overline{X}_0^3), (X_0^4, \overline{X}_0^4),$ for each of which can be recorded:

$$f(X_0^1, \overline{X}_0^1) = 2K + 3K = 5K;\ f(X_0^2, \overline{X}_0^2) = 1{,}5K + 1{,}8K = 3{,}3K;$$
$$C(X_0^1, \overline{X}_0^1) = 2K + 3K = 5K; C(X_0^2, \overline{X}_0^2) = 1{,}5K + 1{,}8K = 3{,}3K;$$
$$f(X_0^3, \overline{X}_0^3) = 2K + 1{,}5K = 3{,}5K;\ f(X_0^4, \overline{X}_0^4) = 3K + 1{,}8K = 4{,}8K;$$
$$C(X_0^3, \overline{X}_0^3) = 2K + 1{,}5K = 3{,}5K; C(X_0^4, \overline{X}_0^4) = 3K + 1{,}8K = 4{,}8K.$$

On the chain of ascending values of the flows on the sections:

(31) $$3{,}3K \le 3{,}5K \le 4{,}8K \le 5K.$$

Corresponds to a similar chain of sections, also arranged in ascending order:

(32) $$(X_0^2, \overline{X}_0^2) \to (X_0^3, \overline{X}_0^3) \to (X_0^4, \overline{X}_0^4) \to (X_0^1, \overline{X}_0^1).$$

This proves the statement that a generalized network flow can have multiple maximal flows with non-matching flow values.

For the generalized network flow of Fig. 1, the value of the maximal final flow V coincides with the stream function on the cut $(X_0^2, \overline{X}_0^2) : V = 3{,}3K.$ The difference ΔV between the resource at the start and end points is:

(33) $$\Delta V = V - V_0 = 5K - 3{,}3K = 1{,}7K,$$

i.e., due to insufficient motivation at the end point t reaches 1.7K less resources than at the start point $s = x_1$. The difference ΔV remains in the intermediate points $\{X \setminus \{s, t\}\}$.

For each of the minimal cuts with indices $r \in \{1, 2, 3, 4\}$, the min-cut-maxflow theorem of Ford and Fulkerson is true:

(34) $$V_{\max}^r = f\left(X_0^r, \overline{X}_0^r\right) - f\left(\overline{X}_0^r, X_0^r\right) = C\left(X_0^r, \overline{X}_0^r\right),$$

where:

(35) $$f\left(\overline{X}_0^r, X_0^r\right) = 0.$$

Some conclusions can be drawn from the proven statement.

If all the coefficients $\{ g_{ij} / (i, j) \in J \}$ take the value of unity, then the generalized network flow will become a classical network flow:

- The values of the resulting minimal cuts will be the same;
- The number of minimal cuts may be different from the number of these cuts in the generalized network flow;
- The value of the flow V_0 in the source, *S* is always equal to *V* in the consumer *t*.

4. Numerical example of maximal generalized network flow with consideration of motivation

In the previous Section 3, the formal description of a generalized network flow is given, with consideration of the motivation – as well as the notations, which are necessary to work with this flow.

I. As an example, Fig. 2 shows a graph with 5 vertices and 7 arcs. Data for the throughput function, coefficients $\left\{ g_{ij} / (i,\ j) \in J \right\}$, and arc estimates $\left\{ a_{ij} / (i,\ j) \in J \right\}$ are shown in Table 1.

Table 1

Parameters \ Arcs	(1,2)	(1,3)	(1,4)	(2,5)	(3,4)	(3,5)	(4,5)
C_{ij}	5	6	2	4,8	4	6	6,84
g_{ij}	1,2	1,4	1,5	0,8	1,6	1,1	0,5
a_{ij}	3	4	3	3	6	3	7

The initial flow V_0 has a value $V_0 = 12{,}8$.

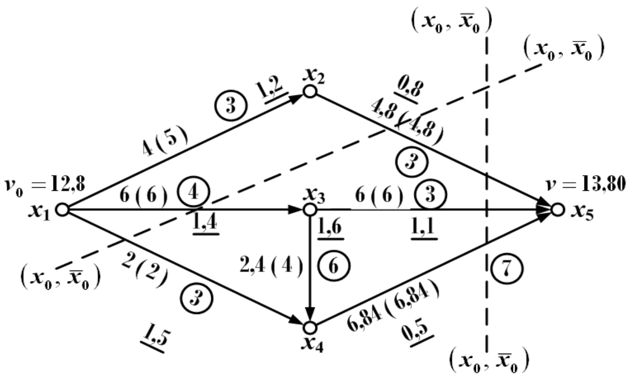

Fig. 2. Throughput data of each arc

The same Fig. 2 shows the throughput data of each arc – in parentheses, of the arc coefficients underlined below, and of the arc ratings in a circle. After the optimal solution, the resulting arc flow functions are shown without additional symbols.

The optimization problem of finding the maximal generalized flow with consideration of the motivation and at initial flow $V_0 = 12{,}8$ is solved using the linear programming package LPSolveIDE. The following equalities and inequalities (22) to (24) are used under goal function (26). For the generalized network flow of Fig. 2, they have the following form:

$$\text{for each } i \in \{1, 2, ..., 4, 5\} \text{ and } (i, j) \in J,$$
$$a_1 : f_{1,2} + f_{1,3} + f_{1,4} = 12{,}8;$$
$$a_2 : f_{2,5} - 1{,}2 f_{1,2} = 0;$$
$$a_3 : f_{3,4} + f_{3,5} - 1{,}4 f_{1,3} = 0;$$
$$a_4 : f_{4,5} - 1{,}6 f_{3,4} - 1{,}5 f_{1,4} = 0;$$
$$a_5 : - 0{,}8 f_{2,5} - 1{,}1 f_{3,5} - 0{,}5 f_{4,5} + v = 0;$$
$$a_6 : f_{1,2} \le 5;$$
$$a_7 : f_{1,3} \le 6;$$
$$a_8 : f_{1,4} \le 2;$$
$$a_9 : f_{2,5} \le 4{,}8;$$
$$a_{10} : f_{3,4} \le 4;$$
$$a_{11} : f_{3,5} \le 6;$$
$$a_{12} : f_{4,5} \le 6{,}84;$$
$$a_{13} : f_{1,2} \ge 0;$$
$$a_{14} : f_{1,3} \ge 0;$$
$$a_{15} : f_{1,4} \ge 0;$$
$$a_{16} : f_{2,5} \ge 0;$$
$$a_{17} : f_{3,4} \ge 0;$$
$$a_{18} : f_{3,5} \ge 0;$$
$$a_{19} : f_{4,5} \ge 0.$$

The goal function of (27) corresponds to the maximal generalized network flow with consideration of motivation. Using the above standard linear programming package leads to the following arc functions and final flow $V = 13{,}86$:

$$f_{1,2} = 4;\; f_{1,3} = 6;\; f_{1,4} = 2;\; f_{2,5} = 4{,}8;\; f_{3,4} = 2{,}4;\; f_{3,5} = 6;\; f_{4,5} = 6{,}84.$$

If the differences $c_{ij} - f_{ij} = \Delta_{ij};\, (i, j) \in J$ are calculated, then those with zero value saturated arcs, namely:

$$\Delta_{1,2} = 1; \Delta_{1,3} = 0; \Delta_{1,4} = 0; \Delta_{2,5} = 0; \Delta_{3,4} = 1; \Delta_{3,5} = 0; \Delta_{4,5} = 0.$$

Only two of the arcs of the graph, namely $\{ x_{1,2} \cup x_{3,4} \}$, are unsaturated with flow values; the remaining arcs are saturated with flow.

This result shows that between the two flows V_0 and V exist two cuts with equality between the maximal flow and the minimal cut. They have the following parameters:

1. $x_0 = \{ x_1 , x_2 \};\; \bar{x}_0 = \{ x_3 , x_4, x_5 \};\; (x_0, \bar{x}_0) = \{ x_{1,3} , x_{1,4}, x_{2,5} \};\; (\bar{x}_0, x_0) = \varnothing$

where the min-cut-maxflow theorem of Ford and Fulkerson is obeyed;

(36) $$C(x_0, \bar{x}_0) = f(x_0, \bar{x}_0) = 2 + 6 + 4{,}8 = 12{,}8 = V_0;$$

(37) $$C(\bar{x}_0, x_0) = f(\bar{x}_0, x_0) = 0.$$

2. The second cut has the following parameters:

$X_0^1 = \{x_1, x_2, x_3, x_4\}$; $\bar{X}_0^1 = \{x_5\}$; $(X_0^1, \bar{X}_0^1) = \{x_{2,5}, x_{3,5}, x_{4,5}\}$; $(\bar{X}_0^1, X_0^1) = \varnothing$;

(38) $$C(X_0^1, \bar{X}_0^1) = f(X_0^1, \bar{X}_0^1) = 3{,}84 + 6{,}6 + 3{,}42 = 13{,}86 = V;$$

(39) $$C(\bar{X}_0^1, X_0^1) = f(\bar{X}_0^1, X_0^1) = 0.$$

The difference ΔV between the two fluxes V_0 and V is $\Delta V = 13{,}86 - 12{,}8 = 1{,}06$ – approximately 8% of the initial flow V_0. The flow increase V is entirely due to motivation. This is because the coefficients used $\{g_{ij}/(i, j) \in G\}$ are mostly above unity and increase the maximal flow. If they are mostly in the range of 0 to 1, motivation will have a decreasing effect on the flow V_0. This confirms the conclusion about the role of motivation in network decision-making systems.

II. Fig. 3 shows a case that corresponds to a maximal flow without motivation. Its data is the same as in Table 1. The difference is that the coefficients $g_{ij} = 1$ for each $(i, j) \in G$.

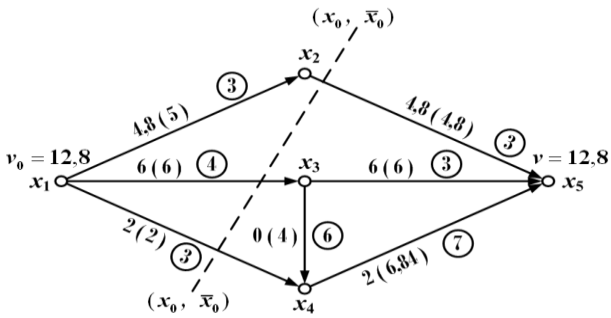

Fig. 3. Throughputs for each arc

In the same Fig. 3, the values of the throughputs for each arc are shown – in parentheses, for the arc estimates – in a circle. The arc flow functions corresponding to the maximal flow are shown without symbols.

The maximal network flow of Fig. 3 can be determined based on the same goal function (26) and the dependencies from a_1 to a_{19} where all the coefficients $\{g_{ij}/(i, j) \in G\}$ from a_1 to a_5 take the value of unity.

The same PSolveIDE linear programming application package is used as in the flow of Fig. 2.

The following results were obtained, which are indicated in the arcs of Fig. 3:

(40) $$f_{1,2} = 4{,}8;\ f_{1,3} = 6;\ f_{1,4} = 2;\ f_{2,5} = 4{,}8;\ f_{3,4} = 0;\ f_{3,5} = 6;\ f_{4,5} = 2.$$

Saturated is the set $\{x_{1,3}, x_{1,4}, x_{2,5}, x_{3,5}\}$.

With the network flow thus defined, there is only one section between the source $s = x_1$ and the consumer $t = x_5$ which is saturated.

(41) $$x_0 = \{x_1, x_2\};\ \bar{x}_0 = \{x_3, x_4, x_5\};\ (x_0, \bar{x}_0) = \{x_{1,3}, x_{1,4}, x_{3,5}\};\ (\bar{x}_0, x_0) = \varnothing.$$

Ford and Fulkerson's mincut-maxflow theorem holds.

(42) $$C\,(x_0,\,\bar{x}_0) = f\,(x_0,\,\bar{x}_0) = 6 + 2 + 4{,}8 = 12{,}8 = V_0 = V;$$

(43) $$C\,(\bar{x}_0,\,x_0) = f\,(\bar{x}_0,\,x_0) = 0.$$

The difference between V_0 and V is zero, as in classical network flow.

This once again confirms the conclusion from the previous problem that there may be a discrepancy between the values of V_0 and V:

$$V_0 = V\ ,$$

can only be if the motivation through the coefficients $\{\,g_{ij}/(i,\ j) \in J\,\}$ is introduced.

The number of minimal cuts can be different for network flows with or without motivation, where $\{\,g_{ij}/(i,\ j) \in G\,\}$ have a single value. In problem **II**, the minimal cuts are two, and in problem **I** – only one. A case where the min-cut-maxflow cut is not equal to V_0 or V may be shown.

In both cases – **I** and **II** – the increase in the maximal flow can be most effectively achieved by increasing the throughput of one of the arcs of the minimal section.

A comparison can be made between the total values of the transported resource – with or without motivation. This is possible because in both cases the values $\{\,a_{ij}/(i,\ j) \in G\,\}$ and $\{\,C_{ij}/(i,\ j) \in G\,\}$ for transportation along the specified sections (arcs) are the same. Considering the motivation, in Fig. 2 the total value of the transported resource is:

(44) $$\sum_{(i,\ j)\in G} a_{ij}\, f_{ij} = 12 + 24 + 6 + 14{,}4 + 14{,}4 + 18 + 47{,}88 = 136{,}68.$$

For the unmotivated network flow of Fig. 3, the same parameter for the total value of the transports at maximal flow is:

(45) $$\sum_{(i,\ j)\in G} a_{ij}\, f_{ij} = 14{,}4 + 24 + 6 + 14{,}4 + 0 + 18 + 14 = 90{,}8.$$

Comparing the two total costs of transportation with and without incentives shows that in the first case, the volume of transportation increases by 8% and the costs increase by 50%. This is related to the fact that as the flow network saturation increases, the increase of a unit of transported resource leads to relatively higher costs.

Conclusion

The results presented in the paper define the need to combine psychological and mathematical models for decision-making in comparison to their separate uses. And that in turn leads to the emergence of new research tasks in the decision-making process. The results can be formed as follows:

1. The particularities of motivation as a psychological process from the point of view of considering influence and in decision-making systems are brought out;
2. The use of a generalized network flow with consideration of the motivation to manage the removal of resources in a complex transport network is assumed;

3. A method for embedding motivation in a generalized network flow with gains and losses is proposed;

4. A method to determine the maximal generalized network flow with motivation has been developed;

5. It is proved that for this flow, in the most general case, the final resource at the consumer does not coincide with the initial resource at the source. The reason for this is the influence of motivation;

6. It is shown that the requirements of the min-cut-maxflow theorem are met at the maximal generalized network flow with motivation;

7. It is proved that in the maximal generalized network flow with motivation, the number of minimal cuts is equal to or greater than in the classical network flow;

8. The results of a numerical example to compare the capabilities of the maximal generalized network flow with motivation and the maximal classical network flow with the same output parameters are shown. It is shown that due to motivation, the capabilities of generalized network flow are generally better than classical network flow;

9. The obtained theoretical and experimental results confirm the possibility of embedding motivation in decision-making systems.

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References

1. F o r d, L. R., D. R. F u l k e r s o n. Maximal Flow through a Network. – Canadian Journal of Mathematics, 1956, No 8, pp. 399-404.
2. D o n P h i l l i p s, G a r c i a-D i a z. Fundamentals of Network Analysis. Prentice Hall, Englewood Cliffs, New York, 1981, 474 pages.
3. C h r i s t o f i d e s, N. Graph Theory: An Algorithmic Approach. Academic Press, London [etc.], 1986.
4. S g u r e v, V., L. D o u k o v s k a. Implication and Inference Rules in Multivalued Logic with Network Configuration. – In: Proc. of 8th IEEE International Conference on Big Data, Knowledge and Control Systems Engineering (BdKCSE'23), 2-3 November 2023, Sofia, Bulgaria, IEEE Xplore, 2023, pp. 1-4. DOI:10.1109/BdKCSE59280.2023.10339696.
5. S g u r e v, V., S. D r a n g a j o v. Risk Estimation and Stochastic Control of Innovation Processes. – Cybernetics and Information Technologies, Vol. **14**, 2014, No 1, pp. 3-10.
6. S g u r e v, V., L. D o u k o v s k a, S. D r a n g a j o v. Intelligent Network-flow Solutions with Risks at Transportation of Products. – In: J. Kacprzyk, Ed. Advances in Intelligent Systems Research and Innovation, Series: Studies in Systems, Decision and Control, Vol. **379**, Springer International Publishing, Switzerland, 2022, pp. 417-439.
7. S g u r e v, V., L. D o u k o v s k a. Multivalued Network Logic with One Real and Two Imaginary Logic Structures. – In: Proc. of IEEE International Conference Automatics and Informatics (ICAI'23), 5-7 October 2023, Varna, Bulgaria, IEEE Xplore, 2023, pp. 395-398. DOI:10.1109/ICAI58806.2023.10339033.
8. S g u r e v, V., L. D o u k o v s k a, S. D r a n g a j o v. Complex Multivalue Logic with Two Sequentially Connected Logic Structure. – In: Proc. of 11th IEEE International Conference on Intelligent Systems (IS'22), 12-14 October 2022, Warsaw, Poland, IEEE Xplore, 2023, pp. 1-5. ISBN: 978-1-6654-5656-2, DOI:10.1109/IS57118.2022.10019677.

9. R. F., Baumeister, K. D. Vohs, Eds. Drive, Psychology of Human Motivation. Psychology Press, 2004.
10. D a v i d, S. Emotional Agility: Get Unstuck, Embrace Change and Thrive in Work and Life. Penguin, 2018.
11. B u r c h a r d, B. E. The Motivation Manifesto. Hay House, 2014.
12. C a r a y o n, P., P. H o o n a k k e r, T. B. W e t t e r n e c k. Motivational Decision Support: Tailoring Interventions to Improve Decision-Making Outcomes. – Ergonomics, Vol. **58**, 2015, No 4, pp. 568-583.
13. D e z e r t, J., A. T c h a m o v a. Involutory Negator of Basic Belief Assignments. – Cybernetics and Information Technologies, Vol. **23**, 2023, No 3, pp. 3-22.

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