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A Real-World Benchmark Problem for Global Optimization

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***Abstract*:** *The paper presents the statement of the problem of dynamical system „crane-load” optimal control. The acceleration period is under consideration and control must meet the minimum duration condition as well as load oscillations elimination. The objective function, which ensures the final condition satisfaction, is developed and analyzed in terms of its topology features. It includes three arguments and their searching is the essence of the benchmark problem. Two variants of the problem are proposed with varied objective function parameters. Twelve agent-based optimization algorithms have been applied to find solutions to a bunch of problems. A brief analysis of the performance of the algorithms reveals their weaknesses and advantages. Thus, the proposed real-world problem may be exploited to estimate the optimization algorithms' search performance.*

***Keywords*:** *Optimization, Metaheuristics methods, Benchmark, Oscillation elimination, Control.*

1. Introduction

There is a great need for efficient optimization methods in a plethora of areas in modern science and production. For some of them, optimization is a “core”, the mandatory procedure. It is impossible to imagine neural network training without the application of an optimization technique [1]; optimally tuned controllers allow to minimize of energy and materials consumption in many productions [2]; optimal scheduling is an important problem for a huge number of organizations [3]; urban traffic optimal control allows to improve transport services, decrease pollutions and fuel consumptions [4]. The full list of examples is enormous.

The efficiency of optimization techniques may be measured in two ways: by application to a class of real-world problems, or with involving of some benchmarks (synthetic objective functions). Both of the ways have drawbacks and advantages, and it is desirable to approve a new optimization algorithm (or a new modification of an existing one) by its application to these two classes of optimization problems [5]. However, most real-world optimization problems, which are presented in scientific works, are quite simple. As a result, the performance of the applied optimization algorithms slightly deviates from each other, i.e., it is almost impossible to reveal optimization algorithms' search features.

In this work, we propose two types of optimization problems, which are extracted from real-world control problems and have practical value. In addition, as it will be shown in further, this is quite complicated to minimize. That is why, we believe, it may be widely used to assess the performance of optimization algorithms.

The article is built in the following manner: the next section presents the literature review on the problem field, and some most important results are noted here. The third section gives the statement of the optimal control problem; in the fourth section, the objective function is derived and some of its topology features are presented; the results of the application of twelve metaheuristic optimization algorithms are discussed in the fifth section, and its brief analysis is provided as well. The article ends with conclusions.

2. Literature review

A solution of duration/time-optimal control may be obtained via Pontryagin's maximum principle [6]. Note, that the mandatory element of this problem is control constraints. Indeed, there is no sense in the problem without control constraints, since the following expression is truthful:

(1) $$\lim_{u\to\infty} t_{ac} = 0,$$

where t_{ac} is duration of a system controlled period, and u – the control function. Expression (1) is quite general and there is a wide range of u interpretation.

The structure of the duration-optimal control problem solution is known: control **u** must switch from an upper constraint to the lower one, and vice versa. One may imagine the simplest case: one-mass system acceleration from the rest state to a steady velocity **v**. For this example, control **u** equals only upper constraint during some period (depending on the steady velocity v value). For more general cases **u** switches between constraints several times. The number of such switches depends on the order of the system only in the case when the system is not oscillatory [7]. Thus, the initial problem may be reduced to the problem of switching moments determination.

The problem of duration-optimal control of a crane with a load is well known and a great number of papers refer to it. In further literature analysis, we focus on the methods of its solution, not on the results and their practical significance.

There are two general classes of control applied to this problem: closed-loop and open-loop. Since the latter is considered in this study, we will give the stress to that class of control. However, some articles [8-11], where closed-loop control is found, are worth mentioning. Here, for instance, the modification of Particle Swarm Optimization (PSO) [8] has been applied to find coefficients of a linear controller of the system „crane-load”. The same method has been applied to train an Artificial Neural Network (ANN) as a controller [9] as well, but with the extended criterion (duration and power RMS), and additional constraints (the rate of drive force is limited). In the work [10], an ANN has been exploited to solve the problem. It has been trained with the Levenberg-Marquardt algorithm with Bayesian regularization. Closed-loop optimal control may be derived based on open-loop control [11] by Pontryagin's maximum principle [6]. In this case, the performance of the controller is slightly worse than for open-loop control.

A class of open-loop problems, as it was mentioned earlier, requires switching moments determination. For this PSO [12] has been applied. In this paper, two types of control constraints are considered and the difference in system dynamics under these controls is shown. Laboratory experiments [13] have confirmed obtained theoretical results. The authors of article [14] have reduced the minimum-time anti-swing motion planning problem of crane motion to a solution of a sequence of fixed-time maximum-range Linear Programming (LP) problems. Theoretical proof of this approach is given as well. Authors of work [15] have considered the duration-optimal control problem as LP one. For its solution, CVX package [16] has been used. In the work [17] the sequential quadratic programming method is used to solve the duration-optimal control problem. To achieve this goal, they have involved MATLAB toolbox GPOPS [18]. In some investigations [19-21] phase plan approach is used to find moments of control switching. They are found as points, where a system changes movement from one phase trajectory to another. Authors of work [22] determine the moment of control switching as a solution to a system of transcendental equations. The method of moments also may be applied to find time-optimal control [20, 23]. However, it requires relatively difficult calculations.

All solutions in the mentioned articles refer to some predetermined set of problem parameters (mass of a load, length of a cable, etc.). Thus, the following question appears: how applied approaches will work on a varied set of parameters? The current article manages with variety of parameters and requires solutions not a single problem, but a bunch of problems (two kinds of them with different parameters). Such an approach may guarantee (to some extent) the generality of results [24] and an optimization algorithm, applied for their solving and may be estimated in relation to its sensitivity.

3. Problem statement

Let us state the optimal control problem. The plant to control is a crane with a load on the flexible suspension (cable). This dynamical system may be presented as a mathematical pendulum with a movable pivot (Fig. 1).

The mathematical model, which corresponds to the Fig. 1, is

$$\begin{cases} m_1\ddot{x}_1 + m_2\ddot{x}_2 = F - W\operatorname{sign}(\dot{x}_1), \\ x_1 = x_2 + \dfrac{l}{g}\ddot{x}_2, \end{cases} \tag{2}$$

where: m_1 and m_2 are the reduced masses of crane and load respectively; x_1 and x_2 are the positions of the masses m_1 and m_2, respectively; l is the length of the cable; g is the free-fall acceleration; F and W are reduced to the mass m_1 drive and resistance forces correspondingly.

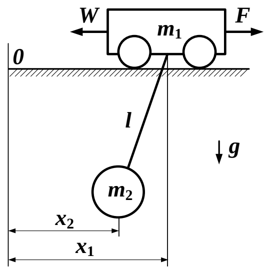


Fig. 1. Dynamical model of the system “crane-load”

System of differential Equations (2) may be reduced to one fourth-order differential equation:

(3) $$\overset{IV}{x}_2 + \ddot{x}_2\Omega^2 = \frac{F-W}{m_1}\Omega_0^2,$$

where Ω and Ω_0 are the frequencies of the natural load oscillation with movable and the stationary pivots, respectively ($\Omega = \sqrt{\frac{m_1+m_2}{m_1}\frac{g}{l}}$, $\Omega_0 = \sqrt{\frac{g}{l}}$).

Acceleration of the system “crane-load” may be described by the following boundary conditions:

(4) $$\begin{cases} x_1(0)=0,\ x_2(0)=0,\ \dot{x}_1(0)=0,\ \dot{x}_2(0)=0, \\ x_1(T)=x_2(T),\ \dot{x}_1(T)=v,\ \dot{x}_2(T)=v, \end{cases}$$

where T is the duration of the system acceleration to the steady velocity, and v is the steady velocity of the system. Taking into account the second line of the system (2) we may rewrite boundary conditions only for one function x_2:

(5) $$\begin{cases} x_2(0)=0,\ \dot{x}_2(0)=0,\ \ddot{x}_2(0)=0,\ \dddot{x}_2(0)=0; \\ \dot{x}_2(T)=v,\ \ddot{x}_2(T)=0,\ \dddot{x}_2(T)=0. \end{cases}$$

Meeting boundary conditions (5) allows the steady velocity v to be reached and pendulum load oscillations eliminated, within time T. Its value must be minimized. Thus, the criterion of the optimal control problem follows:

(6) $$\int_0^T dt = T \to \min.$$

As mentioned above, there is no sense of minimizing (6) without control constraints. For this problem, they are:

(7) $$F_{\min} \le F \le F_{\max},$$

where $F_{\max}$ and $F_{\min}$ are the drive force maximum and minimum values.

4. An objective function and its brief analysis

4.1. Problem solution

Analysis of the multiple works [12-15, 17, 19-23], where the problem (3)-(7) is being solved under similar statements, allows obtaining the structure of its solution: force F must switch from $F_{\max}$ to $F_{\min}$, and vice versa. The number of such switches is two.

Thus, the whole acceleration period may be considered as three subperiods, where force F is of constant, $F_{\max}$ or $F_{\min}$ value. This fact allows obtaining the analytical solution of the differential Equation (3) for i-th subperiod:

$$
(8)\quad
\begin{aligned}
x_2 = {} & \frac{1}{2m_1\Omega^4}(2m_1\Omega^2(\ddot{x}_2(\tau_{i-1}) + \dddot{x}_2(\tau_{i-1})(t-\tau_{i-1})) + (\dot{x}_2(\tau_{i-1})(t-\tau_{i-1}) + \\
& + x_2(\tau_{i-1}))\Omega^2) + (F_i - W)(\Omega^2(t-\tau_{i-1})^2 - 2)\Omega_0^2 - 2(\ddot{x}_2(\tau_{i-1})m_1\Omega^2 - \Omega_0^2 \times \\
& \times (F_i - W))\cos((t-\tau_{i-1})\Omega) - 2m_1\dddot{x}_2(\tau_{i-1})\Omega\sin((t-\tau_{i-1})\Omega)),\ i = 1, 2, 3,
\end{aligned}
$$

where F_i is the the constant value of the drive-force in the i-th subperiod (whether $F_{\max}$ or $F_{\min}$), and τ_{i-1} is the moment of i-th subperiod beginning (Fig. 2).

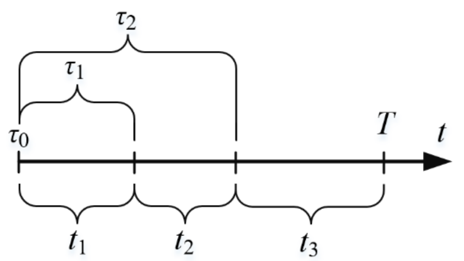


Fig. 2. Time coordinates of the subperiods

The next step of computing is a substitution with the proper expressions for each of the subperiods into the law (8):

$$
(9)\quad
\begin{cases}
x_2(\tau_0) \to 0,\ \dot{x}_2(\tau_0) \to 0,\ \ddot{x}_2(\tau_0) \to 0,\ \dddot{x}_2(\tau_0) \to 0,\ F = F_{\max},\quad \tau_0 \le t \le \tau_1; \\
x_2(\tau_1) \to x_{2.t_1},\ \dot{x}_2(\tau_1) \to \dot{x}_{2.t_1},\ \ddot{x}_2(\tau_1) \to \ddot{x}_{2.t_1},\ \dddot{x}_2(\tau_1) \to \dddot{x}_{2.t_1},\ F = F_{\min},\ \tau_1 < t \le \tau_2; \\
x_2(t_1+t_2) \to x_{2.t_1+t_2},\ \dot{x}_2(t_1+t_2) \to \dot{x}_{2.t_1+t_2},\ \ddot{x}_2(t_1+t_2) \to \ddot{x}_{2.t_1+t_2},\ \dddot{x}_2(t_1+t_2) \to \dddot{x}_{2.t_1+t_2}, \\
F = F_{\max},\ \tau_2 < t \le T,
\end{cases}
$$

$$\tau_0 = 0,\ \tau_1 = t_1,\ \tau_2 = t_1 + t_2,\ t_1 + t_2 + t_3 = T,$$

where t_1, t_2, t_3 are the durations of the first, the second, and the third subperiods, respectively (Fig. 2).

As a result, we have obtained three solutions, which correspond to three subperiods. Let us denote them as $x_{2.1}$, $x_{2.2}$, and $x_{2.3}$ (subscripts 1, 2, and 3 indicate the subperiod). Having all the solutions (for three subperiods) we impose constraints on solutions joints in the moments τ_1, τ_2:

$$
(10)\quad
\begin{cases}
x_{2.1}(\tau_1) = x_{2.\tau_1},\ \dot{x}_{2.1}(\tau_1) = \dot{x}_{2.\tau_1},\ \ddot{x}_{2.1}(\tau_1) = \ddot{x}_{2.\tau_1},\ \dddot{x}_{2.1}(\tau_1) = \dddot{x}_{2.\tau_1}; \\
x_{2.2}(\tau_2) = x_{2.\tau_2},\ \dot{x}_{2.1}(\tau_2) = \dot{x}_{2.\tau_2},\ \ddot{x}_{2.1}(\tau_2) = \ddot{x}_{2.\tau_2},\ \dddot{x}_{2.1}(\tau_2) = \dddot{x}_{2.\tau_2}.
\end{cases}
$$

These calculations bring the solution $x_{2.3} = x_{2.3}(t)$. Substitution $t \to t_1+t_2+t_3$ gives the final coordinate $x_{2.3}(t_1+t_2+t_3)$ and its higher derivatives by time. Thus, we have $\dot{x}_{2.3}(t_1+t_2+t_3)$, $\ddot{x}_{2.3}(t_1+t_2+t_3)$, $\dddot{x}_{2.3}(t_1+t_2+t_3)$. In order to meet final conditions (5) we put into consideration the Terminal Energy (TE) of the system – the function, which reflects (in some sense) the level of the final conditions (5) satisfaction:

$$
\begin{aligned}
\mathrm{TE} &= \frac{m_2}{2}\left(\left(v - \dot{x}_{2.3}(t_1+t_2+t_3)\right)^2 + \left(\ddot{x}_{2.3}(t_1+t_2+t_3)\right)^2 + \left(\frac{\dddot{x}_{2.3}(t_1+t_2+t_3)}{\Omega_0^2} \right)^2 \right) = \\
&= \frac{m_2}{2m_1^2\Omega^6}(\Omega^2\Omega_0^4(F_{\max} - W - (F_{\max} - F_{\min})(\cos(t_3\Omega) - \cos((t_2+t_3)\Omega)) + (W - F_{\max})\times \\
&\times\cos((t_1+t_2+t_3)\Omega)^2 + \Omega^4((F_{\max} - F_{\min})(\sin(t_3\Omega) - \sin((t_2+t_3)\Omega)) + (F_{\max} - W)\times \\
&\times\sin((t_1+t_2+t_3)\Omega)^2 + (m_1 v\Omega^3 - (F_{\min}t_2 + F_{\max}(t_1+t_3) - (t_1+t_2+t_3)W)\Omega\Omega_0^2 + \Omega_0^2\times \\
&\times((F_{\max} - F_{\min})(\sin(t_3\Omega) - \sin((t_2+t_3)\Omega)) + (F_{\max} - W)\sin((t_1+t_2+t_3)\Omega)))^2).
\end{aligned} \tag{11}
$$

The first term in the brackets corresponds to the kinetic „over- or under-energy" of the mass m_2. This value shows the deviation of the mass m_2 energy at the moment $t_1+t_2+t_3$ from the value $m_2v^2/2$. The second and third terms are potential and kinetic energies of the load (pendulum) oscillations.

In addition to that, we should note the values $F_{\min}$ in the function (11). This value influences the duration T: the smaller $F_{\min}$, the smaller T.

On the other hand, switching from $F_{\max}$ to $F_{\min}$ is a severe mode for crane drive and metal structure. Thus, in order to decrease dynamic impacts, we set $F_{\min}=0$ [12]. In the known problem solutions [14, 15, 17-23] symmetrical constraints have been used $F_{\min}= -F_{\max}$. We propose to refer to the problem with constraints $F_{\min} = -F_{\max}$ as Type-1, and refer to the problem with $F_{\min}=0$ as Type-2.

Thus, we have reduced the initial problem to the following one:

$$
\begin{gathered}
\underset{t_1,\, t_2,\, t_3}{\arg\min}\left(\frac{2\mathrm{E}}{m_2 v^2} + (t_1+t_2+t_3)\frac{\Omega}{2\pi} \right), \\
\mathrm{E} = \begin{cases} w\cdot\mathrm{TE} & \text{if} \quad \mathrm{TE} \geq \Delta, \\ 0 & \text{if} \quad \mathrm{TE} < \Delta. \end{cases}
\end{gathered} \tag{12}
$$

Here Δ is some conditional threshold of energy, where the value of TE is insignificant and that is why it may be neglected (in the frame of the current study $\Delta=10^{-2}$ J); w is the weight coefficient, which increases the significance of TE minimization (we set $w=10^6$).

The first term of the expression (12) in the brackets, is the specific energy of the system. The second one – is the specific system acceleration duration. Both of the terms are dimensionless.

The exact solution of the problem reduces the first term of criterion (12) to the value, that is smaller than Δ, and all of the final conditions (5) are almost met (the deviation of final values of phase coordinates from (5) governs by Δ. The smaller Δ, the closer final phase coordinates to the final conditions (5)).

Involving values w and Δ in the objective function (12) may be explained by a two-stage search process. In the first stage, the E value is minimized, in the second one – an algorithm is searching for a minimal T value (since E=0). Such structure of the objective function (12) allows for avoiding false solutions of the initial problem (3)-(7). Indeed, one may imagine at least two examples of this:

a) $w=1$, then $t_1\to 0$, $t_2\to 0$, $t_3\to 0$. For that case, the objective function (12) has a global minimum, which equals 1. The second and third terms of TE (11) in brackets (first line of expression) are zero, only the first term equals to v;

b) $\Delta=0$, then at some stage of the objective function (12) minimization product $w\cdot$TE will be decreased to a value, that is comparable with values t_1, t_2, and t_3. This means $w \cdot$TE and $(t_1+t_2+t_3)\Omega/2\pi$ terms compete in minimization. In this case, there is no guarantee of criterion (6) complete minimization.

4.2. Objective function brief analysis

In order to solve problem (12) we should assign some numerical values of the system (Table 1).

Table 1. Numerical values of the system

Parameter	Unit	Value
m_1	kg	42,000
v	m/s	0.7
F_{max}	N	24,107
F_{min}	N	–24,107 (Type-1) or 0 (Type-2)
W	N	$0.01(m_1+m_2)g$

The objective function (12) has a complicated topology. In order to show it we substitute values (Table 1), m_2=10000 kg and l=6 m into expression (12) and assume $t_3=t_1$ (the case, which is common for most of the problem solutions).

Farther, we study function TE(t_1, t_2) only since it greatly influences objective function (12), at least until condition TE≤Δ is not satisfied (the second term of criterion (12) linearly depends on values t_1, t_2, and t_3. Thus, there are no big difficulties for an algorithm to find its sum minimum).

Illustrations of the function TE(t_1, t_2) features for both problems (Type-1 and Type-2 problems) are given in Fig. 3 and Fig. 4, respectively. In these plots, the red dot shows the location of the global minimum.

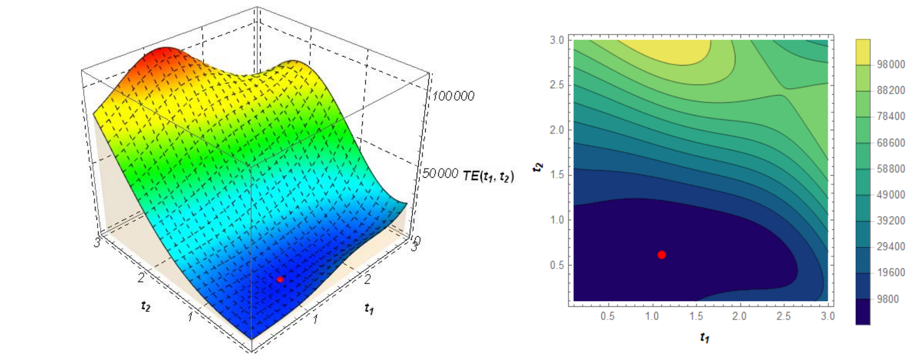
Function TE(t_1, t_2) for both cases is multimodal, non-separable. Its plateau is quite flat (Fig. 3a, and Fig. 4a), and that fact complicates the search for the global minimum location.

The gradient field (Fig. 3b, and Fig. 4b) has two separated canyons. The first one reflects the maximum (hill) of the function TE(t_1, t_2), and the other – its minimum (lower plateau). On the curve of the latter, the global minimum is located (for both cases Type-1 and Type-2). Thus, it is expected, that gradient-based optimization techniques may fail to reach the global minimum.

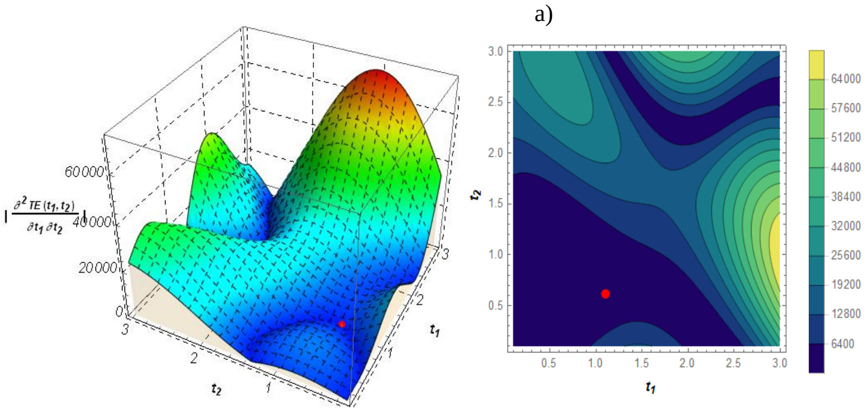
In order to clearly show global minimum location logarithm of TE(t_1, t_2) plots (Fig. 3c and Fig. 4c) are built. The attractor of the global minimum is quite small and an algorithm should make a proper number of iterations to reach it with the needed accuracy.

We should stress, that we set $t_3=t_1$. The presented brief analysis refers to TE(t_1, t_2, t_3) only to some extent. The general case where $t_3 \neq t_1$ is more complex.

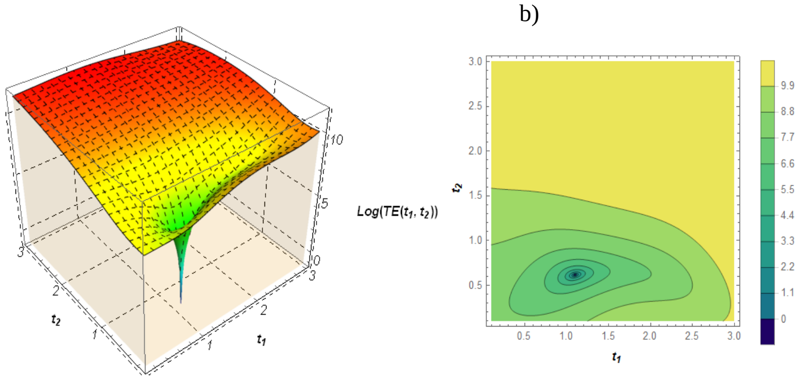
There is a non-convexity at the objective function (12) bottom (Fig. 5). Red dot denotes a minimum location. It is on the edge of the lower pit (Fig. 5). However, as stated earlier, the topology of the lower pit is simple. It is sloped without any nonlinearities (canyons, minimums with narrow attractors, etc.).



a)

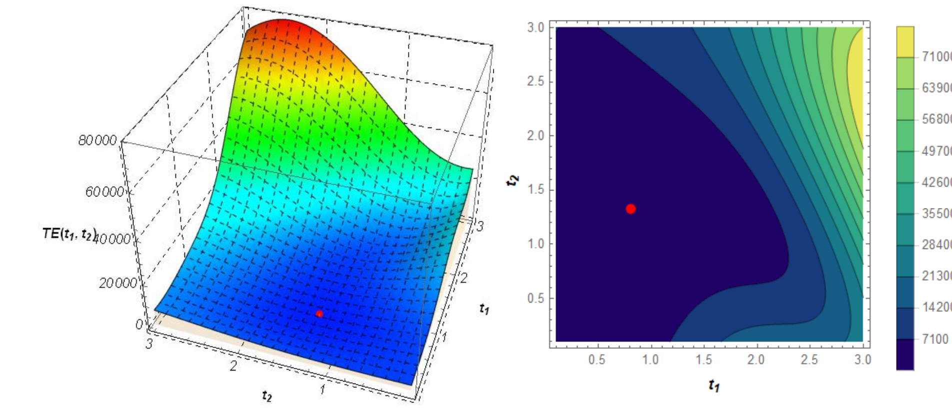


b)

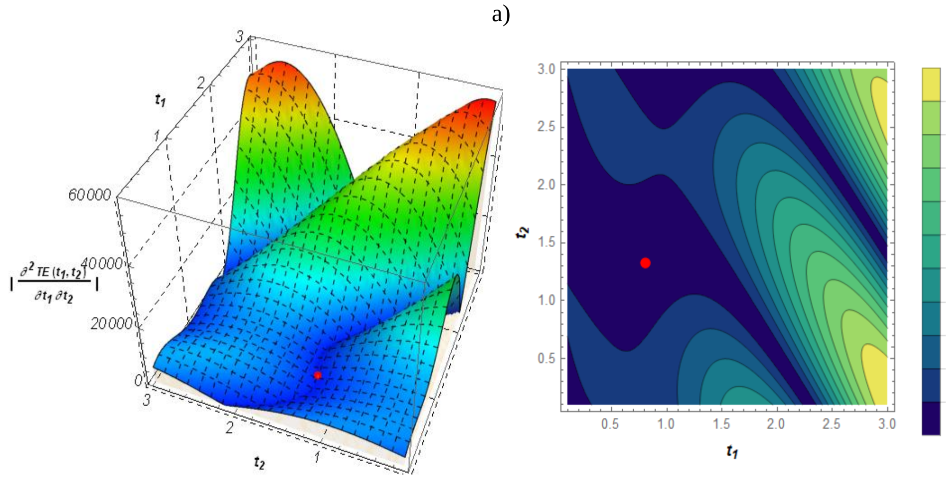


c)

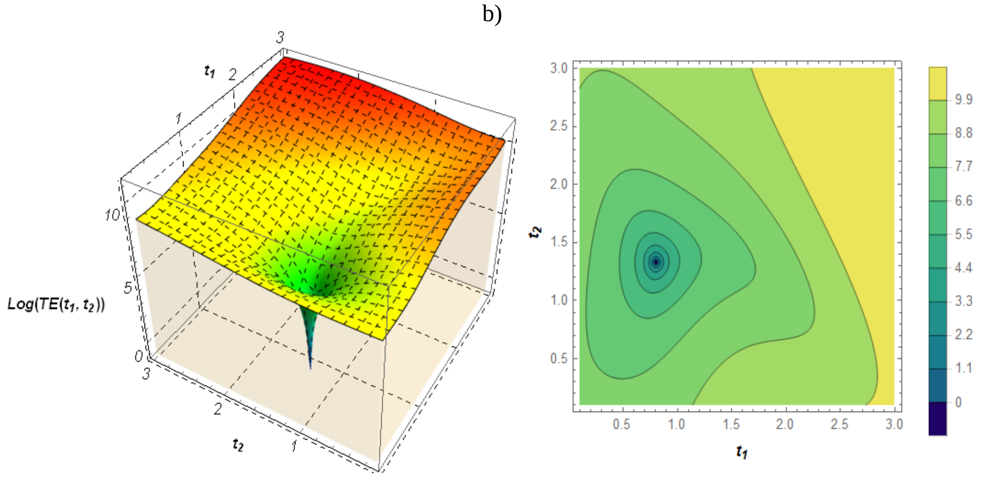
Fig. 3. Function TE (Type-1) topology features (three-dimensions and contour plots): a) TE function; b) $\left|\dfrac{\partial^2 \mathrm{TE}(t_1, t_2)}{\partial t_1 \partial t_2}\right|$ function; c) log(TE) function



a)

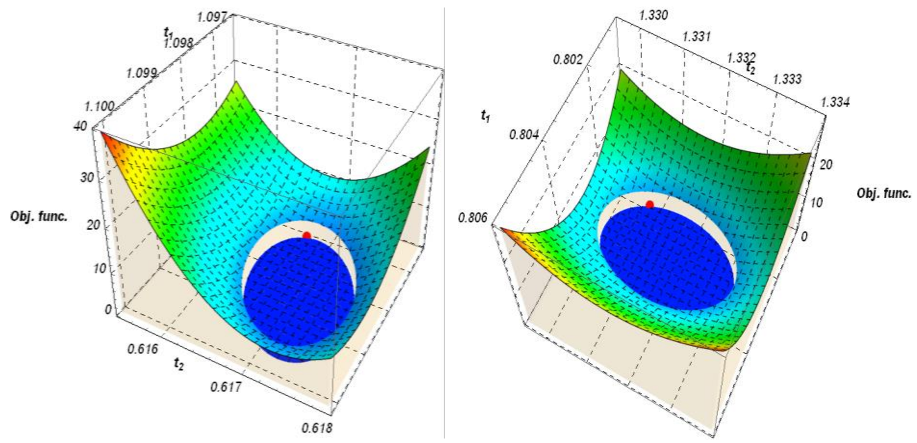


b)



c)

Fig. 4. Function TE (Type-2) topology features (three-dimensions and contour plots): a) TE function; b) $\left|\frac{\partial^2 \mathrm{TE}(t_1,t_2)}{\partial t_1 \partial t_2}\right|$ function; c) log(TE) function



a) b)

Fig. 5. Objective function (12) topology features near its bottom (global minimum): a) Type-1 case; b) Type-2 case

The width of the lower pit is determined by Δ value. As soon as an optimization algorithm has found the objective function (12) lower pit, it slides down to the global minimum, and the optimal control problem is solved.

5. Results and discussion

In order to assess the efficiency of the optimization algorithms we have applied them to problem (12) multiple times with different values of m_2 and l. In algorithms' runs, m_2 is varied from 500, to 25,000 kg with the step 500 kg; l is varied from 3 up to 10 m with the step 1 m. Thus, the problem has been solved 400 times (each time with slightly different parameters m_2 or l). We denote these bunch of solutions as a round.

In the current work, we estimate the search performance of twelve optimization metaheuristic algorithms. The modifications of well-known (parent) metaheuristics (PSO, Differential Evolution (DE), Gray Wolf Optimization (GWO), Harmony Search (HS), and Cuckoo Search (CS)) are under consideration. For each algorithm, the number of its agents (particles – for VCT-PSO [25], LDW-PSO [26] and ME-D-PSO [27]; vectors – for DE best/1/bin [28], DE rand/1/bin [29, 30] and SADE [31]; wolves – for mGWO [32] and GWOEPD [33]; fireflies – for RaFA [34]; harmonies – for ABHS [35] and PAHS [36]; cuckoos – for ACS [37]) is the same for all of the runs. It equals 25.

The parameters of the applied optimization algorithms are given in Table 2.

For each round, the number of algorithm iterations has been increasing from 40 up to 400.

An indicator for algorithms evaluation is a specific number of Failed Algorithm Runs (FAR):

(13)
$$\mathrm{FAR} = \left(\frac{\mathrm{OR}}{\mathrm{SR}} - 1 \right) 100\%,$$

where OR is the Overall algorithm Runs, and SR is the number of Successful algorithm Runs.

Table 2. Parameters of optimization algorithms

Parent algorithm	Common algorithms parameters	Applied algorithms	Specific algorithms parameters
PSO	c_1=c_2=1.19	VCT-PSO	RC=5; w=0.72
		LDW-PSO	w_{max}=0.9; w_{min}=0.4
	-	ME-D-PSO	AR=0.05; w_{max}=2; w_{min}=0; $c_{1.max}$=2; $c_{1.min}$=0; $c_{2.max}$=2; $c_{2.min}$=0
DE	CR=0.5; SF=0.6	best/1/bin	-
		rand/1/bin	-
	-	SADE	τ_1= τ_2=0.1; SF_{low}=0.1; SF_{up}=0.9
GWO	-	mGWO	-
		GWOEPD	-
FA	-	RaFA	a=1
HS	-	ABHS	$HMCR_{max}$=1.0; $HMCR_{min}$=0.9; PAR_{max}=1.0; PAR_{min}=0.3
		PAHS	$HMCR_{max}$=0.99; $HMCR_{min}$=0.7; PAR_{max}=0.99; PAR_{min}=0.01; BW_{min}=0.001
CS	α=1, p_a=0.25	ACS	-

A successful algorithm run must meet the following condition T<$2\pi/\Omega_{min}$, i.e., the final solution of the problem must be lower, that some set in advance threshold (in a practical sense it prevents acceleration of the system during a period of natural load oscillation). We set the most stringent condition and select the smaller period, referred to considered values l and m_2. In the studied case $2\pi/\Omega_{max}$=5.02 s.

All obtained data, which reflect optimization algorithms' performance, are collected in Table 3.

Table 3. FAR values of optimization algorithms performance

Parent algorithm	PSO			DE		GWO		FA	HS
Algorithms / Iterations	VCT-PSO	LDW-PSO	ME-D-PSO	best/1/bin	rand/1/bin	mGWO	GWOEPD	RaFA	ABHS
Type-1 optimization problem									
40	4.8	4.5	15.0	4.2	*	3.5	37.2	64.2	*
100	4.0	3.5	8.8	3.8	8.0	5.2	53.0	19.2	*
200	4.0	3.8	6.0	3.0	5.0	3.8	*	6.2	77.0
400	3.8	3.0	5.8	4.5	7.5	3.2	69.0	4.8	22.8
Type-2 optimization problem									
40	16.8	12.5	24.5	26.2	*	38.0	37.5	45.8	*
100	20.5	13.2	23.2	20.8	*	33.0	45.2	22.0	*
200	15.2	19.2	22.2	15.2	*	42.2	51.8	21.5	61.5
400	14.0	16.0	21.2	21.8	*	35.5	*	12.8	39.5

* Indicator FAR is bigger, than 100%. In some cases, an algorithm did not find all of the needed solutions, and its runs have been interrupted.

Algorithms CS, PAHS, and SADE for both optimization problems have failed. Their efficiency is low; indicator FAR for all of them is bigger than 100%. That is why they are not shown in Table 3.

Successful runs of different algorithms have led to solutions (values t_1, t_2, and t_3), that correspond to the global minimum of the objective function (12). It is supported by the values t_1, t_2, and t_3: different algorithms found almost the same values. In order to illustrate the solutions plots Fig. A1 have been built (Appendix A). Their brief analysis is given there as well.

Analysis of data, presented in Table 3, evidences: Type-2 optimization problem is more complicated than the one of Type-1. Indeed, indicator FAR of the three most successful algorithms for Type-2 problem is 11.5,…, 17.1% greater, than the one for Type-1. However, we should stress some exceptions: overall performance of RaFA, GWOEPD, and ABHS do not support the statement about prevailing complexity of Type-2 problem. Here we observe „no free lunch" theorem [38] consequences.

No one of the algorithms has reached zero value of indicator FAR (perfect case, when each algorithm's run leads to the problem solution). Thus, there are possibilities of improvement of the algorithms in such a way, which decreases FAR up to zero.

The most efficient algorithms for considered problems are DE/best/1/bin, VCT-PSO, and LDWPSO. The latter is a leader in the class of applied metaheuristics (Table 3). Algorithm VCT-PSO is slightly worst, but very close to the leader.

In order to carry out analysis of some algorithms search activity, plots have been built (Fig. 6). Figs 6a and 6c refer to the Type-1 optimization problem, Fig. 6b and 6d refer to the Type-2 problem.

For both cases under poor computational resources (40 iterations) all algorithms, except LDWPSO and DE/best/1/bin, have not found problem (12) solution. At the end of the search, they are at the stage of function TE(t_1, t_2, t_3) minimization, i.e., at the upper pit of the objective function (12). Thus, LDWPSO and DE/best/1/bin have successful runs, the rest of the algorithms shown in Fig. 6 have failed.

An increased number of iterations positively influences the algorithms' performances. All of them (for both cases Type-1 and Type-2) have reached the lower pit of the objective function. The only difference is the number of iterations needed for that. The "fastest" (in terms of spent iterations) algorithms compared are DE/best/1/bin and LDWPSO. For them up to a hundred iterations is enough to find both problems' solutions. The most "slow" algorithm is mGWO. It found problems' solutions almost at the end of the search process: for Type-1 case, mGWO fell down to the objective function lower pit at 396-th iteration, and for Type-2 case – at the 387-th iteration. mGWO restarts may not bring acceptable results (in terms of condition $T<2\pi/\Omega_{min}$ satisfaction). RaFA and ABHS have an average efficiency.

These data lead us to the conclusion: PSO-based algorithms (even relatively simple ones) are fitted for the Type-1 and Type-2 optimization problems.

Comparing plots, which are presented in the left and right columns, we do not note any significant differences. However, here we have illustrated only single runs of the algorithms, which do not reflect the statistical differences in their performance.

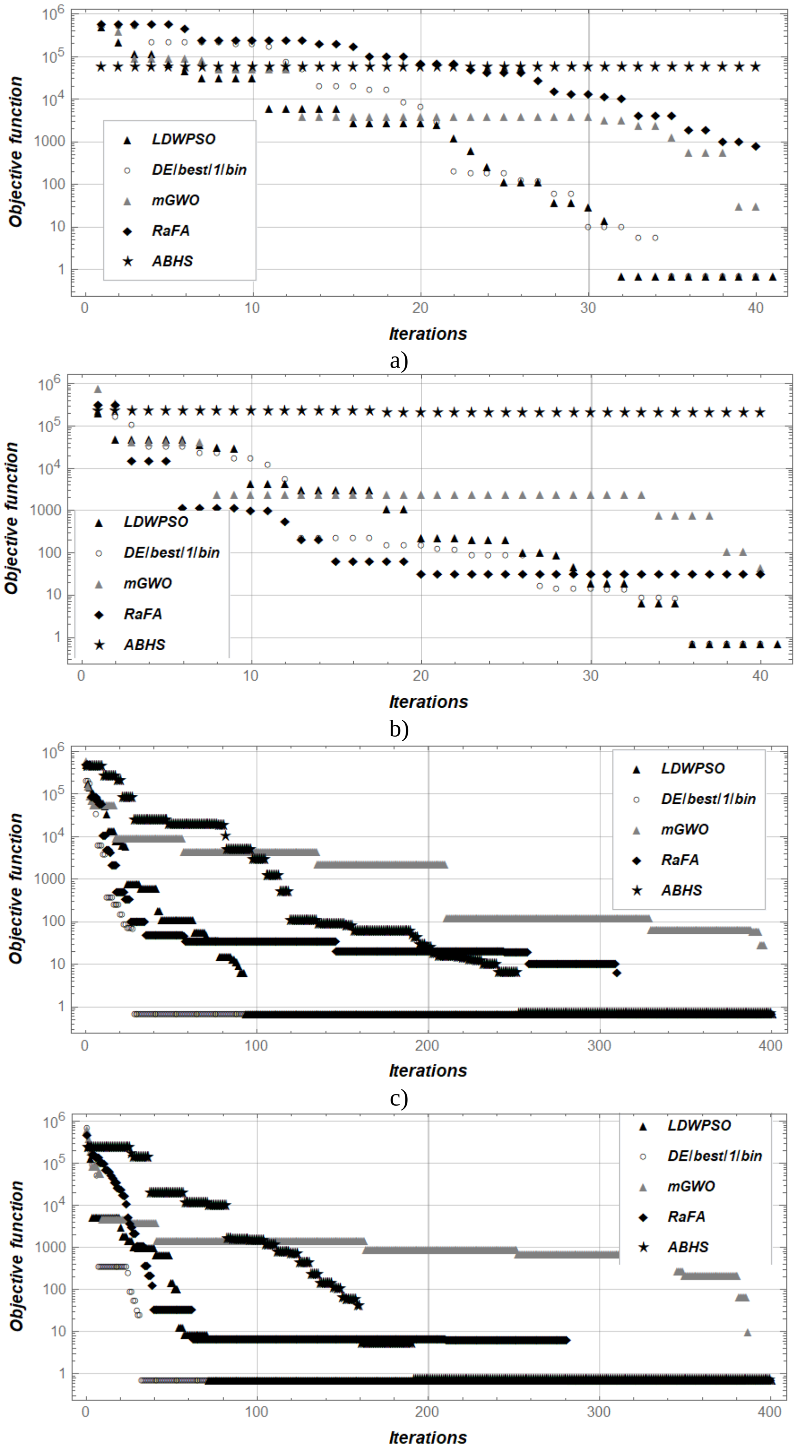


Fig. 6. Algorithms search activity: a) for 40 iterations (Type-1 problem); b) for 40 iterations (Type-2 problem); c) for 400 iterations (Type-1 problem); d) for 400 iterations (Type-2 problem)

6. Conclusion

A real-world optimization problem is proposed to solve in the article. Optimization algorithms performance estimation is used. Since it is connected with pendulum effects, the objective function of the problem includes trigonometric functions. This feature makes criterion topology highly complicated for searching its global minimum: its attractor is relatively narrow, and the function itself is non-separable, multimodal, and non-convex. Two cases of the problem have been proposed, which relate to two values of lower control constraint.

In order to reveal some of the problem's features twenty metaheuristic algorithms have been applied. Three of them have not managed to find all the problem solutions for a variety of objective function parameters. Based on the developed indicator of algorithms efficiency three of the most efficient ones have been determined: DE/best/1/bin, VCT-PSO, and LDWPSO.

Further issues in this field are connected with the extension of a number of optimization algorithms to apply and the estimation of their performance. In addition, there is a possibility of problem modification by taking into account initial non-zero values of the dynamical system phase coordinates. This direction requires some analytical calculations and rebuilding of the control function (the drive-force **F** during the first subperiod may be not positive).

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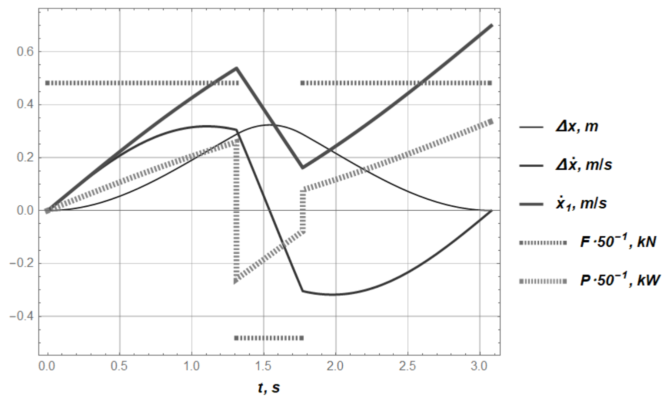
Appendix A. System characteristics under optimal control

Comparative analysis of the plots, presented in Fig. A.1 shows the „on-off" form of the optimal control. However, for the solution of Type-2 problem, there is no driving force **F** sign-changing. In practice, it is desirable, as it does not require a change of electromagnetic torque sign, i.e., dynamical impacts caused by Type-2 solution are much lower, that for the case of Type-1 solution. A maximal gap of the driving force F for Type-2 control is lesser by a factor of two.

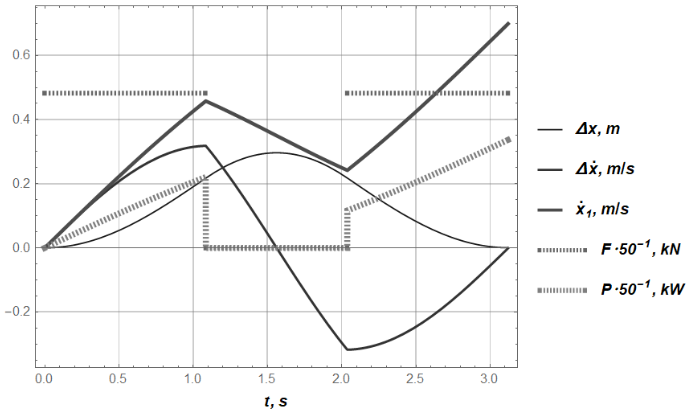
Constant sign for consumed power is another advantage of Type-2 problem solution: this means, that there is no need to embed the drive of the crane with additional power equipment – an inverse inverter.

The steady velocity v is achieved, and load pendulum oscillations are eliminated for both of the cases. There is only a minor (by 1.4%) increase of T for Type-2 control, compared with Type-1 control (Fig. A.1). Magnitudes of load oscillations for Type-1 and Type-2 controls are almost the same. However, for the rest of the m_2 and l values duration T and magnitude of load oscillations variates. In the general case, they are some functions of m_1, m_2, $F_{\max}$, $F_{\min}$, W, v, and l.

Summing everything up, we may state: Type-2 control allows decreasing of power and dynamical loads on crane drive and metal structure by virtue of a slight increase of acceleration duration T.



a)



b)

Fig. A1. Plots of „crane-load” system characteristics under optimal control: a) Type-1 problem solution; b) Type-2 problem solution

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